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Research Article

Revan Index, PI Index and Revan Weighted PI Index of Möbius Function Graphs: A Comparative Study

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Abstract. In mathematical chemistry and graph theory, topological indices help in understanding the structure of molecular graphs. Recently, the Möbius function graph of a finite group has become a topic of interest due to its connections to algebra and topology. Despite their importance, the topological indices of Möbius function graphs have received relatively little attention. In this paper, we calculate and investigate the relationships between certain degree and distance-based topological indices, including the Revan index, the PI index, and the Revan weighted PI index, for the Möbius function graphs of finite groups.

Keywords. First Revan index, Second Revan index, PI index, Revan weighted PI index, Möbius Function Graph

Mathematics Subject Classification (2020). 05C30,68R10

1. Introduction

Topological indices are essential tools in theoretical chemistry, which allow researchers to understand molecular properties. To analyze molecular graphs, various topological indices have been developed, which reveal unique characteristics. Recently, two classes of indices — degree-based and distance-based indices — have attracted significant attention in complex network analysis and chemical graph theory (Akhter *et al*. [2], Ali and Došlić [3], Alimon *et al*. [4], Arockiaraj *et al*. [5], Manju and Somasundaram [19], and Yousefi-Azari *et al*. [22]).

A topological index is a numerical value derived from a graph's structure, and over the years numerous such indices have been explored in theoretical chemistry with various applications. In particular, the Padmakar-Ivan (PI) index has received great attention. The Padmakar-Ivan index was introduced by Khadikar in 2000 and has been extensively studied, e.g., see Khadikar *et al*. [11], Khalifeh *et al*. [12], and Manju and Somasundaram [19]. Let $\Gamma = (V(\Gamma), E(\Gamma))$ be a connected graph with the vertex set $V(\Gamma)$ and the edge set $E(\Gamma)$. Let $ab \in E(\Gamma)$ be an edge between the vertices a and b of Γ. We first state the following definitions. Let

$$n_a(ab|\Gamma) = card\{x \in \Gamma \mid d(a,x) < d(b,x)\},$$

$$n_b(ab|\Gamma) = card\{x \in \Gamma \mid d(b,x) < d(a,x)\}.$$

The two quantities described above first appeared in [21]. The PI index is then defined as:

$$\mathrm{PI}(\Gamma) = \sum_{ab \in E(\Gamma)} (n_a(ab \mid \Gamma) + n_b(ab \mid \Gamma)).$$

The degree of a vertex a refers to the number of edges connected to it and is indicated by $d(a)$. In a graph Γ, $\Delta(\Gamma)$ represents the highest degree, while $\delta(\Gamma)$ represents the minimum degree among its vertices. Kulli [15] introduced the concept of the Revan vertex degree, denoted by $r(a)$, for a vertex a in a graph Γ. It is defined as $r(a) = \Delta(\Gamma) + \delta(\Gamma) - d(a)$. This innovative concept paved the way for the development of two new topological indices: the First Revan Index and the Second Revan Index, which are described as follows:

$$R_1(\Gamma) = \sum_{ab \in E(\Gamma)} (r(a) + r(b)),$$

$$R_2(\Gamma) = \sum_{ab \in E(\Gamma)} r(a)r(b).$$

These indices have been used to derive exact values for various molecular structures (Kulli [14, 16]). Motivated by this work, researchers combined the Revan indices with the Padmakar-Ivan (PI) index to create a more comprehensive measure of graph topology. This led to the introduction of the First Revan Weighted PI index, $\mathrm{PI}_{R_1}(\Gamma)$ and Second Revan Weighted PI index $\mathrm{PI}_{R_2}(\Gamma)$ by Priyadharshini *et al*. [20]. These indices are defined as:

$$\mathrm{PI}_{R_1}(\Gamma) = \sum_{ab \in E(\Gamma)} (r(a) + r(b))(n_a(ab \mid \Gamma) + n_b(ab \mid \Gamma)),$$

$$\mathrm{PI}_{R_2}(\Gamma) = \sum_{ab \in E(\Gamma)} (r(a)r(b))(n_a(ab \mid \Gamma) + n_b(ab \mid \Gamma)).$$

Further research is ongoing in this area.

Algebraic graph theory has experienced significant growth, fueled by investigation into the connections between algebraic structures and graphs (Cameron and Ghosh [7], Cameron [8], Kumar *et al*. [17], and Ma *et al*. [18]). Researchers have made considerable progress in calculating various topological indices for a wide range of graphs, such as zero divisor graphs, coprime graphs and comaximal graphs, which are derived from algebraic structures like groups, rings and semigroups (Ahmadi and Jahani-Nezhad [1], Banerjee [6], Kharkongor *et al*. [13], and Zahidah *et al*. [23]). Recently, the Möbius function graph of finite groups has emerged as a vibrant area of research, owing to its intriguing links with multiple mathematical disciplines.

Definition 1.1 ([10])**.** The Möbius function graph $M(G)$ of a finite group G is a simple graph whose vertex set is same as the elements of G and any two distinct vertices a, b are adjacent in $M(G)$ if and only if $\mu(|a||b|) = \mu(|a|)\mu(|b|)$, where μ is the Möbius function.

The Möbius function graph $M(G)$ of a finite group G is always connected because the vertex representing the identity element is connected to all other vertices. Recently, several workers have studied various properties of $M(G)$, including spectral properties, coloring and Wiener index [9, 10]. This paper calculates and examines key topological indices – Revan index, PI index and Revan weighted PI index – for $M(G)$. By analyzing these indices, we aim to uncover new relationships between the algebraic and topological properties of $M(G)$. We also establish theorems that link these indices.

