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Research Article

Chebyshev Collocation Method for Solving Caputo Fractional Interface Problem

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Abstract. In this study a collocation strategy for the solution of the fractional interface problem is proposed. The spatial derivatives are computed using interpolation with Chebyshev functions. We elaborate the analysis that leads to an *a priori* error estimation. Different numerical examples are presented to show the applicability and effectiveness of the developed approach.

Keywords. Collocation method, Chebyshev polynomials, Interface problem, Fractional differential equations, Caputo fractional derivative

Mathematics Subject Classification (2020). 34A08, 65L60, 65L70, 65L10

1. Introduction

An interface problem is a type of problem in mathematics which involves interface across different or overlapping regions. Such problems usually arise when solving partial differential equations in multiple domains with different boundary conditions or when observing the behavior of solutions in relation to the interface of two domains. In mathematics, these kinds of problems have many uses. Such problems usually involve the usage of differential equations which are defined across interfaces containing discontinuous or non smooth data and solutions.

One specific field of application problems is the conduction of heat through the materials (Cao *et al*. [10]). Another examples relate to fluid motion (Layton [24]), image processing (Li *et al*. [25]), lots of scholars have examined numerical approaches for interface problems solution such as *Finite Difference Method* (FDM) (Han [20]), Chebyshev collocation method (Sameeh and Elsaid [29]), pseudo-spectral collocation method (Hessari *et al*. [21]), and high order difference potentials methods (Epshteyn and Phippen [18]).

There has been increasing importance placed upon *Fractional Differential Equations* (FDEs) due to the wide applicability of these types of equations in research and engineering (Boulaaras *et al*. [9]). The application of fractional calculus is best expressed in modeling processes that exhibit characteristics such as memory and spatial dependency, which cannot be conveniently expressed through the standard integer-order derivatives In particular, the rudimentary steps taken towards calculus, that of differentiation and integration, are extended to non-integer orders through the initial efforts of Riemann-Liouville and Caputo on fractional derivatives. Commonly, these fractional differential equations are used to describe processes like, responding to the corona virus dynamic (Zhang *et al*. [33]), sanitary kinetics and theory of wastewater ongoing restoring models (Alqahtani *et al*. [5]), diffusion processes (Bazhlekov and Bazhlekova [8]), forecasting of economic growth over long periods (Johansyah *et al*. [22]), and rubella disease dynamics modeling (Baleanu *et al*. [7]). The methodologies that have been invented to solve these FDEs include the operational method of wavelets (Neamaty *et al*. [27]), method of lines (Liu *et al*. [26]), finite element method (Adel *et al*. [1]), homotopy (Afrouzi *et al*. [2]), Sumudu transform (Darzi *et al*. [13]), finite difference method (Tadjeran and Meerschaert [31]), and collocation method (Gebril *et al*. [19]).

The numerical analysis of fractional differential equation interface problems presents quite some challenges to analysts because it has a difficult fractional calculus domain. It is often very difficult to model and simulate the behavior of interface problems because of the complex nature of memory and spatial effects in fractional differential equations. Moreover, there is also the problem of lacking sound and adequate methodologies that can be used by the scholars and practitioners in solving interface problems using fractional calculus. Among the few numerical methods designed to solve interface problems involves FDE are the Caputo method (Daneh-Dezfuli *et al*. [12]) and reproducing kernel method (Al-Masaeed *et al*. [4]). Therefore, the Researchers, engineers, and other science workers find it challenging to correctly and quickly simulate any systems with interfaces and described by fractional differential equations. Therefore, there is an urgent need to come up with good numerical schemes, which motivated us in this study to consider the following 1-D fractional interface problem:

$$D^{\gamma}v(\xi)+\beta_1 v(\xi)=g_1(\xi), \quad 0<\xi<\zeta_c,\ 1<\gamma\le 2, \tag{1.1}$$

$$D^{2}v(\xi)+\beta_2 v(\xi)=g_2(\xi), \quad \zeta_c<\xi<1 \tag{1.2}$$

subject to boundary conditions

$$v(0)=\psi_0,\ \ v(1)=\psi_1, \tag{1.3}$$

with interface conditions on σ:

$$v(\sigma^+)=\alpha_1 v(\sigma^-)+\kappa_1, \tag{1.4}$$

$$v'(\sigma^+)=\alpha_2 v'(\sigma^-)+\kappa_2, \tag{1.5}$$

where u is an unidentified function that will be found, andβ_1,β_2 are suitably smooth functions specified in $(0,\zeta_c),(\zeta_c,1)$, respectively. The Caputo fractional derivative has an order $(1<\gamma\le 2)$ can be used for the spatial fractional derivative $D^\gamma v(\xi)$ which is elaborated by Azizi and Loghmani [6] given as

$$D^\gamma v(\xi)=\frac{1}{\Gamma(2-\gamma)}\int_0^\xi(\xi-\varsigma)^{1-\gamma}\frac{\partial^2 v(\xi,\varsigma)}{\partial\varsigma^2}d\varsigma. \tag{1.6}$$

The Chebyshev collocation method is utilized in this study to tackle fractional interface problems. This method is very useful in a broad class of equations such as linear and non-linear ordinary differential equations (El-Gamel and Sameeh [17], Sezer and Kaynak [30]), partial differential equations (Yuksel *et al*. [32]), integro differential equations (Akyüz and Sezer [3], Çerdik-Yaslan *et al*. [11]), partial integro differential equation (Sameeh and Elsaid [28]), Troeschs problem (El-Gamel and Sameeh [15]), eigen value problems (El-Gamel and Sameeh [14,16]). In this paper, six aspects are made to cover. Section 2 lists the main features of Chebyshev basis. The detailed methodology is more clearly specified in Section 3. In Section 4, an *a priori* errors is derived. Numerical simulation results are in Section 5. Finally, in Section 6 the results of the present study are given.

2. Fundamental Relations

The Chebyshev polynomials with degree n, after being shifted, can be represented in relation to ξ within the interval $[a,b]$ as outlined below:

$$\theta_n^*(\xi)=\cos\left(n\arccos\left(\frac{2\xi-(a+b)}{b-a}\right)\right). \tag{2.1}$$

The function $\theta_n^*(\xi)$ attained its highest value $(n+1)$ times for distinct signs over the range $[a,b]$:

$$\|\theta_n\|_\infty=1,\quad \theta_n(\xi_i)=(-1)^i,$$

where the maximum norm $\|\theta_n\|_\infty$ is defined as the supremum of $|\theta_n(\xi)|$, and Chebyshev collocation points denoted by ξ_i, are determined by:

$$\xi_i=\frac{b-a}{2}\left[\left(\frac{a+b}{b-a}\right)+\cos\left(\frac{i\pi}{n}\right)\right],\quad i=0,1,2,\dots,n. \tag{2.2}$$

Utilizing the first kind relocated Chebyshev polynomial $\theta_n^*(\xi)$ over $[a,b]$, the approximation of $v(\xi)$ is achieved through a truncated series of relocated Chebyshev polynomials, expressed as:

$$v_n(\xi)=\sum_{l=0}^N k_l^*\theta_l^*(\xi),\quad a\le\xi\le b. \tag{2.3}$$

The derivatives with integer order $\lambda=0,1,2,3,\dots$, indicated by $v^{(\lambda)}$, are expressed similarly as follows:

$$v_n^{(\lambda)}(\xi)=\sum_{l=0}^N k_l^{*(\lambda)}\theta_l^*(\xi),\quad a\le\xi\le b. \tag{2.4}$$

The matrix representation of the function $v(\xi)$ and its derivatives is provided by

$$v_n(\xi)=\theta^*(\xi)A^*, \tag{2.5}$$

$$v_n^{(\lambda)}(\xi)=\theta^*(\xi)A^{*(\lambda)}, \tag{2.6}$$

where

$$\theta^*(\xi)=[\theta_0^*(\xi)\ \ \theta_1^*(\xi)\ \ \theta_2^*(\xi)\ \ \cdots\ \ \theta_n^*(\xi)],$$

$$A^* = \left[\frac{1}{2}\ \ a_0^*\ \ a_1^*\ \ \cdots\ \ a_n^*\right]^\tau.$$

Lemma 2.1 ([19]). *The computation of vector $A^{*(\lambda)}$ from vector A^* is achieved through the formula:*

