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Research Article

Some Neutrosophic Ideals of BF-algebras

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Abstract. In this paper, we introduce the notions of neutrosophic ideal and neutrosophic subalgebra of BF-algebra with suitable illustrations. Further, we discuss homomorphism and epimorphism mappings of neutrosophic ideals. Also, we introduce the definitions of a neutrosophic implicative ideal and a neutrosophic ideal of BF-algebras and investigate some of their properties.

Keywords. BF-algebra, Ideal, Neutrosophic set, Neutrosophic implicative ideal, Neutrosophic p-ideal

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1. Introduction

The concept of BF-algebra was first developed by Jun *et al*. [8] in 1998, which is a generalisation of BCK/BCI/B algebras. Imai and Iséki [6] presented the BCK and BCI classes of abstract algebras in 1966. The concept of B-algebra, a generalization of BCK-algebras, was first proposed by Neggers *et al*. [11] in 2001. In 2010, fuzzy ideals, fuzzy implicative ideals, and fuzzy ideals in BF-algebra were introduced by Satyanarayana *et al*. [17]. Florentine [4] introduced the definition of neutrosophic set. Ibrahim and Kavitha [5] looked into some properties and presented the notation of a neutrosophic ideal of BN-algebra.

In this work, we first introduce the notion of the neutrosophic ideal of BF-algebras and then look into a number of fundamental properties that are connected to it.

2. Preliminaries

In order to better understand the primary findings, we go over the basic definitions of BF-algebra, ideals, and ideal characteristics in this section. We also go over the concepts of neutrosophic sets and neutrosophic ideals of BF-algebra.

Theorem 2.1 ([17])**.** *A non-empty set X with a constant 0 and a binary operation '$*$' that satisfies the following axioms is called a BF-algebra:*
if for all $\alpha_1, \beta_1 \in X$,

(i) $\alpha_1 * \alpha_1 = 0$,

(ii) $\alpha_1 * 0 = \alpha_1$,

(iii) $0 * (\alpha_1 * \beta_1) = \beta_1 * \alpha_1$, *for all* $\alpha_1, \beta_1 \in X$.

Note. The following is true for any BF-algebra X, for all $\alpha_1, \beta_1 \in X$,

(i) If $0 * \alpha_1 = 0 * \beta_1$, then $\alpha_1 = \beta_1$,

(ii) $0 * (\alpha_1 * \beta_1) = (0 * \alpha_1) * (0 * \beta_1)$.

Example 2.2 ([17])**.** Let R be the set of real numbers and let $X = (R, *, 0)$ be the algebra with the operation '$*$' defined by

$$\alpha_1 * \beta_1 = \begin{cases} \alpha_1, & \text{if } \beta_1 = 0, \\ \beta_1, & \text{if } \alpha_1 = 0, \\ 0, & \text{otherwise.} \end{cases}$$

Then, X is BF-algebra.

Definition 2.3 ([17])**.** A non-empty subset S of a BF-algebra X is called a subalgebra of X if

$$\alpha_1 * \beta_1 \in X, \quad \text{for all } \alpha_1, \beta_1 \in S.$$

Definition 2.4 ([17])**.** A non-empty subset A of X is called ideal of X, if it satisfied the following: for all $\alpha_1, \beta_1 \in X$,

(i) $0 \in A$,

(ii) $\alpha_1 * \beta_1 \in A$ and $\beta_1 \in A$ then $\alpha_1 \in A$.

Definition 2.5 ([17])**.** A subset that is not empty an implicative ideal is one that satisfies the following:
for every $\alpha_1, \beta_1, \gamma_1 \in X$,

(i) $0 \in A$,

(ii) $((\alpha_1 * \beta_1) * \gamma_1)$ and $\beta_1 * \gamma_1 \in A$ implies $\alpha_1 * \gamma_1 \in A$.

Definition 2.6 ([17])**.** A non-empty subset A in a BF-algebra X is called a p-ideal of X, if it satisfied the following:
for all $\alpha_1, \beta_1, \gamma_1 \in X$,

(i) $0 \in A$,

(ii) $(\alpha_1 * \gamma_1) * (\beta_1 * \gamma_1) \in A$ and $\beta_1 \in A$ implies $\alpha_1 \in A$.

Definition 2.7 ([17])**.** An BF-algebra X is referred to as the quasi-right alternative BF-algebra, provided that it met the requirement that, for any $\alpha_1,\beta_1 \in X$, $\alpha_1 * (\beta_1 * \beta_1) = (\alpha_1 * \beta_1) * \beta_1$.

Definition 2.8 ([17])**.** An BF-algebra If, for every $\alpha_1,\beta_1,\gamma_1$ in X, it satisfies the condition $(\alpha_1 * \gamma_1) * (\beta_1 * \gamma_1) = (\alpha_1 * \beta_1) * \gamma_1$, then X is a positive implicative BF-algebra.

Definition 2.9 ([17])**.** A BF-algebra's non-empty subset A. If X met the following conditions, it is referred to be a p-ideal of X, for all $\alpha_1,\beta_1,\gamma_1 \in X$,

(i) $0 \in A$,

(ii) $(\alpha_1 * \gamma_1) * (\beta_1 * \gamma_1) \in A$ and $\beta_1 \in A$ implies $\alpha_1 \in A$.