2. Main Results

This section explores the Revan indices, PI index and Revan weighted PI index of Möbius function graphs associated with finite groups. The following three theorems delve into the specifics of these indices for p-groups, offering an in-depth examination of their properties.

Theorem 2.1. *For any group G of prime order p, the Möbius function graph $M(G)$ has the following topological indices*

(a) $R_1(M(G)) = p(p-1)$,
(b) $R_2(M(G)) = (p-1)^2$,
(c) $\mathrm{PI}(M(G)) = p(p-1)$,
(d) $\mathrm{PI}_{R_1}(M(G)) = p^2(p-1)$,
(e) $\mathrm{PI}_{R_2}(M(G)) = p(p-1)^2$.

Proof. Let G be a group of prime order p. In the Möbius function graph $M(G)$, the vertex corresponding to the identity element e is connected to all other vertices. Since p is prime, every non-identity element a in G generates the entire group, implying $|a| = p$. For any two distinct non-identity elements $a, b \in G - \{e\}$, the Möbius function satisfies $\mu(|a||b|) \neq \mu(|a|)\mu(|b|)$, indicating a and b are not adjacent in $M(G)$. Consequently, $M(G)$ is isomorphic to the star graph $K_{1,p-1}$.

In this graph $\Delta(M(G)) = p-1$ and $\delta(M(G)) = 1$. Hence, for any edge ea in $M(G)$, the degree $d(e) = p-1$ and $d(a) = 1$. Also,

$$r(e) = \Delta(M(G)) + \delta(M(G)) - d(e) = (p-1) + 1 - (p-1) = 1,$$
$$r(a) = \Delta(M(G)) + \delta(M(G)) - d(a) = (p-1) + 1 - 1 = (p-1),$$
$$n_e(ea \mid M(G)) = card\{x \in M(G) \mid d(e,x) < d(a,x)\} = (p-1),$$
$$n_a(ea \mid M(G)) = card\{x \in M(G) \mid d(a,x) < d(e,x)\} = 1.$$

By using equations of above two cases, we obtain

(a) First Revan index of $M(G)$ as

$$R_1(M(G)) = \sum_{ea \in E(M(G))} (r(e) + r(a))$$

$$= \sum_{ea\in E(M(G))} (1+(p-1))$$
$$= p(p-1).$$

(b) Second Revan index of $M(G)$ as

$$R_2(M(G)) = \sum_{ea\in E(M(G))} r(e)r(a)$$
$$= \sum_{ea\in E(M(G))} (p-1)$$
$$= (p-1)^2.$$

(c) PI index of $M(G)$ is

$$\mathrm{PI}(M(G)) = \sum_{ea\in E(M(G))} (n_e(ea \mid M(G)) + n_a(ea \mid M(G)))$$
$$= \sum_{ea\in E(M(G))} ((p-1)+1)$$
$$= p(p-1).$$

(d) First Revan weighted PI index of $M(G)$ is

$$\mathrm{PI}_{R_1}(M(G)) = \sum_{ea\in E(M(G))} (r(e)+r(a))(n_e(ea \mid M(G)) + n_a(ea \mid M(G)))$$
$$= \sum_{ea\in E(M(G))} (1+(p-1))((p-1)+1)$$
$$= p^2(p-1).$$

(e) Second Revan weighted PI index of $M(G)$ is

$$\mathrm{PI}_{R_2}(M(G)) = \sum_{ea\in E(M(G))} (r(e)r(a))(n_e(ea \mid M(G)) + n_a(ea \mid M(G)))$$
$$= \sum_{ea\in E(M(G))} (p-1)((p-1)+1)$$
$$= p(p-1)^2.$$

□

Theorem 2.1 yields several important consequences, which are further discussed in the following two corollaries:

Corollary 2.2. *In a group G of prime order*

(a) $\mathrm{PI}_{R_1}(M(G)) = p(R_1(M(G)))$,

(b) $\mathrm{PI}_{R_2}(M(G)) = p(R_2(M(G)))$.

Corollary 2.3. *For a group of prime order p,*

$$\mathrm{PI}(M(G)) = R_1(M(G)).$$

Theorem 2.4. *Let G be a finite group of order p^k, where p is a prime number and k is a positive integer greater than 1. Then*

(a) $R_1(M(G)) = s^3 + r^2 s + 2rs^2 - rs - s$,

(b) $R_2(M(G)) = r^2s^2 + rs^3 + \frac{s^4}{2} - rs^2 - \frac{s^3}{2}$,

(c) $\mathrm{PI}(M(G)) = r^2s + n^2 + rs - n$,

(d) $\mathrm{PI}_{R_1}(M(G)) = r^3s + 2r^2s^2 + 2s^3 + 2rs^2 - 2s^2 - rs$,

(e) $\mathrm{PI}_{R_2}(M(G)) = r^3s^2 + r^2s^3 + rs^3 - rs^2 + s^4 - s^3$.

Proof. Let $|G| = p^k$, where p prime and $k > 1$. We partition the vertex set of $M(G)$ into two subsets according to the order of an element in the group G : Φ_1, containing vertices of order p and Φ_2, containing vertices of order not equal to p. Applying the definition of $M(G)$, we find that the subgraph induced by Φ_1 is totally disconnected and is isomorphic to $\overline{K}_{|\Phi_1|}$.
Similarly, we get the subgraph induced by Φ_2 is complete, isomorphic to $K_{|\Phi_2|}$. Furthermore, for any $a \in \Phi_1$ and $b \in \Phi_2$, we have $\mu(|a||b|) = \mu(|a|)\mu(|b|)$. This indicates that, in $M(G)$ each element of Φ_1 is adjacent to every element in Φ_2. Consequently, $M(G)$ is a split graph, with Φ_1 forming an independent set of size r and Φ_2 forming a clique of order s(see $r+s = |G|$). In case of $M(G)$, $\Delta(M(G)) = r+s-1$ and $\delta(M(G)) = s$.