$$A^{*(\lambda)} = \left(\frac{4}{b-a}\right)^\lambda \rho^\lambda A^*, \tag{2.7}$$

where

$$\rho = \begin{bmatrix} 0 & \frac{1}{2} & 0 & \frac{3}{2} & 0 & \frac{5}{2} & \dots & r_1 \\ 0 & 0 & 2 & 0 & 4 & 0 & \dots & r_2 \\ 0 & 0 & 0 & 3 & 0 & 5 & \dots & r_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & n \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix},$$

where

$$r_1 = \begin{cases} \frac{n}{2}, & r_2 = 0,\ r_3 = n,\ \text{if } n \text{ is an odd number}, \\ 0, & r_2 = \sigma,\ r_3 = 0,\ \text{if } n \text{ is an even number}. \end{cases}$$

Then, you might explain $v_n^\lambda(\xi)$ as follows:

$$v_n^{(\lambda)}(\xi) = \left(\frac{4}{b-a}\right)^\lambda \rho^\lambda \theta^*(\xi) A^*. \tag{2.8}$$

Lemma 2.2 ([23]). *If the function $v(\xi)$ is approximated in the form* (2.3)*, then $D^\gamma(v_n(\xi))$ can be expressed as*

$$D^\gamma(v_n(\xi)) = \varphi(\xi) A^*, \tag{2.9}$$

where

$$\varphi(\xi) = [\varphi_1(\xi)\ \ \varphi_2(\xi)\ \ \varphi_3(\xi)\ \ \cdots\ \ \varphi_{N+1}(\xi)], \tag{2.10}$$

Consequently, for $\xi \in (0,h)$, the following is a demonstration of φ's vector elements:

$$\varphi_\kappa = \begin{cases} 0, & \kappa = 1,2, \\ \sum\limits_{l=0}^{\kappa-[\gamma]-1} (-1)^l \frac{2^{2(\kappa-l-1)}}{h^{(\kappa-l-1)}} \frac{(\kappa-1)(2\kappa-l-3)!(\kappa-l-1)!}{l!(2\kappa-2l-2)!\Gamma(\kappa-l-\gamma)} \xi^{\kappa-l-\alpha-1}, & 3 \le \kappa \le N+1. \end{cases}$$

3. Description of Method

In this section, we delve into the one-dimensional fractional interface problem described by equations (1.1)-(1.2) and its associated boundary and interface conditions (1.3)-(1.4) using the Chebyshev collocation method. We express the Chebyshev approximation of equations (1.1)-(1.2) through the following composite forms:

$$v(\xi) = \sum_{l=0}^{N} b_l^- \theta_l^-(\xi), \quad 0 \le \xi \le \zeta_c, \tag{3.1}$$

$$v(\xi) = \sum_{k=0}^{M} c_k^+ \theta_k^+(\xi), \quad \zeta_c \le \xi \le 1, \tag{3.2}$$

where θ_l^-, and θ_k^+ represent shifted Chebyshev polynomials on the intervals $(0,\zeta_c)$ and $(\zeta_c,0)$ respectively, defined as:

$$\theta_k^-(\xi) = \cos\left(k \arccos\left(\frac{2\xi - \zeta_c}{\zeta_c}\right)\right), \tag{3.3}$$

$$\theta_l^+(\xi) = \cos\left(l \arccos\left(\frac{2\xi - (1+\zeta_c)}{1-\zeta_c}\right)\right). \tag{3.4}$$

