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Polarimetric Effect of Photon-Scalar Mixing in Astrophysical Situations

Research Article

Avijit K. Ganguly

Department of Physics, MMV, Banaras Hindu University, Varanasi 221005, Uttar Pradesh, India
avijitk@hotmail.com

Abstract. Photon (γ) scalar (ϕ) mixing in field theory involves fields those transform differently under Lorentz transformation. Usually, this takes place through loops involving bosonic fields of unified theories with that of electromagnetism. This type of interaction makes the medium dichoric, and the same induces optical activity, i.e., rotation of plane of polarization of electromagnetic field. In this paper, we study propagation of synchrotron photons in of a magnetized compact star magnetosphere, assuming ϕ γ interaction. Ways to look for polarimetric signatures of scalar photon coupling from the compact star electromagnetic signals is explored.

Keywords. Polarimetry, Dichroism, Axion

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1. Introduction

Photon (γ) scalar $\phi(x)$ or pseudoscalar-photon mixing originates in many theories beyond the framework of standard model of particle physics. Usually these are the unified theories of electromagnetic field and gravity [4]. The scalars which are involve can be moduli fields of string theory or Klien-Kaluza particles from extra dimension, scalar component of the gravitational multiplet in extended supergravity models etc. [1–3, 5, 7–10, 14–17]. The main emphasis of such models had been unification of four types of interactions, however the same idea has also used to solve the dark energy and dark matter problems of the universe [15].

Keeping this in view, in this note, initially, we have studied the optical signatures of dim-5, $\phi F_{\mu\nu}F^{\mu\nu}$ interaction in an ambient magnetic field of strength $\sim 10^{13}$ Gauss.

Following this, we have tried to understand some relevant issues– related to emission energy and altitude [3], [14], for magnetized, rotating cold, compact astrophysical objects like White Dwarf (WD) and Neutron Star (NS)– that affects the produced spectra of electromagnetic radiations (EM) there. Lastly, we have pointed out the potential of the later, in modifying the spectral signatures of the usual dim-5, $\phi F_{\mu\nu}F^{\mu\nu}$ interaction–from similar astrophysical situations of interest. We would like to mention here, that, this model of ours is a toy model. The purpose of its construction is to contrast our observations with those considered in [7]. A detailed analysis of the same for realistic situations is beyond the scope of this work and has to be performed elsewhere.

2. Action for Coupled Photon-Scalar System

Here we are working in flat four dimensional space-time. The action for this coupled photon-scalar system in flat four dimensional space-time is given by

$$S = \int d^4x \left[\frac{1}{2}(\partial_\mu)\phi(\partial^\mu\phi) - \frac{1}{4}g_{\phi\gamma\gamma}\phi F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right], \tag{2.1}$$

where $g_{\gamma\gamma\phi}$ is coupling parameter between scalar and photon field. $F_{\mu\nu}$ is the field strength tensor.

2.1 Equations of Motion for Photon-scalar System

The *equations of motion* (EOM) can be derived by using variational princple from action, which is given in eq. (2.1). EOM for photon field is given as

$$\partial^2\psi - g_{\phi\gamma\gamma}B^2\phi\nabla_\perp^2 = 0. \tag{2.2}$$

The second equation of motion for scalar field can be found, by taking the variation of $\delta\phi$, i.e., given by

$$\partial^2\phi + g_{\phi\gamma\gamma}\psi = 0, \tag{2.3}$$

where variable, $\psi = \left(\frac{f^{\nu\lambda}\tilde{F}_{\nu\lambda}}{2}\right)$ is one degree of freedom of photon. Equations (2.2) and (2.3) show that, the photon field ψ coupled with ϕ and vice versa. Since in case of photon-scalar system only one polarization state of photon coupled with scalar field and other it freely propagates.