Let $\Phi_1 = \{a_1, a_2, \dots, a_r\}$ and $\Phi_2 = \{b_1, b_2, \dots, b_s\}$.

Case 1: Consider the edge b_ib_j, where $b_i, b_j \in \Phi_2$. In this case $d(b_i) = d(b_j) = r+s-1$. Here,

$$r(b_i) = \Delta(M(G)) + \delta(M(G)) - d(b_i) = (r+s-1) + s - (r+s-1) = s,$$
$$r(b_j) = \Delta(M(G)) + \delta(M(G)) - d(b_j) = s,$$
$$n_{b_i}(b_ib_j \mid M(G)) = card\{x \in M(G) \mid d(b_i,x) < d(b_j,x)\} = card\{b_i\} = 1,$$
$$n_{b_j}(b_ib_j \mid M(G)) = card\{x \in M(G) \mid d(b_j,x) < d(b_i,x)\} = card\{b_j\} = 1.$$

Case 2: Next consider the edge a_ib_j, where $a_i \in \Phi_1$ and $b_j \in \Phi_2$. Here $d(a_i) = s$ and $d(b_j) = r+s-1$. Then,

$$r(a_i) = \Delta(M(G)) + \delta(M(G)) - d(a_i) = (r+s-1) + s - s = r+s-1,$$
$$r(b_j) = \Delta(M(G)) + \delta(M(G)) - d(b_j) = s,$$
$$n_{a_i}(a_ib_j \mid M(G)) = card\{x \in M(G) \mid d(a_i,x) < d(b_j,x)\} = card\{a_i\} = 1,$$
$$n_{b_j}(a_ib_j \mid M(G)) = card\{x \in M(G) \mid d(b_j,x) < d(a_i,x)\} = card\{\{b_j\} \cup \{\Phi_1 - \{a_i\}\}\} = r.$$

By using the equations of the above two cases, we get the following:

(a) First Revan index of $M(G)$ as

$$\begin{aligned} R_1(M(G)) &= \sum_{ab \in E(M(G))} (r(a) + r(b)) \\ &= \sum_{b_i, b_j \in \Phi_2} (r(b_i) + r(b_j)) + \sum_{\substack{a_i \in \Phi_1 \\ b_j \in \Phi_2}} (r(a_i) + r(b_j)) \\ &= \binom{s}{2}(s+s) + rs(r+s-1+s) \\ &= s^3 + r^2s + 2rs^2 - rs - s. \end{aligned}$$

(b) Second Revan index of $M(G)$ as

$$R_2(M(G)) = \sum_{ab \in E(M(G))} (r(a)r(b))$$

$$= \sum_{b_i,b_j \in \Phi_2} (r(b_i)r(b_j)) + \sum_{\substack{a_i \in \Phi_1 \\ b_j \in \Phi_2}} (r(a_i)r(b_j))$$

$$= \binom{s}{2}(s^2) + rs^2(r+s-1)$$

$$= r^2s^2 + rs^3 + \frac{s^4}{2} - rs^2 - \frac{s^3}{2}.$$

(c) PI index of $M(G)$ is

$$\begin{aligned}
\mathrm{PI}(M(G)) &= \sum_{ab \in E(M(G))} (n_a(ab \mid M(G)) + n_b(ab \mid M(G))) \\
&= \sum_{b_i,b_j \in \Phi_2} (n_{b_i}(b_ib_j \mid M(G)) + n_{b_j}(b_ib_j \mid M(G))) \\
&\quad + \sum_{\substack{a_i \in \Phi_1 \\ b_j \in \Phi_2}} (n_{a_i}(a_ib_j \mid M(G)) + n_{b_j}(a_ib_j \mid M(G))) \\
&= \sum_{b_i,b_j \in \Phi_2} (1+1) + \sum_{\substack{a_i \in \Phi_1 \\ b_j \in \Phi_2}} (1+r) \\
&= \binom{s}{2} 2 + rs(r+1) \\
&= r^2s + n^2 + rs - s.
\end{aligned}$$

(d) First Revan weighted PI index of $M(G)$ as

$$\begin{aligned}
\mathrm{PI}_{R_1}(M(G)) &= \sum_{ab \in E(M(G))} (r(a)+r(b))(n_a(ab \mid M(G)) + n_b(ab \mid M(G))) \\
&= \sum_{b_i,b_j \in \Phi_2} (r(b_i)+r(b_j))(n_{b_i}(b_ib_j \mid M(G)) + n_{b_j}(b_ib_j \mid M(G))) \\
&\quad + \sum_{\substack{a_i \in \Phi_1 \\ b_j \in \Phi_2}} (r(a_i)+r(b_j))(n_{a_i}(a_ib_j \mid M(G)) + n_{b_j}(a_ib_j \mid M(G))) \\
&= \sum_{b_i,b_j \in \Phi_2} (s+s)(1+1) + \sum_{\substack{a_i \in \Phi_1 \\ b_j \in \Phi_2}} (r+s-1+s)(1+r) \\
&= r^3s + 2r^2s^2 + 2s^3 + 2rs^2 - 2s^2 - rs.
\end{aligned}$$