The discrete Chebyshev system corresponding to equations (1.1) and (1.2) is delineated by the following equations:

$$\varphi(\xi_\nu^-)B^- + \beta_1\theta^-(\xi_\nu^-)B^- = g_1(\xi_\nu^-), \quad 0 < \xi_\nu^- < \zeta_c, \tag{3.5}$$

$$\frac{4}{(1-\zeta_c)}\theta^+(\xi_\mu^+)\rho^2 C^+ + \beta_2\theta^+(\xi_\mu^+)C^+ = g_2(\xi_\mu^+), \quad \zeta_c < \xi_\mu^+ < 1, \tag{3.6}$$

in which ξ_ν^- and ξ_μ^+ are the Chebyshev collocation nodes on the intervals $(0,\zeta_c)$ and $(\zeta_c,0)$, respectively. The matrix form of the discrete Chebyshev system in both intervals is as follows:

$$\omega_1 B^- = G_1, \quad 0 < \xi_\nu^- < \zeta_c, \tag{3.7}$$

$$\omega_2 C^+ = G_2, \quad \zeta_c < \xi_\mu^+ < 1, \tag{3.8}$$

where

$$\omega_1 = \Phi + \beta_1\Theta^-, \quad \omega_2 = \frac{4}{(1-\zeta_c)}\Theta^+\rho^2 + \beta_2\Theta^+,$$

with

$$G_1 = \begin{bmatrix} g_1(\xi_0^-) \\ g_1(\xi_1^-) \\ \vdots \\ g_1(\xi_N^-) \end{bmatrix}, \quad \Theta^- = \begin{bmatrix} \theta_0^-(\xi_0^-) & \theta_1^-(\xi_0^-) & \dots & \theta_N^-(\xi_0^-) \\ \theta_0^-(\xi_1^-) & \theta_1^-(\xi_1^-) & \dots & \theta_N^-(\xi_1^-) \\ \vdots & \vdots & \ddots & \vdots \\ \theta_0^-(\xi_N^-) & \theta_1^-(\xi_N^-) & \dots & \theta_N^-(\xi_N^-) \end{bmatrix},$$

and

$$G_2 = \begin{bmatrix} g_2(\xi_0^+) \\ g_2(\xi_1^+) \\ \vdots \\ g_2(\xi_M^+) \end{bmatrix}, \quad \Theta^+ = \begin{bmatrix} \theta_0^+(\xi_0^+) & \theta_1^+(\xi_0^+) & \dots & \theta_M^+(\xi_0^+) \\ \theta_0^+(\xi_1^+) & \theta_1^+(\xi_1^+) & \dots & \theta_M^+(\xi_1^+) \\ \vdots & \vdots & \ddots & \vdots \\ \theta_0^+(\xi_M^+) & \theta_1^+(\xi_M^+) & \dots & \theta_M^+(\xi_M^+) \end{bmatrix}, \quad \Phi = \begin{bmatrix} \varphi_0(\xi_0^-) & \varphi_1(\xi_0^-) & \dots & \varphi_N(\xi_0^-) \\ \varphi_0(\xi_1^-) & \varphi_1(\xi_1^-) & \dots & \varphi_N(\xi_1^-) \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_0(\xi_N^-) & \varphi_1(\xi_N^-) & \dots & \varphi_N(\xi_N^-) \end{bmatrix}.$$

Both intervals $(0,\zeta_c)$ and $(\zeta_c,1)$ have their discrete Chebyshev systems integrated with interface and boundary conditions to produce a composite matrix. This matrix is recognized as the matrix form of the discrete Chebyshev system throughout the entire domain $(0,1)$. The conditions at both the boundary and the interface are unified in the following manner:

$$\vartheta_1^- B^- = \psi_0, \tag{3.9}$$

$$\vartheta_2^+ C^+ = \psi_1, \tag{3.10}$$

$$\vartheta_3 A = \kappa_1, \tag{3.11}$$

$$\vartheta_4 A = \kappa_2, \tag{3.12}$$

where

$$\vartheta_3 = [\vartheta_3^- \quad \vartheta_3^+], \quad \vartheta_4 = [\vartheta_4^- \quad \vartheta_4^+], \quad A = \begin{bmatrix} B^- \\ C^+ \end{bmatrix},$$

$$\vartheta_1^- = \theta^-(0), \qquad \vartheta_2^+ = \theta^+(1),$$

$$\vartheta_3^- = -\alpha_1\theta^-(\zeta_c), \qquad \vartheta_3^+ = \theta^+(\zeta_c),$$
$$\vartheta_4^- = -\frac{4}{\zeta_c}\alpha_2\theta^-(\zeta_c)\rho, \quad \vartheta_4^+ = \frac{4}{1\zeta_c}\theta^+(\zeta_c)\rho.$$

