



Supersubdivision in the Context of AUM Block Labeling for Coconut Tree and Double Coconut Tree Graph

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Abstract. A graph G with p number of vertices, q number of edges and b number of blocks, where $p, q, b \geq 1$. The set of vertices, edges and blocks are denoted by $V(G) = \{v_1, v_2, \dots, v_p\}$, $E(G) = \{e_1, e_2, \dots, e_q\}$ and $B(G) = \{b_1, b_2, \dots, b_n\}$, respectively. The graph G admits AUM block labeling if there exists a bijection $f : V(G) \rightarrow \mathbb{Z}^+$ induced from f by $f^*(uv) = f(u) * f(v)$ and $f^{**}(B_j) = B(G) \rightarrow \mathbb{Z}^+$ then $f^{**}(B_j) = \sum_{i=1}^n f(v_{ij}) + \sum_{i=1}^n f^*(e_{ij})$ and $f^{**}(B_i) \neq f^{**}(B_j)$. In this paper, we prove that the super subdivision of coconut tree and double coconut tree graph admits AUM block labeling.

Keywords. AUM block labeling, Super subdivision, Coconut tree graph, Double coconut tree graph

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1. Introduction

Graph labeling, an interesting branch of Mathematics, serves as a powerful tool in mathematical modeling for a wide range of applications such as coding theory, topology, psychology, medical, astronomy, geomechanics, data science and communication network. Also, graph theoretical

methods are employed in the field of computational chemistry to search datasets of chemical compounds.

AUM block labeling was introduced by Maheswari and Purnalakshimi [3] in 2022. Obtaining vertex labels, edge labels, or block labels for all graphs under certain constraints remains an open challenge for researchers. Significant contributions to the study of AUM block labeling of graphs have made by Maheswari and Purnalakshimi [3, 4]. Maheswari and Azhagarasi [5] studied AUM block sum labeling for star and bistar graphs, where the blocks are labeled using sum of the vertex and edge labels.

The concept of supersubdivision was introduced by Sethuraman and Selvaraju [8] in 2001. They proved that every supersubdivision of a path is graceful, and that every cycle has some supersubdivision that is graceful.

In this paper, a new AUM block labeling of supersubdivision of coconut tree and double coconut tree graph is determined.

2. Preliminaries

For definitions and terminologies, one can refer Bondy and Murty [1].

Definition 2.1 ([2]). A coconut tree graph is a graph that is created by attaching m new pendant edges to the end vertex of a path P_n .

Definition 2.2 ([2]). A double coconut tree graph is a graph that's created by attaching n pendant edges to the end vertices of a path graph P_n .

Definition 2.3 ([5]). The supersubdivision of a graph G denoted by $SS(G)$ is obtained from G by replacing every edge of G by a complete bipartite graph $k_{2,m}$ (where m is an integer).

3. Main Results

Theorem 3.1. *The supersubdivision of a coconut tree graph admits AUM block labeling.*

Proof. Let G be the supersubdivision of the coconut tree graph. Let w_1 be the vertex of the star connected to the path. Let u_i for $1 \leq i \leq n$ be the pendent vertices of the star. Let the u_j^i for $1 \leq j \leq m$ and $1 \leq i \leq n$ be the vertices of the supersubdivision of the star. Let w_i for $2 \leq i \leq s$ be the other vertices of the path. Let w_j^i for $1 \leq j \leq m$ and $1 \leq i \leq (s-1)$ be the vertices of the supersubdivision of the path. The generalized supersubdivision of a coconut tree graph is shown in Figure 1.

Define the number of vertices and edges as follow:

$$|V(G)| = 2n + mn + s + m(s-1),$$

$$|E(G)| = 2mn + 2m(s-1).$$

Define the vertex labeling $f : V(G) \rightarrow \{1, 2, 3, \dots, p\}$,

$$f(w_1) = 1$$

$$f(u_i) = 1 + mn + i; \quad 1 \leq i \leq n,$$

and second star respectively. Let w_1 and w_s be the first and last vertices of the path, and let w_i for $2 \leq i \leq (s-1)$ be the other vertices of the path. Let w_j^i for $1 \leq j \leq m$ and $1 \leq i \leq (s-1)$ are the vertices of the supersubdivision of the path. The generalized supersubdivision of a double coconut tree graph is shown in Figure 3.