(e) Second Revan weighted PI index of $M(G)$ as

$$\begin{aligned}
\mathrm{PI}_{R_2}(M(G)) &= \sum_{ab \in E(M(G))} (r(a)r(b))(n_a(ab \mid M(G)) + n_b(ab \mid M(G))) \\
&= \sum_{b_i,b_j \in \Phi_2} (r(b_i)r(b_j))(n_{b_i}(b_ib_j \mid M(G)) + n_{b_j}(b_ib_j \mid M(G))) \\
&\quad + \sum_{\substack{a_i \in \Phi_1 \\ b_j \in \Phi_2}} (r(a_i)r(b_j))(n_{a_i}(a_ib_j \mid M(G)) + n_{b_j}(a_ib_j \mid M(G))) \\
&= \sum_{b_i,b_j \in \Phi_2} s^2(1+1) + \sum_{\substack{a_i \in \Phi_1 \\ b_j \in \Phi_2}} s(r+s-1)(1+r) \\
&= r^3s^2 + r^2s^3 + rs^3 - rs^2 + s^4 - s^3.
\end{aligned}$$

□

In the subsequent theorem, we explore a special case of Theorem 2.4, where the prime number p is set to 2, considering an abelian group. This particular scenario yields interesting characterization theorems, potentially leading to future research.

Theorem 2.5. *Let G be a finite abelian group of order 2^k, for some $k \in \mathbb{Z}^+$. Then*

(a) $R_1(M(G)) = 2^k(2^k-1)^2$,

(b) $R_2(M(G)) = 2^{k-1}(2^k-1)^3$,

(c) $\mathrm{PI}(M(G)) = 2^k(2^k-1)$,

(d) $\mathrm{PI}_{R_1}(M(G)) = 2^{k+1}(2^k-1)^2$,

(e) $\mathrm{PI}_{R_2}(M(G)) = 2^k(2^k-1)^3$.

Proof. Let G be a finite abelian group of order $n = 2^k$. We claim that the graph $M(G)$ is complete. Since G is a 2-group, it contains an element a of order 2, which is self-inverse. Moreover, every non-identity element in G, except for a, has order 2^m for some $m > 1$. This implies that every vertex in $M(G)$ satisfies the Möbius function graph condition. Therefore, $M(G)$ is a complete graph. In $M(G)$, $\Delta(M(G)) = n-1$ and $\delta(M(G)) = n-1$.
For any edge ab in $M(G)$, we can say that $d(a) = d(b) = n-1$.
Hence, we can calculate Revan vertex degree of $r(a)$ and $r(b)$ as

$$r(a) = \Delta(M(G)) + \delta(M(G)) - d(a) = (n-1)+(n-1)-(n-1) = n-1,$$

$$r(b) = \Delta(M(G)) + \delta(M(G)) - d(b) = n-1.$$

Also,

$$n_a(ab \mid M(G)) = card\{x \in M(G) \mid d(a,x) < d(b,x)\} = card\{a\} = 1,$$

$$n_b(ab \mid M(G)) = card\{x \in M(G) \mid d(b,x) < d(a,x)\} = card\{b\} = 1.$$

By using the above values, we get

(a) First Revan index of $M(G)$ as

$$\begin{aligned} R_1(M(G)) &= \sum_{ab \in E(M(G))} (r(a)+r(b)) \\ &= \sum_{ab \in E(M(G))} ((n-1)+(n-1)) \\ &= \binom{n}{2} 2(n-1) = 2^k(2^k-1)^2. \end{aligned}$$

(b) Second Revan index of $M(G)$ as

$$\begin{aligned} R_2(M(G)) &= \sum_{ab \in E(M(G))} r(a)r(n) \\ &= \sum_{ab \in E(M(G))} (n-1)^2 \\ &= \binom{n}{2}(n-1)^2 = 2^{k-1}(2^k-1)^3. \end{aligned}$$

(c) PI index of $M(G)$ is

$$\begin{aligned}\mathrm{PI}(M(G)) &= \sum_{uv\in E(M(G))} (n_u(uv \mid M(G)) + n_v(uv \mid M(G)))\\ &= \binom{n}{2}2 = 2^k(2^k-1).\end{aligned}$$

(d) First Revan weighted PI index of $M(G)$ is

$$\begin{aligned}\mathrm{PI}_{R_1}(M(G)) &= \sum_{ab\in E(M(G))} (r(a)+r(b))(n_a(ab \mid M(G)) + n_b(ab \mid M(G)))\\ &= \sum_{ab\in E(M(G))} ((n-1)+(n-1))(1+1)\\ &= \binom{n}{2}4(n-1) = 2^{k+1}(2^k-1)^2.\end{aligned}$$

(e) Second Revan weighted PI index of $M(G)$ is

$$\begin{aligned}\mathrm{PI}_{R_1}(M(G)) &= \sum_{ab\in E(M(G))} (r(a)r(b))(n_a(ab \mid M(G)) + n_b(ab \mid M(G)))\\ &= \sum_{ab\in E(M(G))} (n-1)^2(1+1)\\ &= \binom{n}{2}2(n-1)^2 = 2^k(2^k-1)^3.\end{aligned}$$

□

From Theorem 2.5, we arrive at a characterization theorem for Möbius function graph when the group is an abelian 2-group.

Corollary 2.6. *In Möbius function graph of an abelian group of order* 2^k,

(a) $\mathrm{PI}_{R_1}(M(G)) = 2R_1(M(G))$,

(b) $\mathrm{PI}_{R_2}(M(G)) = 2R_2(M(G))$.

