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Fixed Point and Common Fixed Point Results of D_F-Contractions via Measure of Non-compactness with Applications

Research Article

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Abstract. In this paper, we study a new contraction mapping inspired by the concept of F-contraction, which was recently introduced by Wardowski [20]. We find common fixed points for a sequence of mappings by introducing D_F-contractive operators in Banach space using the concept of measure of non-compactness. As an application, we prove some results on the existence of solutions for a system of an infinite fractional order differential equations in the space c, where space c consists of real sequences having the finite limits.

Keywords. D_F-contraction; Fixed point; Common fixed points; Measure of non-compactness

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1. Introduction

The most well known and fundamental result in the theory of fixed points is the Banach's contraction principle [5], which was published in 1922. It states that every self mapping T

defined on a complete metric space (X,d) satisfying

$$\forall\ x,\ y \in X\ [d(Tx,Ty) \le kd(x,y);\ k \in (0,1)]$$

has a unique fixed point. Because of its importance, this result has been extended and generalized in many directions (see [1,6–9,16–19]) and the references therein.

The concept of measure of non-compactness was first introduced by Kuratowski [15] in 1930. Darbo [10] in 1955 used this measure to generalize Banach's contraction mapping principle. Another measure of non-compactness is so-called Hausdorff measure. The Hausdorff measure of non-compactness $\varkappa$ was introduced by Goldenstein *et al.* [12] in 1957. It was further studied by Goldenstein and Markus [11] in 1964, and the measure of non-compactness β by Istratescu [13] in 1972.

Measure of non-compactness is very useful tool used in fixed point theory, to study the existence of solutions of differential, integral, integro-differential equations and optimization problems.

For instance, Aghajani *et al.* [3], extended Darbo's fixed point theorem and used it to study the problem of existence of solutions for a general system of nonlinear integral equations.

One of the most interesting generalization of Banach contraction principle was given by Wardowski [20] in 2012. In which author introduced a new concept of contraction on a complete metric space and proved a new fixed point theorem concerning F-contractions, which generalizes Banach contraction principle in a different way.

Inspired by the work mentioned above, in this article we intoduce a new contraction mapping, called D_F-contraction. A fixed point theorem and a common fixed point theorem concerning D_F-contraction is proved. Furthermore, we study the existence of solutions for a system of an infinite fractional order differential equations as an application of our results.

The rest of paper is organized as follows. In section 2, we give some preliminary material which will be used to establish our main results. We give our main results in Section 3. Section 4 is concerned about the solution of a system of an infinite fractional differential equations of order $0 < \alpha \le 1$, in the space c of real sequences having the finite limits.

2. Preliminaries

Throughout this paper, we assume E be the Banach space with a norm $\|\cdot\|$ and 0 be the zero element of E. If $Y \subset E$, then we denote $\overline{Y}$, $\operatorname{conv}(Y)$ and $\operatorname{co}(Y)$, the closure, closed convex hull and the convex hull of Y, respectively. Moreover, $\mathfrak{M}_E$ denotes the family of all non-empty and bounded subsets of E.

2.1 Definition ([4]). A mapping $\mu: \mathfrak{M}_E \to^+ \cup \{0\}$ is called measure of non-compactness in E if it satisfies:

(i) The subfamily $\ker\mu = \{Y \in \mathfrak{M}_E : \mu(Y) = 0\}$ is a non-empty and $\ker\mu \subset \mathfrak{M}_E$;

(ii) $Z \subset Y$ implies $\mu(Z) \le \mu(Y)$;

(iii) $\mu(\overline{Y}) = \mu(Y)$;

(iv) $\mu(\mathrm{co}(Y)) = \mu(Y)$;

(v) $\mu(\lambda Y + (1-\lambda)Z) \le \lambda\mu(Y) + (1-\lambda)\mu(Z)$ for $\lambda \in [0,1]$;

(vi) if $\{Y_n\}$ is a sequence of closed sets from $\mathfrak{M}_E$ such that $Y_{n+1} \subset Y_n$, (for $n = 1,2,3,\cdots$), and $\lim_{n\to\infty} \mu(Y_n) = 0$, then the intersection set $Y_\infty = \bigcap_{n=1}^{\infty} Y_n$ is non-empty. As

$$\mu(Y_\infty) = \mu\left(\bigcap_{n=1}^{\infty} Y_n\right) \le \mu(Y_n) \quad \text{for all } n$$

which implies that

$$\mu\left(\bigcap_{n=1}^{\infty} Y_n\right) = 0.$$

Therefore, $Y_\infty \in \ker\mu$.

2.2 Lemma ([4]). *If (Y_n) is a decreasing sequence of non-empty, closed and bounded subsets of a complete metric space X such that $\lim_{n\to\infty} \mu(Y_n) \to 0$ then $Y_\infty \subset Y$ is non-empty and compact.*

2.3 Theorem (Schauder's [2]). *Let $Y \subset E$ be a non-empty, bounded, closed and convex set. Then each continuous and compact mapping $T : Y \to Y$ has a fixed point in Y.*

2.4 Theorem (Darbo's [10]). *Let $Y \subset E$ be a non-empty, bounded, closed, and $T : Y \to Y$ be a continuous function. If there exists $k \in [0,1)$ such that*

$$\mu(T(A)) \le k\mu(A)$$

for any non-empty $A \subset Y$. Then T has a fixed point in Y.