Define the number of vertices and edges as follow:

$$|V(G)| = 2nm + 2n + s + m(s-1),$$

$$|E(G)| = 4mn + 2m(s-1).$$

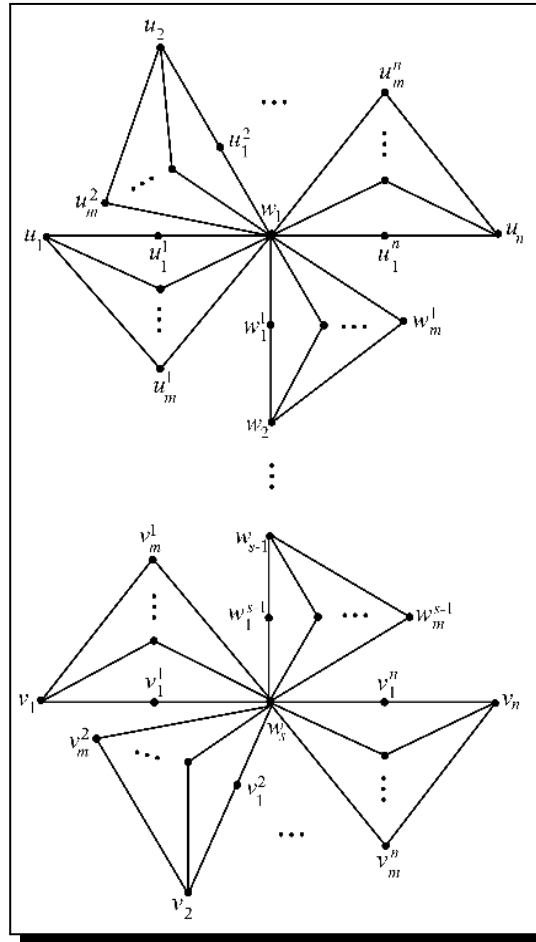


Figure 3. AUM block labeling of SSD of double coconut tree graph

Define the vertex labeling $f : V(G) \rightarrow \{1, 2, 3, \dots, p\}$,

$$f(w_1) = 1,$$

$$f(w_s) = 2,$$

$$f(u_i) = 2 + 2mn + m(s-1) + i; \quad 1 \leq i \leq n,$$

$$f(u_j^i) = 2 + j + m(i-1); \quad 1 \leq i \leq n, 1 \leq j \leq m,$$

$$f(w_i) = 2mn + n + m(s-1) + i + 1; \quad 2 \leq i \leq s-1,$$

$$f(w_j^i) = 2 + mn + j + m(i-1); \quad 1 \leq i \leq s-1, 1 \leq j \leq m,$$

$$f(v_i) = s + 2mn + m(s-1) + n + i; \quad 1 \leq i \leq n,$$

$$f(v_j^i) = 2 + mn + m(s-1) + j + m(i-1); \quad 1 \leq i \leq n, 1 \leq j \leq m. \quad (3.4)$$

Define the edge labeling $f^* : E(G) \rightarrow \mathbb{Z}^+$,

$$\begin{aligned} f^*(w_1 u_j^i) &= 2 + j + m(i-1); & 1 \leq i \leq n, 1 \leq j \leq m, \\ f^*(u_i u_j^i) &= (2 + 2mn + m(s-1) + i)(2 + j + m(i-1)); & 1 \leq i \leq n, 1 \leq j \leq m, \\ f^*(w_s v_j^i) &= 2(2 + mn + m(s-1) + j + m(i-1)); & 1 \leq i \leq n, 1 \leq j \leq m, \\ f^*(v_i v_j^i) &= (s + 2mn + m(s-1) + n + i)(2 + mn + m(s-1) + j + m(i-1)); & 1 \leq i \leq n, 1 \leq j \leq m, \\ f^*(w_i w_j^i) &= (2mn + n + m(s-1) + i + 1)(2 + mn + j + m(i-1)); & 1 \leq i \leq s-1, 1 \leq j \leq m. \end{aligned} \quad (3.5)$$

Define the block by adding the vertices and edges $f^{**} : B(G) \rightarrow \mathbb{Z}^+$,

$$\begin{aligned} f^{**}(B_i) &= f(u_i) + \sum_{j=1}^m f(u_j^i) + f(w_1) + \sum_{j=1}^m f^*(w_1 u_j^i) + \sum_{j=1}^m f^*(u_j^i u_i); & 1 \leq i \leq n, \\ f^{**}(B_j) &= f(w_i) + f(w_{i+1}) + \sum_{j=1}^m f(w_j^i) + \sum_{j=1}^m f^*(w_i w_j^i) + \sum_{j=1}^m f^*(w_{i+1} w_j^{i+1}); & 1 \leq i \leq s, \\ f^{**}(B_r) &= f(v_i) + \sum_{j=1}^m f(v_j^i) + f(w_s) + \sum_{j=1}^m f^*(w_s v_j^i) + \sum_{j=1}^m f^*(v_j^i v_i); & 1 \leq i \leq n. \end{aligned} \quad (3.6)$$

Thus, the supersubdivision of double coconut tree graph admits AUM block labeling. The illustration of Theorem 3.2 is shown in Figure 4.