In the next theorem, we examine the distance- based and weighted indices of the Möbius function graph of a group of order pq.

Theorem 2.7. *Let G be a group with $|G| = pq$, where p and q are distinct primes. Then*

(a) $R_1(M(G)) = r^2s + r^2 + rs^2 + 2rst + 2rt + r + s^2 + 2st + s + t^2 + t$,

(b) $R_2(M(G)) = r^2s^2 + r^2st + r^2 + rs^2t + rst^2 + 2rt + s^2 + 2st + t^2$,

(c) $\mathrm{PI}(M(G)) = r^2s + r^2 + rs^2 + 2rt + r + s^2 + 2st + s + t^2 + t$,

(d) $\mathrm{PI}_{R_1}(M(G)) = r^3s + r^3 + 2r^2s^2 + 2r^2st + 3r^2t + 2r^2 + rs^3 + 2rs^2t + 2rst + 3rt^2 + 4rt + r + s^3 + 3s^2t + 2s^2 + 3st^2 + 4st + s + t^3 + 2t^2 + t$

(e) $\mathrm{PI}_{R_2}(M(G)) = r^3s^2 + r^3st + r^3 + r^2s^3 + 2r^2s^2t + r^2st^2 + 3r^2t + r^2 + rs^3t + rs^2t^2 + 2rst + 3rt^2 + 2rt + s^3 + 3s^2t + s^2 + 3st^2 + 2st + t^3 + t^2$

Proof. Let us consider a group G with $|G| = pq$, then we can partition the vertex set of Möbius function graph as $\Phi_1 = \{e\}$, $\Phi_2 = \{a : |a| = p\}$, $\Phi_3 = \{b : |b| = q\}$, $\Phi_4 = \{c : |c| = pq\}$. By the definition of Möbius function graph of finite group Φ_2, Φ_3, Φ_4 are independent sets which are represented by $\overline{K}_r, \overline{K}_s, \overline{K}_t$ respectively where r, s, t denote the number of elements of order p, q, pq.

For any $a \in \Phi_2$ and $b \in \Phi_3$, $\mu(|a||b|) = \mu(|a|)\mu(|b|)$. Hence, sets Φ_2 and Φ_3 forms a complete bipartate graph in $M(G)$ and its represented by $\overline{K}_r + \overline{K}_s$. Hence, we can say that the graph so obtained is isomorphic to $(\overline{K}_r + \overline{K}_s) + K_1 + \overline{K}_t$. In $M(G)$, $\Delta(M(G)) = r+s+t$ and $\delta(M(G)) = 1$. Now we can find all the indices of $M(G)$.

Case 1: Consider the edge ea, where $e \in \Phi_1$, $a \in \Phi_2$.
In this case $d(e) = r+s+t$ and $d(a) = s+1$. Hence, Revan vertex degree is calculated as:

$$r(e) = \Delta(M(G)) + \delta(M(G)) - d(e) = (r+s+t)+(1)-(r+s+t) = 1,$$
$$r(a) = \Delta(M(G)) + \delta(M(G)) - d(a) = (r+s+t)+(1)-(s+1) = r+t.$$

Also,

$$n_e(ea \mid M(G)) = card\{x \in M(G) \mid d(e,x) < d(a,x)\} = card\{\Phi_1 \cup \Phi_2 - \{a\} \cup \Phi_4\} = r+t,$$
$$n_a(ea \mid M(G)) = card\{x \in M(G) \mid d(a,x) < d(e,x)\} = card\{a\} = 1.$$

Case 2: Consider the edge eb, where $e \in \Phi_1, b \in \Phi_3$. Here $d(e) = r+s+t$ and $d(b) = r+1$,

$$r(e) = \Delta(M(G)) + \delta(M(G)) - d(e) = (r+s+t)+(1)-(r+s+t) = 1,$$
$$r(b) = \Delta(M(G)) + \delta(M(G)) - d(b) = (r+s+t)+(1)-(r+1) = s+t.$$

Also,

$$n_e(eb \mid M(G)) = card\{x \in M(G) \mid d(e,x) < d(b,x)\} = card\{\Phi_1 \cup \Phi_3 - \{b\} \cup \Phi_4\} = s+t,$$
$$n_v(ev \mid M(G)) = card\{x \in M(G) \mid d(b,x) < d(e,x)\} = card\{b\} = 1.$$

Case 3: Consider the edge ec, where $e \in \Phi_1$, $c \in \Phi_4$. In this case $d(e) = r+s+t$ and $d(c) = 1$. Here

$$r(e) = \Delta(M(G)) + \delta(M(G)) - d(e) = (r+s+t)+(1)-(r+s+t) = 1,$$
$$r(c) = \Delta(M(G)) + \delta(M(G)) - d(c) = (r+s+t)+(1)-(1) = r+s+t.$$

Also,

$$n_e(ec \mid M(G)) = card\{x \in M(G) \mid d(e,x) < d(c,x)\} = card\{\Phi_1 \cup \Phi_2 \cup \Phi_3 \cup \Phi_4 - \{c\}\} = r+s+t,$$
$$n_c(ec \mid M(G)) = card\{x \in M(G) \mid d(c,x) < d(e,x)\} = card\{c\} = 1.$$

Case 4: Consider the edge ab, where $a \in \Phi_2$, $b \in \Phi_3$. Here $d(a) = s+1$ and $d(b) = r+1$. Also, we can find

$$r(a) = \Delta(M(G)) + \delta(M(G)) - d(a) = (r+s+t)+(1)-(s+1) = r+t,$$
$$r(b) = \Delta(M(G)) + \delta(M(G)) - d(b) = (r+s+t)+(1)-(r+1) = s+t.$$