We would assume that $\nabla_\perp^2 = \nabla_x^2 + \nabla_y^2$ and other $\partial^2 = \partial_t^2 - \nabla^2$, where $\nabla^2 = \nabla_x^2 + \nabla_y^2 + \nabla_z^2$. Substituting the value of ∇^2, we get the following expression:

$$\partial^2 = \partial_t^2 - \partial_z^2 - \nabla_\perp^2. \tag{2.4}$$

Taking the Fourier transformation along the time component and perpendicular component, i.e., ∂_t and $\nabla_\perp$. The equations of motion take following form:

$$(-\omega^2 - \partial_z^2 + K_\perp^2)\psi - g_{\phi\gamma\gamma}B^2K_\perp^2\phi = 0 \tag{2.5}$$

and other equation is

$$(-\omega^2 - \partial_z^2 + K_\perp^2)\phi + g_{\phi\gamma\gamma}\psi = 0. \tag{2.6}$$

The equation does not look symmetric because the dimension of ϕ and the dimensions of ψ are different. To bring the same in symmetric form, we multiply by $BK_\perp$ in ϕ equation. The equations of motion are defined as:

$$(-\omega^2 - \partial_z^2 + K_\perp^2)\psi - g_{\phi\gamma\gamma}BK_\perp(BK_\perp\phi) = 0, \tag{2.7}$$

$$(-\omega^2 - \partial_z^2 + K_\perp^2)(BK_\perp\phi) + g_{\phi\gamma\gamma}BK_\perp\psi = 0, \tag{2.8}$$

where, we assume that $\mathscr{B}K_\perp\phi = \Phi$. The new equations of motion take the following form:

$$(K_\perp^2 - \omega^2)\psi - \partial_z^2\psi - g_{\phi\gamma\gamma}BK_\perp\Phi = 0$$

and

$$(K_\perp^2 - \omega^2)\Phi - \partial_z^2\Phi + g_{\phi\gamma\gamma}BK_\perp\psi = 0.$$

We can write the perpendicular component of propagation vector, i.e., $K_\perp = K\sin\theta \simeq \omega\sin\theta$, where θ is the angle between the magnetic field B and the propagation direction K. Therefore, $(K_\perp^2 - \omega^2) = -\omega^2\cos\theta$.

In terms of these, we can rewrite the equations of motion for photon field

$$-\partial_z^2\psi - \omega^2\cos\theta\psi - g_{\phi\gamma\gamma}BK_\perp\Phi = 0$$

and for scalar field is:

$$-\partial_z^2\Phi - \omega^2\cos\theta\Phi - g_{\phi\gamma\gamma}BK_\perp\psi = 0.$$

We assume that $\partial_z^2 = \nabla_\parallel^2$. Therefore, both equations of motion are written in following form:

$$\nabla_\parallel^2\psi + g_{\phi\gamma\gamma}B_\perp\omega\Phi + \omega^2\cos\theta\psi = 0 \tag{2.9}$$

and

$$\nabla_\parallel^2\Phi - g_{\phi\gamma\gamma}B_\perp\omega\psi + \omega^2\cos\theta\Phi = 0 \tag{2.10}$$

We can write both equations of motion in the matrix form:

$$\left[\nabla_\parallel^2\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - \begin{pmatrix}0 & -g_{\phi\gamma\gamma}B_\perp\omega\\ g_{\phi\gamma\gamma}B_\perp\omega & 0\end{pmatrix} + \begin{pmatrix}\omega^2\cos^2\theta & 0\\ 0 & \omega^2\cos^2\theta\end{pmatrix}\right]\left[\begin{pmatrix}\psi\\ \Phi\end{pmatrix}\right] = 0 \tag{2.11}$$

where, we define a matrix is $U = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}$ and its inverse is $\mathrm{U}^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}$. By use of these matrices we take the similrity transformation of eq. (2.11) and diagonalize it, we get a new matrix in following form:

$$\begin{pmatrix}\nabla_\parallel^2 + g_{\phi\gamma\gamma}B_\perp\omega + \omega^2\cos^2\theta & 0\\ 0 & \nabla_\parallel^2 - g_{\phi\gamma\gamma}B_\perp\omega + \omega^2\cos^2\theta\end{pmatrix}\begin{bmatrix}\Phi+\psi\\ \Phi-\psi\end{bmatrix} = 0. \tag{2.12}$$

If we denote the two mixing modes $(\Phi+\psi)$ and $(\Phi-\psi)$ by X and Y respectively, the two equations of motion turn out to be:

$$(\partial_z^2 + K_1^2)X = 0 \tag{2.13}$$

and

$$(\partial_z^2 + K_2^2)Y = 0, \tag{2.14}$$

where $K_1^2 = \omega^2\cos^2\theta + g_{\phi\gamma\gamma}B_\perp\omega$ and $K_2^2 = \omega^2\cos^2\theta - g_{\phi\gamma\gamma}B_\perp\omega$.