Throughout the whole domain $(0,1)$, the discrete Chebyshev system is as follows

$$T\,A = G, \tag{3.13}$$

where

$$T = \begin{bmatrix} \omega_1 & 0 \\ \hline 0 & \omega_2 \\ \hline \vartheta_1^- & 0 \\ \hline 0 & \vartheta_2^+ \\ \hline \vartheta_3^- & \vartheta_3^+ \\ \hline \vartheta_4^- & \vartheta_4^+ \end{bmatrix}, \quad G = \begin{bmatrix} G_1 \\ \hline G_2 \\ \hline 0 \\ \hline \kappa_1 \\ \hline \kappa_2 \end{bmatrix}.$$

4. Error Estimation

Two procedures are involved in estimating the error for the interface problem (1.1)-(1.5). Estimating the error in the subinterval $(0,\zeta_c)$ is the first stage, and estimating the error in the subinterval $(\zeta_c,1)$ is the second. In order to establish the error resulting from employing Chebyshev polynomials to approximate the solution of equation (1.1) in $(0,\zeta_c)$. It was assumed that the exact solution of this equation can be stated in the following form:

$$\upsilon(\xi) = L_N^{\Upsilon^-}\upsilon(\xi) + R_N^{\Upsilon^-}(\xi). \tag{4.1}$$

The interpolating error $R_N^{\Upsilon^-}(\xi)$, delineated in [2], corresponds to the Nth order Lagrange approximating polynomial $L_N^{\Upsilon^-}\upsilon(\xi)$ that interpolates $\upsilon(\xi)$ on the Chebyshev collocation node mesh Υ^- within subinterval $(0,\zeta_c)$.

$$R_N^{\Upsilon^-}(\xi) = L_N^{\Upsilon^-}\upsilon(\xi) - \upsilon(\xi) = \frac{\upsilon^{(N+1)}(\eta)}{(N+1)!}\psi_{N+1}^{\Upsilon^-}(\xi), \tag{4.2}$$

η falls within the range $(0,\zeta_c)$, while the polynomials $\psi_{N+1}^{\Upsilon^-}(\xi)$ are precisely described as

$$\psi_{N+1}^{\Upsilon^-}(X) = \prod_{r=0}^{N}(\xi - \xi_r).$$

By expanding $\upsilon(\xi)$ as Lagrange interpolating polynomials, as described in equation (4.1), $L_N^{\Upsilon^-}\upsilon(\xi)$ is considered to be a solution to the next equation:

$$D^\gamma(L_N^{\Upsilon^-}\upsilon(\xi)) + \beta_1(L_N^{\Upsilon^-}\upsilon(\xi)) = g_1(\xi) + \Delta g_1(\xi), \tag{4.3}$$

where

$$\Delta g_1(\xi) = -D^\gamma(R_N^{\Upsilon^-}(\xi)) - \beta_1(R_N^{\Upsilon^-}(\xi)). \tag{4.4}$$

The representation of $L_N^{\Upsilon^-}\upsilon(\xi)$ through Chebyshev series as $L_N^{\Upsilon^-}\upsilon(\xi) = \theta^- B^{-\prime}$ results in the discrete Chebyshev system of equation (4.3).

$$\omega_1^- B^{-\prime} = G_1 + \Delta G_1. \tag{4.5}$$

The difference between (3.7) and (4.5) yields

$$B^{-\prime} - B^- = (\omega_1)^{-1}\Delta G_1 \tag{4.6}$$

Theorem 4.1. *If υ and υ_n stand for the accurate and Chebyshev approximated solutions of* (1.1)*, respectively, and assuming υ exhibits sufficient smoothness, then*

$$|\upsilon-\upsilon_n| \leq |R_N^{\Upsilon_n^-}| + \|\theta^-(\xi)\| \, \|(\omega_1)^{-1}\| \, \|\Delta G_1\|. \tag{4.7}$$

Proof. For assessing the upper limit of error, we are equipped

$$\begin{aligned}|\upsilon-\upsilon_n| &\leq |\upsilon - L_N^{\Upsilon^-}\upsilon(\xi)| + |L_N^{\Upsilon^-}\upsilon(\xi)-\upsilon_n| \leq |R_N^{\Upsilon_n^-}| + |\theta^-(\xi)B^{-\prime} - \theta^-(\xi)B^-| \\ &\leq |R_N^{\Upsilon_n^-}| + \|\theta^-(\xi)\| |B^{-\prime} - B^-| \leq |R_N^{\Upsilon_n^-}| + \|\theta^-(\xi)\| \, \|(\omega_1)^{-1}\| \, \|\Delta G_1\|.\end{aligned}$$ □

Similarly, the error is evaluated within the subinterval $(\zeta_c, 1)$.