Also,

$$n_a(ab \mid M(G)) = card\{x \in M(G) \mid d(a,x) < d(b,x)\} = card\{\{a\} \cup \Phi_3 - \{b\}\} = s,$$
$$n_b(ab \mid M(G)) = card\{x \in M(G) \mid d(b,x) < d(a,x)\} = card\{\{b\} \cup \Phi_2 - \{a\}\} = r.$$

By using the above four case, we can find indices:

(a) First Revan index of $M(G)$ as

$$\begin{aligned} R_1(M(G)) &= \sum_{ab \in E(M(G))} (r(a)+r(b)) \\ &= \sum_{\substack{e\in\Phi_1 \\ a\in\Phi_2}} (r(e)+r(a)) + \sum_{\substack{e\in\Phi_1 \\ b\in\Phi_3}} (r(e)+r(b)) + \sum_{\substack{e\in\Phi_1 \\ c\in\Phi_4}} (r(e)+r(c)) + \sum_{\substack{a\in\Phi_2 \\ b\in\Phi_3}} (r(a)+r(b)) \end{aligned}$$

$$
\begin{aligned}
&= \sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}} (1+r+t) + \sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}} (1+s+t) + \sum_{\substack{e\in\Phi_1\\ c\in\Phi_4}} (r+s+t+1) + \sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}} (r+s+2t)\\
&= r(1+r+t)+s(1+s+t)+t(r+s+t+1)+rs(r+s+2t)\\
&= r^2s+r^2+rs^2+2rst+2rt+r+s^2+2st+s+t^2+t.
\end{aligned}
$$

(b) Second Revan index of $M(G)$ as

$$
\begin{aligned}
R_2(M(G)) &= \sum_{ab\in E(M(G))} r(a)r(b)\\
&= \sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}} r(e)r(a) + \sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}} r(e)r(b) + \sum_{\substack{e\in\Phi_1\\ c\in\Phi_4}} r(e)r(c) + \sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}} r(a)r(b)\\
&= \sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}} (r+t) + \sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}} (s+t) + \sum_{\substack{e\in\Phi_1\\ c\in\Phi_4}} (r+s+t) + \sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}} (r+t)(s+t)\\
&= r(r+t)+s(s+t)+t(r+s+t)+rs(r+t)(s+t)\\
&= r^2s^2+r^2st+r^2+rs^2t+rst^2+2rt+s^2+2st+t^2.
\end{aligned}
$$

(c) PI index of $M(G)$ is

$$
\begin{aligned}
PI(M(G)) &= \sum_{uv\in E(M(G))} (n_u(uv\mid M(G))+n_v(uv\mid M(G)))\\
&= \sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}} (n_e(ea\mid M(G))+n_a(ea\mid M(G))) + \sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}} (n_e(eb\mid M(G))+n_b(eb\mid M(G)))\\
&\quad + \sum_{\substack{e\in\Phi_1\\ c\in\Phi_4}} (n_e(ec\mid M(G))+n_c(ec\mid M(G))) + \sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}} (n_a(ab\mid M(G))+n_b(ab\mid M(G)))\\
&= \sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}} (r+t+1) + \sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}} (s+t+1) + \sum_{\substack{e\in\Phi_1\\ c\in\Phi_4}} (r+s+t+1) + \sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}} (r+s)\\
&= r(r+t+1)+s(s+t+1)+t(r+s+t+1)+rs(r+s)\\
&= r^2s+r^2+rs^2+2rt+r+s^2+2st+s+t^2+t.
\end{aligned}
$$

(d) First Revan weighted PI index of $M(G)$ is

$$
\begin{aligned}
PI_{R_1}(M(G)) &= \sum_{ab\in E(M(G))} (r(a)+r(b))(n_a(ab\mid M(G))+n_b(ab\mid M(G)))\\
&= \sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}} (r(e)+r(a))(n_e(ea\mid M(G))+n_a(ea\mid M(G)))\\
&\quad + \sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}} (r(e)+r(b))(n_e(eb\mid M(G))+n_b(eb\mid M(G)))\\
&\quad + \sum_{\substack{e\in\Phi_1\\ c\in\Phi_4}} (r(e)+r(c))(n_e(ec\mid M(G))+n_c(ec\mid M(G)))\\
&\quad + \sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}} (r(a)+r(b))(n_a(ab\mid M(G))+n_b(ab\mid M(G)))\\
&= \sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}} (1+r+t)(r+t+1) + \sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}} (1+s+t)(s+t+1)
\end{aligned}
$$

$$+\sum_{\substack{e\in\Phi_1\\ c\in\Phi_4}}(r+s+t+1)(r+s+t+1)+\sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}}(r+s+2t)(r+s)$$
$$=r(r+t+1)^2+s(s+t+1)^2+t(r+s+t+1)^2+rs(r+s+2t)(r+s)$$
$$=r^3s+r^3+2r^2s^2+2r^2st+3r^2t+2r^2+rs^3+2rs^2t+2rst+3rt^2+4rt$$
$$+r+s^3+3s^2t+2s^2+3st^2+4st+s+t^3+2t^2+t.$$