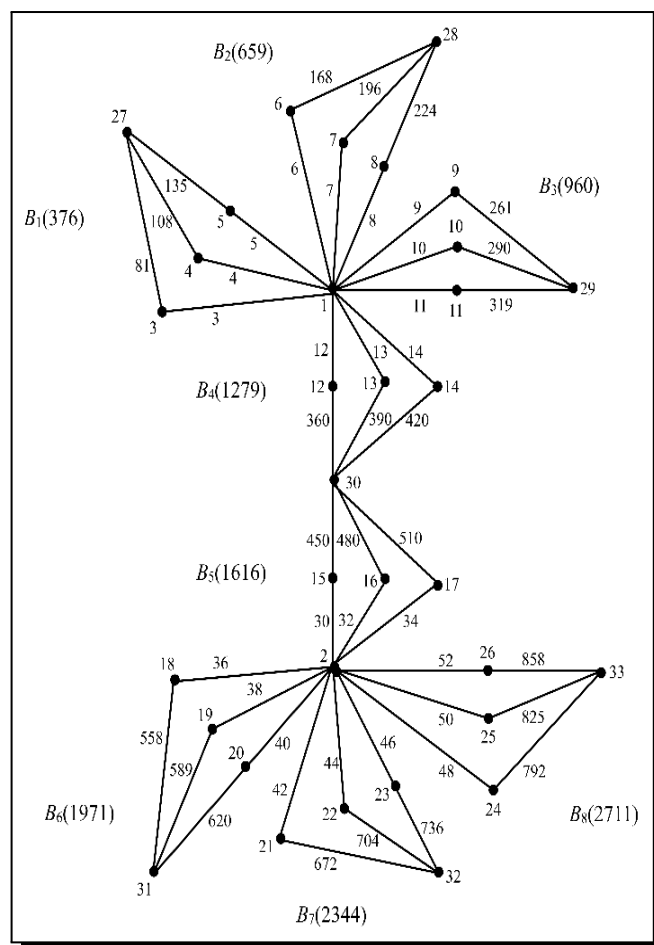


Figure 4. AUM block labeling of SSD of double coconut tree graph for $n = 3$, $s = 3$, $m = 3$

4. Conclusion

Super subdivision of coconut tree and double coconut tree graph admits AUM block labeling is proved. MATLAB program is developed for labeling the graph coconut tree and double coconut tree graph, which has applications in many other fields including medical science, health science, chemical science, physical science, computer science.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

- [1] J. A. Bondy and U. S. R. Murty, *Graph Theory with Applications*, North-Holland, New York, x + 264 pages (1976).

- [2] J. A. Gallian, A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics* **DS6v25** (2022), 1 – 623, URL: <https://www.combinatorics.org/files/Surveys/ds6/ds6v25-2022.pdf>.
- [3] A. U. Maheswari and A. A. Purnalakshimi, AUM Block lablling for star, bi-star and sunlet graph, *Neuro Quantology* **20**(9) (2022), 3967 – 3975.
- [4] A. U. Maheswari and A. S. Purnalakshimi, AUM block labeling for snake graphs and dutch windmill graph, *Neuro Quantology* **20**(9) (2022), 414 – 421.
- [5] A. U. Maheswari and S. Azhagarasi, AUM block sum labelling for some special graphs, *International Journal of Mechanical Engineering* **7**(Special issue 5) (2022), 102 – 110.
- [6] B. Mondal and K. De, An overview applications of graph theory in real field, *International Journal of Scientific Research in Computer Science, Engineering and Information Technology* **2**(5) (2017), 751 – 759.
- [7] A. Rosa, On certain valuations of the vertices of a graph, in: *Theory of Graphs* (An International Symposium, Rome, Italy, pp. 349 – 355, (1966).
- [8] G. Sethuraman and P. Selvaraju, Gracefulness of arbitrary supersubdivisions of graphs, *Indian Journal of Pure and Applied Mathematics* **32**(7) (2001), 1059 – 1064.