2.2 Solotions of Wave Equations

The solutions for the dynamical degrees of freedom is written as:

$$X = A_1 e^{iK_1.x},$$
$$Y = A_2 e^{iK_2.x}$$

or we can write it

$$\Phi + \psi = A_1 e^{iK_1.x}, \tag{2.15}$$
$$\Phi - \psi = A_2 e^{iK_2.x}, \tag{2.16}$$

where A_1 and A_2 are constant. We can solve eqs. (2.15) and (2.16), we get the value of $\psi(x)$ and $\Phi(x)$. The expression are given as:

$$\Phi(x) = \frac{1}{2} A_1 e^{iK_1.x} + \frac{1}{2} A_2 e^{iK_2.x},$$
$$\psi(x) = \frac{1}{2} A_1 e^{iK_1.x} - \frac{1}{2} A_2 e^{iK_2.x},$$

where we consider the suitable boundary condition $\Phi(0,0) = 0$. With this boundary condition, we get $\frac{A_1}{A_2} = -1$, i.e., $A_1 = -A_2$ and the equation takes new form:

$$\Phi(x) = \frac{A_1}{2}(e^{iK_1.x} - e^{iK_2.x}), \tag{2.17}$$
$$\psi(x) = \frac{A_1}{2}(e^{iK_1.x} + e^{iK_2.x}). \tag{2.18}$$

Since Parallel and perpendicular components of amplitudes of the electromagnetic radiation, from synchrotron radiation, are given as,

$$A_{\parallel} = \frac{3e\gamma}{4\pi^{\frac{3}{2}} R} \xi K_{\frac{2}{3}}(\xi),$$
$$A_{\perp} = \frac{3e\gamma}{8\sqrt{2}\pi^{\frac{3}{2}} R} \xi K_{\frac{1}{3}}(\xi),$$

where $K_{\frac{1}{3}}$ and $K_{\frac{2}{3}}$ are the fractional order modified Bessel's functions. Here the parallel and perpendicular component of amplitude is opposite to the parallel and perpendicular component of polarization state of photon. At $x = 0$ the parallel component of polarization of photon is equal to perpendicular component of amplitude, i.e., $A_1 = 2A_{\perp}$. Therefore, the expressions for solutions of in terms of modified Bessel's functions are:

$$\psi(x) = \frac{3e\gamma}{4\sqrt{2}\pi^{\frac{3}{2}} R} \xi K_{\frac{1}{3}}(\xi)(e^{iK_1.x} + e^{iK_2.x}) \tag{2.19}$$

and the other

$$\Phi(x) = \frac{3e\gamma}{4\sqrt{2}\pi^{\frac{3}{2}} R} \xi K_{\frac{1}{3}}(\xi)(e^{iK_1.x} - e^{iK_2.x}) \tag{2.20}$$

Since the photon has two degree of freedom i.e ψ and $\tilde{\psi}$, which is the parallel and perpendicular component of polarization state. $\tilde{\psi}$ does not mix with ϕ, Therefore the equation of motion for $\tilde{\psi}$ is:

$$\partial^2 \tilde{\psi} = 0 \tag{2.21}$$

or we can write

$$(\partial_t^2 - \partial_z^2 - \nabla_\perp^2)\tilde{\psi} = 0 .$$

The equation of motion is written in the momentum space by taking Fourier transformation. The equation of motion is:

$$(-\omega^2 - \partial_z^2 - K_\perp^2)\tilde{\psi} = 0$$

on putting the value of $K_\perp^2 \simeq \omega^2 \sin^2\theta$ in above equation, the modified equation of motion is:

$$(\partial_z^2 + K^2)\tilde{\psi} = 0 , \tag{2.22}$$

where $K^2 = \omega^2 \cos^2\theta$. The solution for $\tilde{\psi}$ is defined as:

$$\tilde{\psi}(x) = A_3 e^{iK.x} \tag{2.23}$$

Apply the boundary condition at $x = 0$, $\tilde{\psi}(x) = A_3 = A_\parallel$. Therefore, the modified solution is written as:

$$\tilde{\psi}(x) = \frac{3e\gamma}{4\pi^{\frac{3}{2}} R} \xi K_{\frac{2}{3}}(\xi) e^{iKx} . \tag{2.24}$$

3. Polarimetric Variables for Coupled Photon-Scalar System

We can find the polarimetric variables for coupled photon-scalar system by using solutions from eqs. (2.24) and (2.19). Those are given below.