Definition 2.10 ([17])**.** Given two BF-algebras $(X,*,0)$ and $(X',*',0)$, let f be a function from X into X'. The fuzzy set in X defined by if υ is a fuzzy set in f is the preimage of υ under f in X', for all $\alpha_1 \in X$, $f^{-1}(\upsilon)(\alpha_1) = \upsilon(f(\alpha_1))$.

Definition 2.11 ([17])**.** Let $(X,*,0)$ and $(X',*',0')$ be two BF-algebras and let f is a mapping from X into X' is called homomorphism if $f(\alpha_1 * \beta_1) = f(\alpha_1) *' f(\beta_1)$, for all $\alpha_1,\beta_1 \in X$.

Note. A onto homomorphism is called epimorphism.

Definition 2.12 ([4])**.** Let U be the discourse universe. A neutrosophic set N of U is characterized by a truth membership function T_N, an indeterminacy membership function I_N, and a falsity membership function F_N, where T_N, I_N, and F_N are real standard elements of $[0,1]$. It can be written as $N = \{(\alpha_1, T_N(\alpha_1), I_N(\alpha_1), F_N(\alpha_1))/\lambda \in U\}$, $T_N, I_N, F_N \in \,]0,1[$. There is no restriction on the sum of $T_N(\alpha_1)$, $I_N(\alpha_1)$ and $F_N(\alpha_1)$, and thus,

$$0^- \le T_N(\alpha_1) + I_N(\alpha_1) + F_N(\alpha_1) \le 3^+.$$

Definition 2.13 ([5])**.** A neutrosophic set $N = \{T_N, I_N, F_N\}$ of BN-algebra $(A,*,0)$, if N is called an neutrosophic ideal of BN-algebra A, if it satisfied the following:
for all $\alpha_1,\beta_1 \in A$,

(i) $T_N(0) \ge T_N(\alpha_1), I_N(0) \ge I_N(\alpha_1), F_N(0) \le F_N(\alpha_1)$,

(ii) $T_N(\alpha_1) \ge \min\{T_N(\alpha_1 * \beta_1), T_N(\beta_1)\}$; $I_N(\beta_1) \ge \min\{I_N(\alpha_1 * \beta_1), I_N(\beta_1)\}$;
$F_N(\alpha_1) \le \max\{F_N(\alpha_1 * \beta_1), F_N(\beta_1)\}$.

3. Main Results

The main results of the research are presented in this section beginning with an explanation of neutrosophic ideals of BF-algebra with an example. In addition, it contains several basic properties which are related to neutrosophic ideal in BF-algebra.

Definition 3.1. Let $(X,*,0)$ be a BF-algebra, and let N be a non-empty neutrosophic set of a BF-algebra X is called neutrosophic ideal of X, if it satisfies the following:
for all $\alpha_1,\beta_1 \in N$,

(i) $T_N(0) = T_N(\alpha_1)$; $I_N(0) = I_N(\alpha_1)$; $F_N(0) = F_N(\alpha_1)$,

(ii) $T_N(\alpha_1) = \min\{T_N(\alpha_1 * \beta_1), T_N(\beta_1)\}$; $I_N(\alpha_1) = \min\{I_N(\alpha_1 * \beta_1), I_N(\beta_1)\}$;
$F_N(\alpha_1) \le \max\{F_N(\alpha_1 * \beta_1), F_N(\beta_1)\}$.

Example 3.2. Consider a set $A = \{0, \alpha_1, 1\}$. Define a binary operation '$*$' on N given by Table 3.1, and neutrosophic set by Table 3.2 as shown below:

Table 3.1. '$*$' Operation

$*$	0	α_1	1
0	0	α_1	1
α_1	α_1	0	1
1	1	1	0

Table 3.2. Neutrosophic set

$N \backslash A$	0	α_1	1
T_N	0.9	0.7	0.7
I_N	0.8	0.6	0.6
F_N	0.3	0.2	0.2

It is easily verified that N is a neutrosophic ideal of X, and that it satisfies the conditions of Definition 3.1.

Definition 3.3. Let $(X,*,0)$ be a BF-algebra, and let be N a non-empty neutrosophic subset N of X, N is called neutrosophic sub-algebra of X, if it satisfies the following:
for all $\alpha_1, \beta_1 \in X$,

(i) $T_N(\alpha_1 * \beta_1) \geq \min\{T_N(\alpha_1), T_N(\beta_1)\}$;

(ii) $I_N(\alpha_1 * \beta_1) = \min\{I_N(\alpha_1), I_N(\beta_1)\}$;

(iii) $F_N(\alpha_1 * \beta_1) = \max\{F_N(\alpha_1), F_N(\beta_1)\}$.

Definition 3.4. Consider a set $A = \{0, \alpha_1, 1\}$ be a neutrosophic subset of X. Define a binary operation '$*$' on N given by Table 3.3, and neutrosophic set by Table 3.4 as shown below:

Table 3.3. '$*$' Operation

$*$	0	α_1	1
0	0	α_1	1
α_1	α_1	0	1
1	1	1	0

Table 3.4. Neutrosophic set

$N \backslash A$	0	α_1	1
T_N	0.7	0.5	0.5
I_N	0.6	0.4	0.4
F_N	0.2	0.1	0.1

It is easily verified that N is neutrosophic sub-algebra of X, and that it satisfies the conditions of Definition 3.3.