Theorem 4.2. *If υ and υ_m stand for the accurate and Chebyshev approximated solutions of* (1.2)*, respectively, and assuming υ exhibits sufficient smoothness, then*

$$|\upsilon-\upsilon_m| \leq |R_N^{\Upsilon_m^+}| + \|\theta^+(\xi)\| \, \|(\omega_2)^{-1}\| \, \|\Delta G_2\|. \tag{4.8}$$

The mesh Υ_m^+ consists of Chebyshev collocation nodes within the subinterval $(\zeta_c, 1)$.

5. Numerical Examples

In this portion of the article, we provide three numerical examples to demonstrate the effectiveness of the Chebyshev collocation method in handling fractional order 1-D interface problems. The results achieved via this approach, are compared to the results obtained via other methods so we can emphasize the benefits and efficiency of Chebyshev collocation in solving such complex problems.

Example 5.1. Take into consideration the following problem:

$$\kappa D^{\gamma}\upsilon(\xi) = 12\xi^2, \quad \kappa = \begin{cases} 1, & \xi \in (0, 0.5), \gamma \in (1,2], \\ 2, & \xi \in (0.5, 1), \gamma = 2 \end{cases}$$

subject to boundary conditions

$$\upsilon(0) = 0, \quad \upsilon(1) = \frac{17}{32},$$

with interface conditions

$$\upsilon(0.5^-) = \upsilon(0.5^+), \quad \upsilon'(0.5^-) = \upsilon'(0.5^+) + \frac{1}{2}.$$

As $\gamma = 2$, the exact solution is

$$\upsilon(\xi) = \begin{cases} \xi^4, & \xi \in (0, 0.5), \\ \frac{1}{2}\left(\xi^4 + \frac{1}{16}\right), & \xi \in (0.5, 1). \end{cases}$$

Table 1 shows the absolute error computed by both the Chebyshev collocation technique and the modified reproducing kernel (Al-Masaeed *et al.* [4]) for Example 5.1 in different locations.

Figure 1 depict Chebyshev fitted graphs of the solution for Example 5.1 indicating different values of γ.

Table 1. Absolute error for Example 5.1 at $\gamma = 2$

ξ	Chebyshev collocation method $N+M=10$	Reproducing kernel [4] $N+M=50$
0.1	2.70643e-17	5.33167e-7
0.2	2.90566e-17	1.85062e-6
0.3	2.77556e-17	3.94932e-6
0.4	3.12250e-17	6.82927e-6
0.5	2.77556e-17	1.04000e-5
0.6	6.93889e-17	7.61874e-6
0.7	2.77556e-17	5.12922e-6
0.8	2.77556e-17	3.02654e-6
0.9	5.55112e-17	1.31528e-6
1	0	0