3.1 Stokes Parameters

The definition of stokes parameters is defined as:

$$\left.\begin{aligned}
I &= \langle \tilde{\psi}^*(x)\tilde{\psi}(x)\rangle + \langle \psi^*(x)\psi(x)\rangle , \\
Q &= \langle \tilde{\psi}^*(x)\tilde{\psi}(x)\rangle - \langle \psi^*(x)\psi(x)\rangle , \\
U &= 2Re\langle \tilde{\psi}^*(x)\psi(x)\rangle , \\
V &= 2Im\langle \tilde{\psi}^*(x)\psi(x)\rangle .
\end{aligned}\right\} \tag{3.1}$$

In eq. (3.1), I denotes total intensity of the electromagnetic radiation. Q and U represent the linear polarization and V is the circular polarization of the electromagnetic radiation.

$$\left.\begin{aligned}
I(\omega,z) &= \frac{9e^2\gamma^2}{16\pi^3R^2}\xi^2 K_{\frac{2}{3}}^2(\xi) + \frac{9e^2\gamma^2}{32\pi^3R^2}\xi^2 K_{\frac{1}{3}}^2(\xi)[2+2\cos(k_1-k_2)x], \\
Q(\omega,z) &= \frac{9e^2\gamma^2}{16\pi^3R^2}\xi^2 K_{\frac{2}{3}}^2(\xi) - \frac{9e^2\gamma^2}{32\pi^3R^2}\xi^2 K_{\frac{1}{3}}^2(\xi)[2+2\cos(k_1-k_2)x], \\
U(\omega,z) &= \frac{9e^2\gamma^2}{8\sqrt{2}\pi^3R^2}\xi^2 K_{\frac{1}{3}} K_{\frac{2}{3}}(\xi)[\cos(k_1-k)x + \cos(k_2-k)x], \\
V(\omega,z) &= \frac{9e^2\gamma^2}{8\sqrt{2}\pi^3R^2}\xi^2 K_{\frac{1}{3}} K_{\frac{2}{3}}(\xi)[\sin(k_1-k)x + \sin(k_2-k)x].
\end{aligned}\right\} \tag{3.2}$$

where, we put the value of K, K_1, K_2 and $\theta = \frac{\pi}{4}$, the modified expression for Stokes parameters turn out to be into new forms, given as:

$$\left.\begin{aligned}
I(\omega,z) &= \frac{9e^2\gamma^2}{16\pi^3R^2}\xi^2K^2_{\frac{2}{3}}(\xi) + \frac{9e^2\gamma^2}{16\pi^3R^2}\xi^2K^2_{\frac{1}{3}}(\xi)\left[1+\cos\left[gB\left(1+\frac{g^2B^2}{2\omega^2}\right)x\right]\right]\\
Q(\omega,z) &= \frac{9e^2\gamma^2}{16\pi^3R^2}\xi^2K^2_{\frac{2}{3}}(\xi) - \frac{9e^2\gamma^2}{16\pi^3R^2}\xi^2K^2_{\frac{1}{3}}(\xi)\left[1+\cos\left[gB\left(1+\frac{g^2B^2}{2\omega^2}\right)x\right]\right]\\
U(\omega,z) &= \frac{9e^2\gamma^2}{8\sqrt{2}\pi^3R^2}\xi^2K_{\frac{1}{3}}K_{\frac{2}{3}}(\xi)\cos\left[\frac{gB}{2}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\omega+\frac{g^2B^2}{4\sqrt{2}\omega}\right)x\right]\cos\left[\frac{gB}{2}\left(1+\frac{g^2B^2}{2\omega^2}\right)x\right]\\
V(\omega,z) &= -\frac{9e^2\gamma^2}{8\sqrt{2}\pi^3R^2}\xi^2K_{\frac{1}{3}}K_{\frac{2}{3}}(\xi)\sin\left[\frac{gB}{2}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\omega+\frac{g^2B^2}{4\sqrt{2}\omega}\right)x\right]\cos\left[\frac{gB}{2}\left(1+\frac{g^2B^2}{2\omega^2}\right)x\right].
\end{aligned}\right\} \tag{3.3}$$

3.2 Polarization and Ellipticity Angle

The polarization and the ellipticity angles for coupled $\gamma\phi$ system can be obtained from the stokes parameters. The polarization angle Ψ define as,

$$\Psi = \frac{1}{2}\tan^{-1}\left[\frac{\mathbf{U}(\omega,z)}{\mathbf{Q}(\omega,z)}\right]. \tag{3.4}$$