Theorem 3.5. *If a neutrosophic set N in a BF-algebra X is a neutrosophic ideal, then $(\alpha_1 * \beta_1) * \gamma_1 = 0$, for all $\alpha_1, \beta_1, \gamma_1 \in N$ implies*

(i) $T_N(\alpha_1) \geq \min\{T_N(\beta_1), T_N(\gamma_1)\}$;

(ii) $I_N(\alpha_1) = \min\{I_N(\beta_1), I_N(\gamma_1)\}$;

(iii) $F_N(\alpha_1) \leq \max\{F_N(\beta_1), F_N(\gamma_1)\}$.

Proof. Let N be a neutrosophic ideal of X.
Then

$$T_N(\alpha_1) = \min\{T_N((\alpha_1 * \beta_1) * \gamma_1), T_N(\beta_1)\}, \quad \text{for all } \alpha_1, \beta_1, \gamma_1 \in X,$$

$$
\begin{aligned}
&= \min\{\min\{T_N((\alpha_1 * \beta_1) * \gamma_1), T_N(\beta_1)\}, T_N(\beta_1)\} \\
&= \min\{\min\{T_N(0), T_N(\gamma_1)\}, T_N(\beta_1)\} \\
&= \min\{T_N(\gamma_1), T_N(\beta_1)\}.
\end{aligned}
$$

Thus,

$$T_N(\alpha_1) = \min\{T_N(\beta_1), T_N(\gamma_1)\}, \quad \text{for all } \alpha_1, \beta_1, \gamma_1 \in X.$$

Similarly, we can prove for $I_N(\alpha_1)$. Next,

$$
\begin{aligned}
F_N(\alpha_1) &= \max\{F_N((\alpha_1 * \beta_1) * \gamma_1), F_N(\beta_1)\}, \quad \text{for all } \alpha_1, \beta_1, \gamma_1 \in X, \\
&\le \max\{\max\{F_N((\alpha_1 * \beta_1) * \gamma_1), F_N(\beta_1)\}, F_N(\beta_1)\} \\
&= \max\{\max\{F_N(0), F_N(\gamma_1)\}, F_N(\beta_1)\} \\
&= \max\{F_N(\gamma_1), F_N(\beta_1)\}.
\end{aligned}
$$

Thus,

$$F_N(\alpha_1) = \max\{F_N(\beta_1), F_N(z)\}, \quad \text{for all } \alpha_1, \beta_1, \gamma_1 \in X. \qquad \square$$

Theorem 3.6. *A subset N of BF-algebra that is neutrosophic if and only if X is a neutrosophic ideal of X, then for every $\tau \in [0,1]$, $N_\tau = \{\alpha_1/\alpha_1 \in X, T_N(\alpha_1) = \tau\}$ is X's ideal, when $N_\tau \neq \emptyset$.*

Proof. Let N be a neutrosophic subset of a BF-algebra X.
If N is a neutrosophic ideal of X, and $y \in N_\tau$.
Then, from Definition 3.1,

$$
\begin{aligned}
T_N(0) &= T_N(\alpha_1), \quad \text{for all } \alpha_1 \in X \\
&= \tau.
\end{aligned}
$$

Therefore, $0 \in N_\tau$.
Let $\alpha_1 * \beta_1 \in N_\tau$, $\beta_1 \in N_\tau$. Then

$$T_N(\alpha_1 * \beta_1) = \lambda \ \text{ and } \ T_N(\beta_1) = \tau, \quad \text{for all } \alpha_1, \beta_1 \in X.$$

Thus,

$$
\begin{aligned}
T_N(\alpha_1) &= \min\{T_N(\alpha_1 * \beta_1), T_N(\beta_1)\} \\
&= \tau.
\end{aligned}
$$

Therefore, $x \in N_\tau$. Similarly, we can prove for I_N and F_N.
Hence, N_τ is X's ideal.
Conversely, if for every $\tau \in [0,1]$, $N_\tau = \{\alpha_1/\alpha_1 \in X, T_N(\alpha_1) = \tau\}$ is X's ideal, when $N_\tau \neq \emptyset$.

To prove: A neutrosophic subset N of BF-algebra X is neutrosophic ideal of X.
It is enough to prove the following:
for all $\alpha_1, \beta_1 \in X$,

(i) $T_N(0) \ge T_N(\alpha_1)$, and
(ii) $T_N(\alpha_1) \ge \min\{T_N(\alpha_1 * \beta_1), T_N(\beta_1)\}$.

If $T_N(0) = T_N(\alpha_1)$, for all $\alpha_1 \in X$ is not true, that implies

$$T_N(0) < T_N(\alpha_1), \quad \text{for some } \alpha_1 \in X.$$

Then, there exists $\alpha_{10} \in X$ such that $T_N(0) < T_N(\alpha_{10})$.