(e) Second Revan weighted PI index of $M(G)$ is

$$PI_{R_2}(M(G))=\sum_{ab\in E(M(G))}(r(a)r(b))(n_a(ab\mid M(G))+n_b(ab\mid M(G)))$$
$$=\sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}}(r(e)r(a))(n_e(ea\mid M(G))+n_a(ea\mid M(G)))$$
$$+\sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}}(r(e)r(b))(n_e(eb\mid M(G))+n_b(eb\mid M(G)))$$
$$+\sum_{\substack{e\in\Phi_1\\ c\in\Phi_4}}(r(e)r(c))(n_e(ec\mid M(G))+n_c(ec\mid M(G)))$$
$$+\sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}}(r(a)r(b))(n_a(ab\mid M(G))+n_b(ab\mid M(G)))$$
$$=\sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}}(r+t)(r+t+1)+\sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}}(s+t)(s+t+1)$$
$$+\sum_{\substack{e\in\Phi_1\\ c\in\Phi_4}}(r+s+t)(r+s+t+1)+\sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}}(r+t)(s+t)(r+s)$$
$$=r(r+t)(r+t+1)+s(s+t)(s+t+1)+t(r+s+t)(r+s+t+1)$$
$$+rs(r+t)(s+t)(r+s)$$
$$=r^3s^2+r^3st+r^3+r^2s^3+2r^2s^2t+r^2st^2+3r^2t+r^2$$
$$+rs^3t+rs^2t^2+2rst+3rt^2+2rt+s^3+3s^2t+s^2+3st^2+2st+t^3+t^2. \qquad \square$$

Theorem 2.8. *$M(D_{2p})$, $p>2$ prime number, then*

(a) $R_1(M(D_{2p}))=4p^3-p^2-3p+2$,

(b) $R_2(M(D_{2p}))=4p^4-6p^3+3p^2$,

(c) $\mathrm{PI}(M(D_{2p}))=2p^3-p^2+p$,

(d) $PI_{R_1}(M(D_{2p}))=8p^4-12p^3+10p^2-2p$,

(e) $PI_{R_2}(M(D_{2p}))=8p^5-22p^4+29p^3-15p^2+2p$.

Proof. Consider the dihedral group $G=D_{2p}=\{e,u,u^2,\dots,u^{p-1},v,uv,u^2v,\dots,u^{p-1}v\}$, for a prime $p>2$. Then $|e|=1$, $|u|=|u^2|=\dots=|u^{p-1}|=p$, $|v|=|uv|=|u^2v|=\dots=|u^{p-1}v|=2$. We can partition the vertex set of $M(G)$ into three sets, say $\Phi_1=\{e\}$, $\Phi_2=\{u^i:1\le i\le p-1\}$ and $\Phi_3=\{u^jv:0\le j\le p-1\}$ with $|\Phi_1|=1,|\Phi_2|=p-1,|\Phi_3|=p$. In the graph $M(G)$, the vertex corresponding to identity element e is connected to all $(2p-1)$ other vertices. Specifically, every vertex in Φ_1 is adjacent to every vertex in Φ_2 and Φ_3.

For any two vertices $a,b \in \Phi_2$, $\mu(|a||b|) \neq \mu(|a|)\mu(|b|)$. Consequently, no two elements of Φ_2 are adjacent. A similar argument also shows that the two vertices in Φ_3 are not adjacent.

Now, consider the adjacency relationship between vertices $a \in \Phi_2$ and $c \in \Phi_3$. Then,

$$\mu(|a|)\mu(|c|) = \mu(p)\mu(2) = (-1)(-1) = 1, \tag{2.1}$$

$$\mu(|a||c|) = \mu(p.2) = (-1)^2 = 1. \tag{2.2}$$

From eqs. 2.1 and 2.2, $\mu(|a||c|) = \mu(|a|)\mu(|c|)$. Therefore, each vertex in Φ_2 is connected to every vertex in Φ_3. Thus, we can conclude that $M(D_{2p})$, $p > 2$ prime number, is a complete tripartite graph. Here in $M(G)$, $\Delta(M(G)) = 2p-1$ and $\delta(M(G)) = p$.

Case 1: Consider the edge ea, where $e \in \Phi_1$, $a \in \Phi_2$. In this case $d(e) = 2p-1$ and $d(a) = p+1$. Hence, Revan vertex degree is calculated as:

$$r(e) = \Delta(M(G)) + \delta(M(G)) - d(e) = (2p-1) + (p) - (2p-1) = p,$$

$$r(a) = \Delta(M(G)) + \delta(M(G)) - d(a) = (2p-1) + (p) - (p+1) = 2(p-1).$$

Also,

$$n_e(ea \mid M(G)) = card\{x \in M(G) \mid d(e,x) < d(a,x)\} = card\{\Phi_1 \cup \Phi_2 - \{a\}\} = (p-1),$$

$$n_a(ea \mid M(G)) = card\{x \in M(G) \mid d(a,x) < d(e,x)\} = card\{a\} = 1.$$

Case 2: Consider the edge eb, where $e \in \Phi_1$, $b \in \Phi_3$. Here $d(e) = 2p-1$ and $d(b) = p$. Hence, Revan vertex degree is calculated as:

$$r(e) = \Delta(M(G)) + \delta(M(G)) - d(e) = (2p-1) + (p) - (2p-1) = p,$$

$$r(b) = \Delta(M(G)) + \delta(M(G)) - d(b) = (2p-1) + (p) - (p) = (2p-1).$$

Also,

$$n_e(eb \mid M(G)) = card\{x \in M(G) \mid d(e,x) < d(b,x)\} = card\{\Phi_1 \cup \Phi_3 - \{b\}\} = p,$$

$$n_b(eb \mid M(G)) = card\{x \in M(G) \mid d(b,x) < d(e,x)\} = card\{b\} = 1.$$

Case 3: Consider the edge ab, where $a \in \Phi_2$, $b \in \Phi_3$. In this case $d(a) = p+1$ and $d(b) = p$. Hence, Revan vertex degree is calculated as:

$$r(a) = 2(p-1) \quad \text{and} \quad r(b) = 2p-1.$$

Also,

$$n_a(ab \mid M(G)) = card\{x \in M(G) \mid d(a,x) < d(b,x)\} = card\{\{a\} \cup \Phi_3 - \{b\}\} = p,$$

$$n_b(ab \mid M(G)) = card\{x \in M(G) \mid d(b,x) < d(a,x)\} = card\{\{b\} \cup \Phi_2 - \{a\}\} = p-1.$$