Similarly, the ellipticity angle, which is denoted by χ is given by

$$\chi = \frac{1}{2}\tan^{-1}\left[\frac{\mathbf{V}(\omega,z)}{\sqrt{\mathbf{Q}^2(\omega,z)+\mathbf{U}^2(\omega,z)}}\right]. \tag{3.5}$$

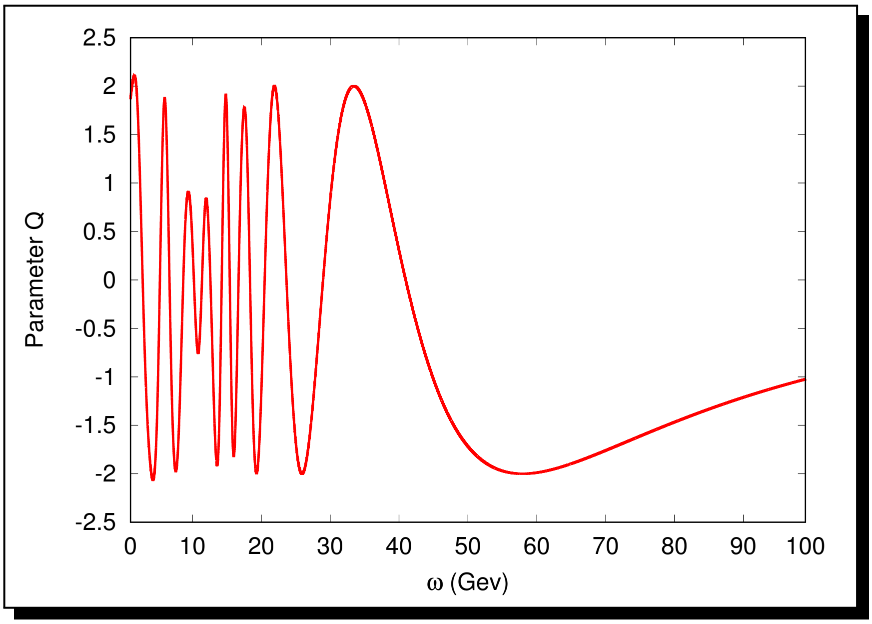

Figure 1. Stokes parameter $\mathbf{Q}$ vs Energy

3.3 Total Degree of Polarization

The degree of linear polarization can be expressed, in terms of stokes parameters as,

$$P_{lin} = \frac{\sqrt{\mathbf{Q}^2(\omega,z)+\mathbf{U}^2(\omega,z)}}{\mathbf{I}(\omega,z)}, \tag{3.6}$$

and other is total degree of polarization for coupled system is given as

$$P_{tot} = \frac{\sqrt{\mathbf{Q}^2(\omega,z)+\mathbf{U}^2(\omega,z)+\mathbf{V}^2(\omega,z)}}{\mathbf{I}(\omega,z)}. \tag{3.7}$$