Let

$$\tau_0 = \frac{T_N(\alpha_{10}) + T_N(0)}{2}$$

$$T_N(0) < \tau_0, \quad 0 = \tau_0 < T_N(\alpha_{10}) = 1$$

$$T_N(\alpha_{10}) > \tau_0.$$

Then,

$$\alpha_{10} \in N_{\tau_0} \quad \text{and} \quad N_{\tau_0} \neq \emptyset.$$

But, N_{τ_0} is an ideal of X, so $0 \in N_{\tau_0}$.
Therefore, from the definition of N_{τ_0}, we have

$$T_N(0) = \tau_0$$

which is contradiction to $T_N(0) < \tau_0$.
Thus,

$$T_N(0) = T_N(\alpha_1), \quad \text{for all } \alpha_1 \in X.$$

If $T_N(\alpha_1) = \min\{T_N(\alpha_1 * \beta_1), T_N(\beta_1)\}$ is not true.
Then, there exists $\alpha_{10}, \beta_{10} \in X$ such that

$$T_N(\alpha_{10}) < \min\{T_N(\alpha_{10} * \beta_{10}), T_N(\beta_{10})\}.$$

Let $\tau_1 \in [0,1]$ such that

$$\tau_1 = \frac{T_N(\alpha_{10}) + \min\{T_N(\alpha_{10} * \beta_{10}), T_N(\beta_{10})\}}{2}.$$

Then

$$T_N(\alpha_{10}) < \tau_1, \quad 0 = \tau_1 < \min\{T_N(\alpha_{10} * \beta_{10}),\ T_N(\beta_{10})\} = 1,$$

$$\Rightarrow \quad \min\{T_N(\alpha_{10} * \beta_{10}),\ T_N(\beta_{10})\} > \tau_1$$

$$\Rightarrow \quad T_N(\alpha_{10} * \beta_{10}) > \tau_1 \text{ and } T_N(\beta_{10}) > \tau_1$$

$$\Rightarrow \quad \alpha_{10} * \beta_{10} \in N_{\tau_1} \text{ and } \beta_{10} \in N_{\tau_1}$$

$$\Rightarrow \quad N_{\lambda_1} \neq \emptyset \text{ and } \alpha_{10} \in N_{\tau_1} \quad (\text{since } N_{\tau_1} \text{ is an ideal})$$

Thus, we have

$$T_N(\alpha_{10}) > \tau_1,$$

which is contradiction to $T_N(\alpha_{10}) < \tau_1$.
Hence,

$$T_N(\alpha_1) = \min\{T_N(\alpha_1 * \beta_1), T_N(\beta_1)\}, \quad \text{for all } \alpha_1, \beta_1 \in X.$$

Similarly, we can prove for I_N and F_N. Therefore, N is an neutrosophic ideal of X. □

Lemma 3.7. *If N is a neutrosophic BF-subalgebra of X, then $T_N(0) \geq T_N(\alpha_1)$, $I_N(0) \geq I_N(\alpha_1)$, and $F_N(0) \leq F_N(\alpha_1)$, for all $\alpha_1 \in X$.*

Proof. Proof of the lemma is straight forward. □

Theorem 3.8. *A neutrosophic subset N of a quasi-right alternative BF-algebra X is neutrosophic sub-algebra if and only if N is a neutrosophic ideal.*

Proof. Let N be neutrosophic sub-algebra of quasi-right alternative BF-algebra X. Then from Lemma 3.7, we have

$$T_N(0) = T_N(\alpha_1), \quad \text{for all } \alpha_1 \in X.$$

For all $\alpha_1 \in X$ and

$$\begin{aligned} T_N(\alpha_1) &= T_N(\alpha_1 * 0) \\ &= T_N(\alpha_1 * (\beta_1 * \beta_1)) \\ &= T_N((\alpha_1 * \beta_1) * \beta_1), \quad \alpha_1 \neq \beta_1 \end{aligned}$$

implies

$$T_N(\alpha_1) = \min\{T_N(\alpha_1 * \beta_1), T_N(\beta_1)\}, \quad \alpha_1 \neq \beta_1, \text{ for all } \alpha_1, \beta_1 \in X.$$

If $\alpha_1 = \beta_1$,

$$T_N(\alpha_1) = \min\{T_N(\alpha_1 * \alpha_1), T_N(\alpha_1)\} = \min\{T_N(0), T_N(\alpha_1)\} = T_N(\alpha_1).$$

Therefore, we have

$$T_N(\alpha_1) = \min\{T_N(\alpha_1 * \beta_1), T_N(\alpha_1)\}, \quad \text{for all } \alpha_1, \beta_1 \in X.$$

Similarly, we can prove for I_N and F_N. Thus, N is a neutrosophic ideal of X.
Conversely, if N is neutrosophic ideal, then

$$\begin{aligned} T_N(\alpha_1 * \beta_1) &= \min\{T_N(\alpha_1 * \beta_1) * \beta_1, T_N(\beta_1)\}, \quad \text{for all } \alpha_1, \beta_1 \in X \\ &= \min\{T_N(\alpha_1 * (\beta_1 * \beta_1), T_N(\beta_1)\}, \quad \alpha_1 \neq \beta_1 \\ &= \min\{T_N(\alpha_1), T_N(\beta_1)\}, \quad \alpha_1 \neq \beta_1 \\ &= \min\{T_N(\alpha_1), T_N(\beta_1)\}, \quad \text{when } \alpha_1 \neq \beta_1. \end{aligned}$$