By using all the above cases, we get

(a) First Revan index of $M(G)$ as

$$\begin{aligned} R_1(M(G)) &= \sum_{ab \in E(M(G))} (r(a) + r(b)) \\ &= \sum_{\substack{e \in \Phi_1 \\ a \in \Phi_2}} (r(e) + r(a)) + \sum_{\substack{e \in \Phi_1 \\ b \in \Phi_3}} (r(e) + r(b)) + \sum_{\substack{a \in \Phi_2 \\ b \in \Phi_3}} (r(a) + r(b)) \\ &= \sum_{\substack{e \in \Phi_1 \\ a \in \Phi_2}} (3p-2) + \sum_{\substack{e \in \Phi_1 \\ b \in \Phi_3}} (3p-1) + \sum_{\substack{a \in \Phi_2 \\ b \in \Phi_3}} (4p-3) \\ &= (p-1)(3p-2) + p(3p-1) + p(p-1)(4p-3) \end{aligned}$$

$$= 4p^3 - p^2 - 3p + 2.$$

(b) Second Revan index of $M(G)$ as

$$\begin{aligned}
R_2(M(G)) &= \sum_{ab\in E(M(G))} r(a)r(b) \\
&= \sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}} r(e)r(a) + \sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}} r(e)r(b) + \sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}} r(a)r(b) \\
&= \sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}} p(2p-2) + \sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}} p(2p-1) + \sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}} (2p-2)(2p-1) \\
&= (p-1)p(2p-2) + p^2(2p-1) + p(p-1)(2p-2)(2p-1) \\
&= 4p^4 - 6p^3 + 3p^2.
\end{aligned}$$

(c) PI index of $M(G)$ is

$$\begin{aligned}
\mathrm{PI}(M(G)) &= \sum_{ab\in E(M(G))} (n_a(ab\mid M(G)) + n_b(ab\mid M(G))) \\
&= \sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}} (n_e(ea\mid M(G)) + n_a(ea\mid M(G))) + \sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}} (n_e(eb\mid M(G)) + n_b(eb\mid M(G))) \\
&\quad + \sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}} (n_a(ab\mid M(G)) + n_b(ab\mid M(G))) \\
&= p(p-1) + p(p+1) + p(p-1)(2p-1) \\
&= 2p^3 - p^2 + p.
\end{aligned}$$

(d) First Revan weighted PI index of $M(G)$ as

$$\begin{aligned}
PI_{R_1}(M(G)) &= \sum_{ab\in E(M(G))} (r(a)+r(b))(n_a(ab\mid M(G)) + n_b(ab\mid M(G))) \\
&= \sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}} (r(e)+r(a))(n_e(ea\mid M(G)) + n_a(ea\mid M(G))) \\
&\quad + \sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}} (r(e)+r(b))(n_e(eb\mid M(G)) + n_b(eb\mid M(G))) \\
&\quad + \sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}} (r(a)+r(b))(n_a(ab\mid M(G)) + n_b(ab\mid M(G))) \\
&= \sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}} (3p-2)p + \sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}} (3p-1)(p+1) + \sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}} (4p-3)(2p-1) \\
&= (p-1)(3p-2)p + p(3p-1)(p+1) + p(p-1)(4p-3)(2p-1) \\
&= 8p^4 - 12p^3 + 10p^2 - 2p.
\end{aligned}$$

(e) Second Revan weighted PI index of $M(G)$ as

$$PI_{R_1}(M(G)) = \sum_{ab\in E(M(G))} (r(a)r(b))(n_a(ab\mid M(G)) + n_b(ab\mid M(G)))$$

$$\begin{aligned}
&= \sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}} (r(e)r(a))(n_e(ea\mid M(G))+n_a(ea\mid M(G)))\\
&\quad + \sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}} (r(e)r(b))(n_e(eb\mid M(G))+n_b(eb\mid M(G)))\\
&\quad + \sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}} (r(a)r(b))(n_a(ab\mid M(G))+n_b(ab\mid M(G)))\\
&= \sum_{\substack{e\in\Phi_1\\ a\in\Phi_2}} 2p^2(p-1) + \sum_{\substack{e\in\Phi_1\\ b\in\Phi_3}} p(2p-1)(p+1) + \sum_{\substack{a\in\Phi_2\\ b\in\Phi_3}} 2(p-1)(2p-1)^2\\
&= 2p^2(p-1)+p^2(2p-1)(p+1)+2p(p-1)^2(2p-1)^2\\
&= 8p^5-22p^4+29p^3-15p^2+2p. \qquad \square
\end{aligned}$$

3. Conclusion

This paper presents the computation of the Revan index, PI index and Revan weighted PI index for the Möbius function graph of finite groups. These calculations enable the identification of similarities between the Möbius function graph and other graphs obtained from algebraic structures. Future research directions include a comparative analysis of the Möbius function graph with other graphs, leveraging their respective indices.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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