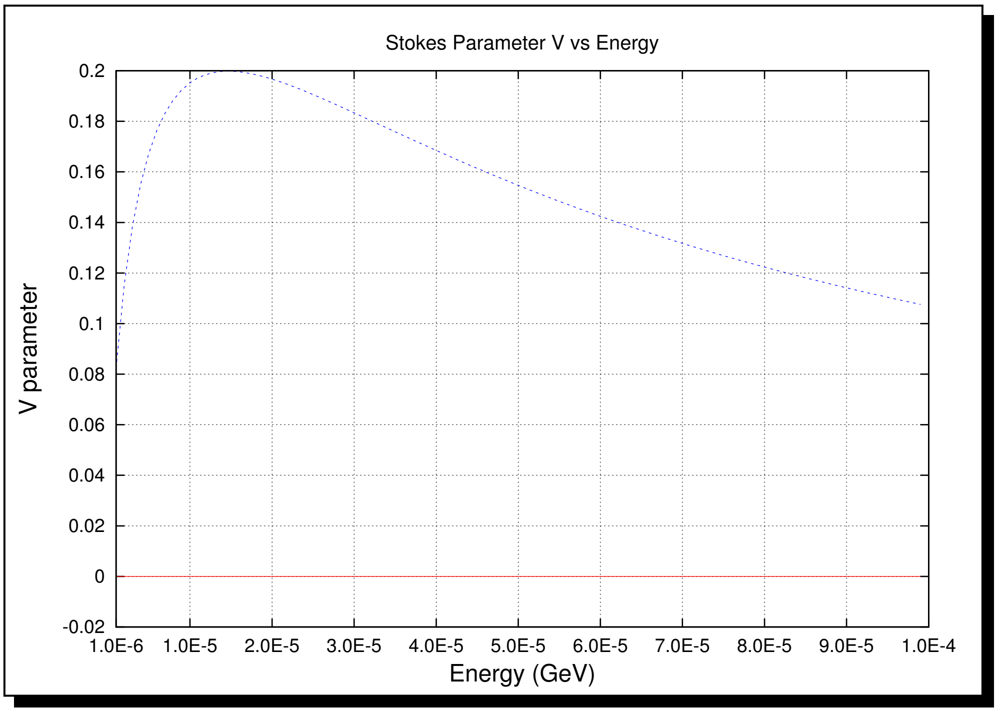

Figure 2. Stokes **V** vs Energy

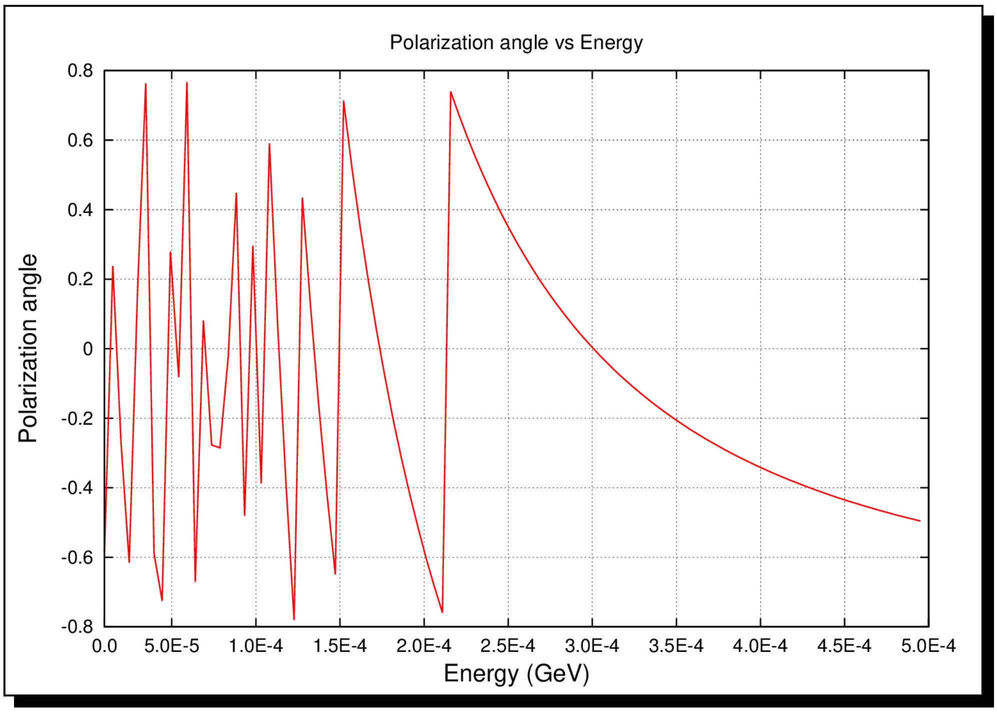

Figure 3. Polarization angle vs Energy

4. Conclusions

In this work, we have considered the mixing effect of photon worth dm candidate. The mixing effect may be capable of producing (a) Elliptic, (b) Circular polarization from initially Plane Polarized beam generated from synchrotron radiation. The amount of polarization generated at different energies are dependent on the energy bands, for the beams traveling through the some distance. The Polarization and Ellipticity angles are multivalued functions of energy and there may be several energy bands where the angles repeat themselves. A significant amount of scalars may be produced in the radiation beam once they are out of the stellar environment through oscillation.

To summarise, in this work we have developed framework using which one can explore the possibility of detecting scalar ALP from synchrotron radiation. Usually the stellar environment with high dipole magnetic field generates plane polarized radiation with orthogonal polarization planes lying on and orthogonal to the trajectory of the charge particles. The amplitudes of the same are expressed in terms of modified Bessel functions. We have provided the expression for the polarimetric variablers those can be estimated using our framework. Eventually the same can be used to model the astrophysical environment of compact stars using the astrophysical observational data.

Competing Interests

The author declares that he has no competing interests.

Authors' Contributions

The author wrote, read and approved the final manuscript.

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