If $\alpha_1 = \beta_1$,

$$T_N(\alpha_1 * \beta_1) = T_N(0) = T_N(\alpha_1) = \min\{T_N(\alpha_1), T_N(\alpha_1)\}.$$

Therefore, we have

$$T_N(\alpha_1 * \beta_1) = \min\{T_N(\alpha_1), T_N(\beta_1)\}, \quad \text{for all } \alpha_1, \beta_1 \in X.$$

Similarly, we can prove for I_N and F_N. Hence, N is a neutrosophic sub-algebra of X. □

Next, we discuss homomorphism and epimorphism mappings on neutrosophic ideal.

Theorem 3.9. *For BF-algebra $(X, *, 0)$ let f be a homomorphic mapping onto BF-algebra $(X_1, *', 0')$, when N_1 is a neutrosophic ideal of X_1, then X is a neutrosophic ideal of N_1's homomorphic preimage N under f.*

Proof. It is easy to see that

$$T_N(0) = T_N(\alpha_1), \quad \text{for all } \alpha_1 \in X.$$

For all $\alpha_1, \beta_1 \in X$,

$$\begin{aligned} T_N(\alpha_1) &= T_{N_1}(f(\alpha_1)) \\ &= \min\{T_{N_1}(f(\alpha_1 *' \beta_1))\}, T_{N_1}(f(\beta_1))\} \\ &= \min\{T_N(\alpha_1 *' \beta_1), T_N(\beta_1)\}, \quad \text{for all } \alpha_1, \beta_1 \in X. \end{aligned}$$

Similarly, we can prove for I_N and F_N. Hence, N is neutrosophic ideal of X. □

Theorem 3.10. *For BF-algebras, $f : X \to X_1$ be an epimorphism. This means that $f^{-1}(N_1)$ is a neutrosophic ideal of X if N_1 is a neutrosophic ideal in X_1.*

Proof. Let $f : X \to X_1$ be a BF's epimorphism and let N_1 be a neutrosophic ideal in X_1. It is easy to see that

$$f^{-1}\{T_{N_1}(0)\} \geq f^{-1}\{T_{N_1}(\alpha_1)\}, \quad \text{for all } \alpha_1 \in X.$$

For all $x, y \in X$,

$$\begin{aligned} f^{-1}\{T_{N_1}(\alpha_1)\} &= T_{N_1}\{f(\alpha_1)\} \\ &= \min\{T_{N_1}(f(\alpha_1 * \beta_1), T_{N_1}(f(\beta_1)))\} \\ &= \min(f^{-1}\{T_{N_1}(\alpha_1 * \beta_1)\}, f^{-1}\{T_{N_1}(\beta_1)\}). \end{aligned}$$

Similarly, we can prove for I_N and F_N. Thus, $f^{-1}\{T_{N_1}\}$ is a neutrosophic ideal in X. □

Definition 3.11. Let X be a BF-algebra, a neutrosophic subset N of X is called a neutrosophic implicative ideal of X, if it satisfies the following:
for all $\alpha_1, \beta_1, \gamma_1 \in X$,

(i) $T_N(0) = T_N(\alpha_1)$; $I_N(0) = I_N(\alpha_1)$; $F_N(0) = F_N(\alpha_1)$,

(ii) $T_N(\alpha_1 * \gamma_1) = \min\{T_N((\alpha_1 * \beta_1) * \gamma_1), T_N(\beta_1 * \gamma_1)\}$;
$I_N(\alpha_1 * \gamma_1) = \min\{I_N((\alpha_1 * \beta_1) * \gamma_1), I_N(\beta_1 * \gamma_1)$;
$F_N(\alpha_1 * \gamma_1) \leq \max\{F_N((\alpha_1 * \beta_1) * \gamma_1), F_N(\beta_1 * \gamma_1)\}$.

Example 3.12. Consider a set $A = \{0, \alpha_1, 1\}$ be a neutrosophic subset of X. Define a binary operation '$*$' on N given by Table 3.5, and neutrosophic set by Table 3.6 as shown below:

Table 3.5. '$*$' Operation

$*$	0	α_1	1
0	0	α_1	1
α_1	α_1	0	1
1	1	1	0

Table 3.6. Neutrosophic set

N \ A	0	α_1	1
T_N	0.9	0.7	0.7
I_N	0.8	0.6	0.6
F_N	0.3	0.2	0.2

It is easily verified that N is a neutrosophic implicative ideal of X, and that it satisfies the conditions of Definition 3.11.

Definition 3.13. A neutrosophic positive implicative ideal of X is defined as a neutrosophic subset N of X that meets the following:
For all $\alpha_1, \beta_1, \gamma_1 \in X$,

(i) $T_N(0) = T_N(\alpha_1)$; $I_N(0) = I_N(\alpha_1)$; $F_N(0) = F_N(\alpha_1)$,

(ii) $T_N(\alpha_1 * \gamma_1) = \min\{T_N(\alpha_1 * \beta_1) * \gamma_1), T_N(\beta_1 * \gamma_1)\}$;
$I_N(\alpha_1 * \gamma_1) = \min\{I_N(\alpha_1 * \beta_1) * \gamma_1), I_N(\beta_1 * \gamma_1)\}$;
$F_N(\alpha_1 * \gamma_1) \leq \max\{F_N(\alpha_1 * \beta_1) * \gamma_1), F_N(\beta_1 * \gamma_1)\}$.