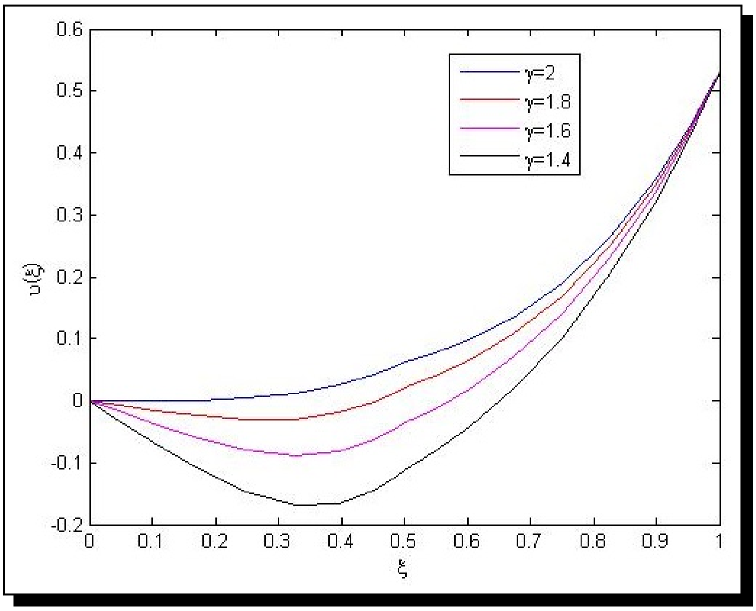

Figure 1. Chebyshev fitted graphs of the solution for Example 5.1 indicating different values of γ

Example 5.2. Take into consideration the following problem:

$$D^{\gamma}\upsilon(\xi) = \begin{cases} e^{\xi}, \gamma \in (1,2], & \xi \in (0,0.5), \\ 2\sec^2(\xi - 0.5)\tan(\xi - 0.5), \gamma = 2, & \xi \in (0.5,1), \end{cases}$$

subject to boundary conditions

$$\upsilon(0) = 1, \quad \upsilon(1) = e^{0.5} + \tan(0.5),$$

with interface condition

$$\upsilon(0.5^-) = \upsilon(0.5^+), \quad \upsilon'(0.5^-) = e^{0.5}\upsilon'(0.5^+).$$

As $\gamma = 2$, the exact solution is

$$\upsilon(\xi) = \begin{cases} e^{\xi}, & \xi \in (0,0.5), \\ e^{0.5} + \tan(\xi - 0.5), & \xi \in (0.5,1). \end{cases}$$

Table 2 determines the absolute error computed using both Chebyshev collocation method as well as the modified reproducing kernel [4] which was generated from Example 5.2 in various positions. Figure 2 show Chebyshev fitted curves for the solutions of Example 5.2 at various values of γ.

Table 2. Absolute error for Example 5.2 at $\gamma = 2$

ξ	Chebyshev collocation method $N+M=10$	Reproducing kernel [4] $N+M=50$
0.1	4.25644e-11	1.38358e-8
0.2	8.51285e-11	5.07672e-8
0.3	1.276925e-10	1.13166e-7
0.4	1.702567e-10	2.03710e-7
0.5	2.128213e-10	3.25359e-7
0.6	1.70125e-10	9.03652e-7
0.7	1.45797e-10	1.10708e-7
0.8	6.71069e-11	2.35579e-7
0.9	4.21019e-11	2.27158e-7
1	0	0

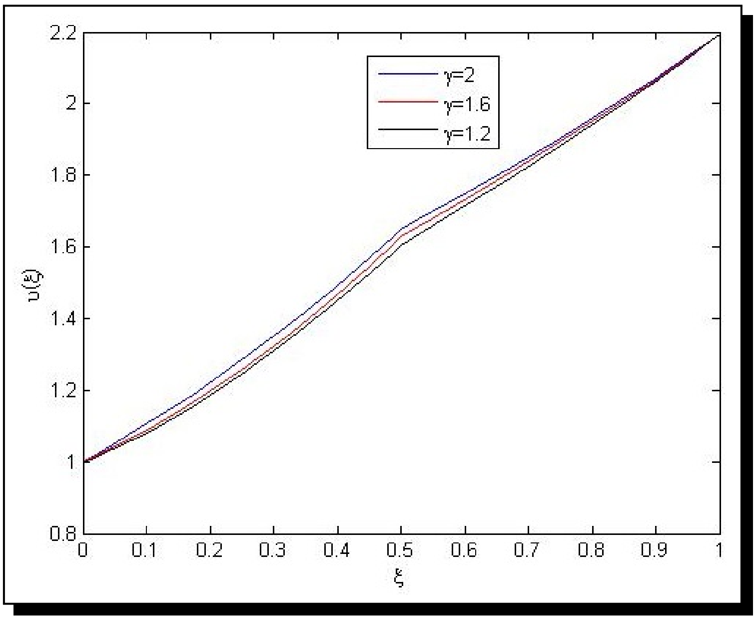

Figure 2. Chebyshev fitted graphs of the solution for Example 5.2 indicating different values of γ