Example 3.14. Consider a set $A = \{0, \alpha_1, 1\}$ be a neutrosophic subset of X. Define a binary operation '$*$' on N given by Table 3.7, and neutrosophic set by Table 3.8 as shown below:

Table 3.7. '$*$' Operation

$*$	0	α_1	1
0	0	α_1	1
α_1	α_1	0	1
1	1	1	0

Table 3.8. Neutrosophic set

N \ A	0	α_1	1
T_N	0.8	0.8	0.8
I_N	0.6	0.6	0.6
F_N	0.2	0.2	0.2

It is easily verified that N is neutrosophic positive implicative ideal of X, and that it satisfies the conditions of Definition 3.13.

Theorem 3.15. *The neutrosophic ideal N of BF-algebra X must be a neutrosophic implicative ideal. Alternatively, the neutrosophic implicative ideal must be the BF-algebra X positive implicative ideal.*

Proof. Let the implicative ideal N be neutrosophic.
Then from Definition 3.11, we have

(i) $T_N(0) = T_N(\alpha_1)$,

(ii) $T_N(\alpha_1 * \gamma_1) = \min\{T_N((\alpha_1 * \beta_1) * \gamma_1), T_N(\beta_1 * \gamma_1)\}$, for all $\alpha_1, \beta_1, \gamma_1 \in X$.

Put $\gamma_1 = 0$ in (ii), we get

$$T_N(\alpha_1 * 0) = \min\{T_N((\alpha_1 * \beta_1) * 0), T_N(\beta_1 * 0)\} \quad (\text{since } \alpha_1 * 0 = \alpha_1)$$
$$T_N(\alpha_1) = \min\{T_N((\alpha_1 * \beta_1)), T_N(\beta_1)\}.$$

Similarly, we can prove for I_N and F_N.
Hence, N is a neutrosophic ideal.
On the other hand, if X is positive implicative and N is a neutrosophic ideal. Then,

$$(\alpha_1 * z) * (\beta_1 * \gamma_1) = (\alpha_1 * \beta_1) * \gamma_1, \quad \text{for all } \alpha_1, \beta_1, \gamma_1 \in X,$$
$$T_N((\alpha_1 * \gamma_1) * (\beta_1 * \gamma_1)) = T_N((\alpha_1 * \beta_1) * \gamma_1).$$

Since, N is neutrosophic ideal,

$$T_N(0) = T_N(\alpha_1), \quad \text{for all } \alpha_1 \in X$$

and

$$T_N(\alpha_1 * \gamma_1) = T_N\{(\alpha_1 * \gamma_1) * (\beta_1 * \gamma_1)), T_N(\beta_1 * \gamma_1)\},$$
$$T_N(\alpha_1 * \gamma_1) = \min\{T_N((\alpha_1 * \beta_1) * \gamma_1))\}, T_N(\beta_1 * \gamma_1)\} \quad (\text{by Definition 2.8})$$

Similarly, we can prove for I_N and F_N.
Hence, N is a neutrosophic implicative ideal. □

Theorem 3.16. *Let N be a neutrosophic set in X defined for $\alpha_1 \in X$ by and let I be a subset of BF-algebra,*

$$T_N(\alpha_1) = \begin{cases} \mu_0, & \text{if } \alpha_1 \in I, \\ \delta_0, & \text{otherwise}, \end{cases}$$

where $0 = \delta_0 < \mu_0$ in $[0,1]$, if and only if I is an implicative ideal of X, then N is a neutrosophic implicative ideal of X.

Proof. Let N be neutrosophic implicative ideal of X.
Let $\alpha_1 \in I$, since

$$T_N(0) = T_N(\alpha_1) = \mu_0$$
$$\Rightarrow \quad T_N(0) = \mu_0$$
$$\Rightarrow \quad 0 \in I$$

Let $(\alpha_1 * \beta_1) * \gamma_1 \in I$, for all $\alpha_1, \beta_1, \gamma_1 \in X$.
Since

$$\begin{aligned} T_N(\alpha_1 * \gamma_1) &= \min\{T_N((\alpha_1 * \beta_1) * \gamma_1), T_N(\beta_1 * \gamma_1)\} \\ &= \min\{\mu_0, \mu_0\} \\ &= \mu_0 \end{aligned}$$
$$\Rightarrow \quad T_N(\alpha_1 * \gamma_1) = \mu_0,$$

therefore, $\alpha_1 * \gamma_1 \in I$.
I is therefore an implicative ideal of X. On the other hand, suppose that I is X's implicative ideal.

To prove: N is neutrosophic implicative ideal of X.
Let $\alpha_1 \in X$, if $\alpha_1 \in I \Rightarrow T_N(\alpha_1) = \mu_0$ since $0 \in I$, we have $T_N(0) = \mu_0$, therefore $T_N(0) = T_N(\alpha_1)$.
If $\alpha_1 \notin I \Rightarrow T_N(x) = \delta_0$ since $T_N(0) = \mu_0 > \delta_0 = T_N(\alpha_1)$, therefore $T_N(0) > T_N(\alpha_1)$.
Hence, $T_N(0) = T_N(\alpha_1)$, for all $\alpha_1 \in X$.
Let $(\alpha_1 * \beta_1) * \gamma_1 \in X$ and $\beta_1 * \gamma_1 \in X$, for all $\alpha_1, \beta_1, \gamma_1 \in X$.
If $(\alpha_1 * \beta_1) * \gamma_1 \in I$ and $\beta_1 * \gamma_1 \in I$ implies $(\beta_1 * \gamma_1) \in X$ (since I is implicative ideal).
Therefore,

$$T_N(\alpha_1 * \gamma_1) = \mu_0 = \min\{\mu_0, \mu_0\} = \min\{T_N((\alpha_1 * \beta_1) * \gamma_1), T_N(\beta_1 * \gamma_1)\}.$$

Let $(\alpha_1 * \beta_1) * \gamma_1 \notin I$ or $\beta_1 * \gamma_1 \notin I$, then

$$\min\{T_N((\alpha_1 * \beta_1) * \gamma_1), T_N(\beta_1 * \gamma_1)\} = \delta_0 = T_N(\alpha_1 * \gamma_1).$$

Therefore,

$$T_N(\alpha_1 * \gamma_1) = \min\{T_N((\alpha_1 * \beta_1) * \gamma_1), T_N(\beta_1 * \gamma_1)\}, \quad \text{for all } \alpha_1, \beta_1, \gamma_1 \in X.$$

Similarly, we can prove for I_N and F_N. Hence N is neutrosophic implicative ideal of X. □

Definition 3.17. Let N be a neutrosophic subset of BF-algebra X. For $t \in [0,1]$ the set $N_t = \{\alpha_1 \in X / N(\alpha_1) = t\}$ is called a level subset of N.

Definition 3.18. Suppose that X is a BF-algebra. A neutrosophic subset N in X is called a neutrosophic p-ideal if it satisfies the following:
For all $\alpha_1, \beta_1, \gamma_1 \in X$,

(i) $T_N(0) = T_N(\alpha_1)$; $I_N(0) = I_N(\alpha_1)$; $F_N(0) = F_N(\alpha_1)$,

(ii) $T_N(\alpha_1) = \min\{T_N((\alpha_1 * \gamma_1) * (\beta_1 * \gamma_1)), T_N(\beta_1)\}$;
$I_N(\alpha_1) \geq \min\{I_N((\alpha_1 * \gamma_1) * (\beta_1 * \gamma_1)), I_N(\beta_1)\}$;
$F_N(\alpha_1) = \max\{F_N((\alpha_1 * \gamma_1) * (\beta_1 * \gamma_1)), F_N(\beta_1)\}$.

Example 3.19. Consider a set $N = \{0, \alpha_1, 1\}$ be a neutrosophic subset of X. Define a binary operation '$*$' on N given by Table 3.9, and neutrosophic set by Table 3.10 as shown below:

Table 3.9. '$*$' Operation

$*$	0	α_1	1
0	0	α_1	1
α_1	α_1	0	1
1	1	1	0

Table 3.10. Neutrosophic set

N \ A	0	α_1	1
T_N	0.8	0.7	0.7
I_N	0.7	0.5	0.5
F_N	0.2	0.1	0.1

It is easily verified that N is p-ideal of X, and that it satisfies the conditions of Definition 3.18.

Note. Every neutrosophic p-ideal is neutrosophic ideal.

Theorem 3.20. *Suppose that X is a BF-algebra. If a BF-algebra's neutrosophic ideal N is also a neutrosophic p-ideal, then*

$$T_N(\alpha_1) = T_N(0 * (0 * \alpha_1)).$$

Proof. Let X be a BF-algebra and N be a neutrosophic p-ideal.
Then, we have

$$T_N(\alpha_1) = \min\{T_N((\alpha_1 * \gamma_1) * (\beta_1 * \gamma_1)), T_N(\beta_1)\}, \quad \text{for all } \alpha_1, \beta_1, \gamma_1 \in X.$$

Putting $\beta_1 = 0$ and $\gamma_1 = \alpha_1$ then, we get

$$T_N(\alpha_1) = T_N(0 * (0 * \alpha_1)).$$

Similarly, we can prove for I_N and F_N. □

Theorem 3.21. *Assume that BF-algebra X has neutrosophic ideals N and N_1 be such that $N = N_1$ and $T_N(0) = T_{N_1}(0)$ if N is neutrosophic p-ideal of X, then T_{N_1} is also neutrosophic p-ideal of X.*

Proof. Given that N and N_1 be neutrosophic ideals of BF-algebra X such that $N = N_1$ and $T_N(0) = T_{N_1}(0)$. Let N is neutrosophic p-ideal of X. We have to prove N_1 is neutrosophic p-ideal of X. It is enough to show that

$$T_{N_1}(\alpha_1) = T_{N_1}(0 * (0 * \alpha_1)), \quad \text{for each } \alpha_1 \in X.$$