Example 5.3. Take into consideration the following problem:

$$\kappa D^{\gamma} \upsilon(\xi) = 56\xi^6, \quad \kappa = \begin{cases} 1, & \gamma \in (1,2], \xi \in (0,0.5), \\ 2, & \gamma = 2, \xi \in (0.5,1), \end{cases}$$

subject to boundary conditions

$$\upsilon(0) = -1, \quad \upsilon(1) = \frac{257}{512},$$

with interface conditions

$$\upsilon(0.5^-) = \upsilon(0.5^+) - 2, \quad \upsilon'(0.5^-) = 2\upsilon'(0.5^+).$$

As $\gamma = 2$, the exact solution is

$$\upsilon(\xi) = \begin{cases} \xi^8 - 1, & \xi \in (0, 0.5), \\ \frac{1}{2}\left(\xi^8 + \frac{1}{256}\right), & \xi \in (0.5, 1). \end{cases}$$

Table 3 presents a comparison of the absolute error, as calculated by the Chebyshev collocation technique and by modified reproducing kernel [4] for Example 5.3 at the various points. Figure 3 depict various Chebyshev fitted graphs of the solution for Example 5.3 distinguishing numerous values of γ.

Table 3. Absolute error for Example 5.3 at $\gamma = 2$

ξ	Chebyshev collocation method	Reproducing kernel [4]
	$N+M=10$	$N+M=50$
0.1	3.33067e-16	1.29807e-10
0.2	1.11022e-16	8.28191e-9
0.3	1.11022e-16	9.42817e-8
0.4	2.22045e-20	5.29630e-7
0.5	2.22045e-16	2.02019e-6
0.6	2.22045e-16	9.30563e-6
0.7	2.22045e-16	1.80000e-5
0.8	2.77556e-20	2.18000e-5
0.9	2.22045e-16	1.75000e-5
1	0	0

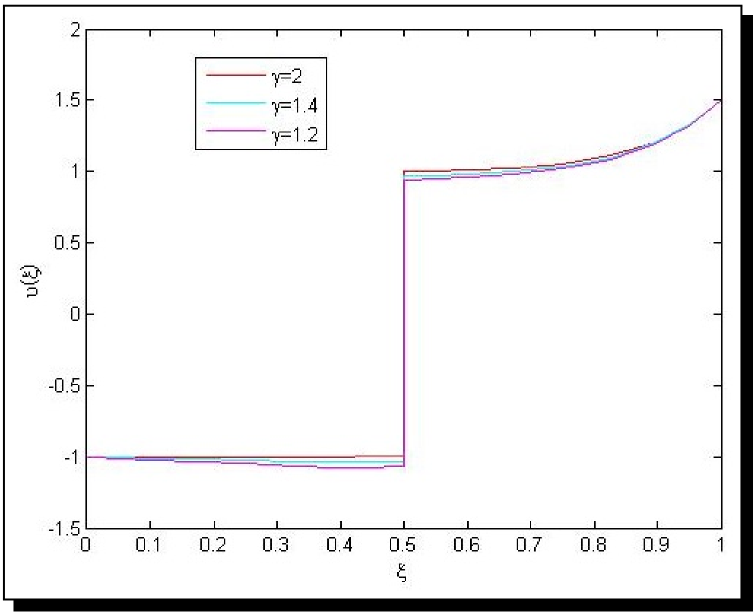

Figure 3. Chebyshev fitted graphs of the solution for Example 5.3 indicating different values of γ

6. Conclusion

This paper has employed a computational method to solve the fractional order 1-D interface problems in the Caputo sense. The main essence of the methodology involves approximating the solution via Chebyshev polynomials. An estimate of the *a priori* error was obtained. One of the advantages of this method is that it has a fast convergence to a solution. The results from different examples show that the method is valid and consistent. Moreover, the simulation results are found to be closer to the exact solution of the proposed case. The achievements of the proposed approach are consistent and better than that realized through the reproducing kernel method as the Chebyshev approach attains results with moderate repetitions.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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