Take $s = 0 * (0 * \alpha_1)$ then

$$\begin{aligned} T_N\{0 * (0 * (\alpha_1 * s))\} &= T_N\{0 * ((0 * \alpha_1) * (0 * s))\} \\ &= T_N\{(0 * (0 * \alpha_1)) * (0 * (0 * s))\} \\ &= T_N\{(0 * (0 * \alpha_1)) * (0 * (0 * (0 * (0 * \alpha_1)))\} \\ &= T_N\{(0 * (0 * \alpha_1)) * (0 * (0 * \alpha_1))\} = 0 \quad (\text{since } \alpha_1 * \alpha_1 = 0) \end{aligned}$$

Hence,

$$T_N(0 * (0 * (\alpha_1 * s)) = T_N(0) = T_{N_1}(0).$$

Since N is a neutrosophic p-ideal of X and by using Theorem 3.20, we get

$$T_N(\alpha_1 * s) = T_N(0 * (0 * (\alpha_1 * s)) = T_{N_1}(0)$$

implies

$$T_{N_1}(\alpha_1 * s) = T_N(\alpha_1 * s) = T_{N_1}(0) = T_{N_1}(s).$$

Since T_{N_1} is neutrosophic ideal of X, we have

$$T_{N_1}(\alpha_1) = T_{N_1}(0 * (0 * \alpha_1)).$$

Similarly, we can prove for I_N and F_N. Hence, T_{N_1} is neutrosophic p-ideal of X. □

Theorem 3.22. *A BF-algebra X's neutrosophic set N is a neutrosophic p-ideal of X if and only if N_t is either empty or a p-ideal of X for every 't' in* $[0,1]$.

Proof. Let N be a neutrosophic p-ideal in X and $N_t \neq \varphi$, for $t \in [0,1]$.
Since

$$T_N(0) = T_N(\alpha_1) = t, \quad \text{for all } t \in [0,1].$$

Thus, $0 \in N_t$. Now let

$$(\alpha_1 * \gamma_1) * (\beta_1 * \gamma_1) \in N_t \text{ and } \beta_1 \in N_t$$

$$\Rightarrow \quad T_N((\alpha_1 * \gamma_1) * (\beta_1 * \gamma_1)) = t \text{ and } T_N(\beta_1) = t$$

by (ii) of Definition 3.18, we have

$$T_N(\alpha_1) = \min\{T_N((\alpha_1 * \gamma_1) * (\beta_1 * \gamma_1)), T_N(\beta_1)\} = t.$$

Hence $\alpha_1 \in N_t$, thus N_t is a p-ideal of X.
Conversely, suppose that for each $t \in [0,1]$, N_t is either empty or a p-ideal of X. For all $\alpha_1 \in X$, let $T_N(\alpha_1) = t$, then $\alpha_1 \in N_t$ since $N_t \neq \emptyset$ is a p-ideal of X, so $0 \in N_t$ implies $T_N(0) = t = T_N(\alpha_1)$ implies $T_N(0) = T_N(\alpha_1)$, for all $\alpha_1 \in X$.
Now, we have to prove that N satisfies

$$T_N(\alpha_1) = \min\{T_N((\alpha_1 * \gamma_1) * (\beta_1 * \gamma_1)), T_N(\beta_1)\},$$

for all $\alpha_1, \beta_1, \gamma_1 \in X$. Suppose

$$T_N(\alpha_1) = \min\{T_N((\alpha_1 * \gamma_1) * (\beta_1 * \gamma_1)), T_N(\beta_1)\}$$

is not true.
Then, there exists $\alpha_{10}, \beta_{10}, \gamma_{10} \in X$ such that

$$T_N(\alpha_{10}) < \min\{T_N((\alpha_{10} * \gamma_{10}) * (y_0 * \gamma_{10})) * T_N(y_0)\}.$$

Let

$$t_0 = \frac{1}{2}[T_N(\alpha_{10}) + \min\{T_N((\alpha_{10} * \gamma_{10}) * (\beta_{10} * \gamma_{10})) * T_N(\beta_{10})\}]$$

$$\Rightarrow \quad T_N(\alpha_{10}) < t_0 < \min\{T_N((\alpha_{10} * \gamma_{10}) * (\beta_{10} * \gamma_{10})) * T_N(\beta_{10})\}.$$

Hence $(\alpha_{10} * z_0) * (\beta_{10} * \gamma_{10}) \in N_{t_0}$ and $\beta_{10} \in N_{t_0}$ but $\alpha_{10} \notin N_{t_0}$.
Thus, N_{t_0} is not a p-ideal of X. This is the contradiction.
Similarly, we can prove for I_N and $F_{N.}$
Therefore, N is neutrosophic p-ideal of X. □

4. Conclusions

This paper begins by considering the notions of the neutrosophic ideal of BF-algebra with suitable illustrations, and we investigate several basic properties related to the neutrosophic ideal in BF-algebra. Additionally, we discuss and examine some of the features of neutrosophic implicative ideals, neutrosophic positive implicative ideals, and neutrosophic ideals in BF-algebras. In future work, we will consider the concepts of neutrosophic implicative ideal and neutrosophic completely closed ideal in BH and BE-algebras and investigate some of their properties.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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