3. Main Results

In this section, we give our main results. We introduce the following definition of D_F-contraction inspired by the concept of F-contraction given in Wardowski [20]

3.1 Definition. Let Γ denotes the family of all functions $F : \mathbb{R}^+ \to \mathbb{R}$ satisfying the following conditions:

($\mathrm{D_{F_1}}$) F is strictly increasing;

($\mathrm{D_{F_2}}$) for each sequence $\{\beta_n\}_{n\in\mathbb{N}} \subset \mathbb{R}^+$, $\lim_{n\to\infty} F(\beta_n) = -\infty$ implies $\lim_{n\to\infty} \beta_n = 0$.

Let $Y \subset E$ be a non-empty, bounded, closed and convex set, and $T : Y \to Y$ be a continuous mapping. Then T is said to be D_F-contraction if there exists $\tau > 0$ such that $\mu(A) > 0$ implies

$$\tau + F(\mu(T(A))) \le F(\mu(A))$$

for any non-empty $A \subset Y$, where μ is a measure of non-compactness defined on E and $F \in \Gamma$.

Now we prove our first main results which states that every D_F contraction has a fixed point, this theorem generalize the Theorem 2.4.

3.2 Theorem. *Let $Y \subset E$ be a non-empty, bounded, closed and convex set, and $T : Y \to Y$ be a D_F-contraction. Then T has a fixed point in Y.*

Proof. Define a sequence Y_n of subsets of Y as:

$$Y_0 = Y,\ Y_1 = \operatorname{conv}(T(Y_0)), Y_2 = \operatorname{conv}(T(Y_1)), \cdots, Y_{n+1} = \operatorname{conv}(T(Y_n)).$$

Consider

$$\mu(Y_n) = \mu(\operatorname{conv}(T(Y_{n-1}))) = \mu(T(Y_{n-1})).$$

Therefore

$$F\left(\mu(Y_n)\right) \le F\left(\mu(Y_{n-1})\right) - \tau \le F\left(\mu(Y_{n-2})\right) - 2\tau \le \cdots \le F\left(\mu(Y_0)\right) - n\tau.$$

Thus, we have

$$\lim_{n\to\infty} F\left(\mu(Y_n)\right) = -\infty. \tag{1}$$

Eq. (1) together with (D_{F_2}) gives

$$\lim_{n\to\infty} \mu(Y_n) = 0.$$

Clearly,

$$Y_1 = \operatorname{conv}(T(Y_0)) \subset Y_0.$$

Suppose $Y_n \subset Y_{n-1}$ holds.
Consider

$$Y_{n+1} = \operatorname{conv}(T(Y_n)) \subset Y_n.$$

Thus by using Principle of Mathematical Induction, we have

$$Y_{n+1} \subset Y_n, \quad \text{for } n = 1, 2, 3, \cdots.$$

As $T : Y_n \to Y_n$ $(n = 0, 1, 2, \ldots)$, thus by Lemma 2.2, we conclude

$$Y_\infty = \bigcap_{n=1}^{\infty} Y_n.$$

is a non-empty and compact subset of E. Consequently, using Theorem 2.3, T has a fixed point in $Y_\infty = \bigcap\limits_{n=1}^{\infty} Y_n \subset Y$. □

Clearly for $F(t) = \ln t$, the main result of Darbo will be obtained.

Now we present a common fixed point theorem in which common fixed point of a self mapping with a sequence of self mappings on a closed and convex subset of a Hilbert space is proved.

3.3 Theorem. *Let $Y \subset E$, be a non-empty, bounded, closed and convex set, and S, $T_i : Y \to Y$ be continuous mappings for each $i \in \mathbb{N}$ such that ;*

(i) $ST_i = T_iS$ *for each* $i \in \mathbb{N}$.

(ii) $T_i(conv(A)) \subset conv(T_i(A))$ *for each* $i \in \mathbb{N}$ *and any* $A \subset Y$.

(iii) *There exists* $F \in \Gamma$ *and* $\tau > 0$ *such that*

$$\tau + F\left(\mu(S(A))\right) \le F\left(\mu(T_i(A))\right), \text{ for any } A \subset Y..$$

Then the following hold:

(a) $S_{\text{fix}} = \{x \in Y : S(x) = x\}$ *is non-empty and compact.*

(b) *For any $i \in \mathbb{N}$, T_i has a fixed point and the set $\{x \in Y : T_i(x) = x\}$ is closed and invariant by S.*

(c) *If T_i is affine and commuting family, then S and the family T_i have a common fixed point and the set $\{x \in Y : T_i(x) = S(x) = x\}$ is compact.*

Proof. (a): Define a sequence Y_n of subsets of Y

$$Y_0 = Y,\ Y_1 = \operatorname{conv}(S(Y_0)),\ Y_2 = \operatorname{conv}(S(Y_1)), \cdots,\ Y_{n+1} = \operatorname{conv}(S(Y_n)).$$

Obviously

$$Y_1 = \operatorname{conv}(S(Y_0)) \subset Y_0.$$

Suppose $Y_n \subset Y_{n-1}$ holds.
Consider

$$Y_{n+1} = \operatorname{conv}(S(Y_n)) \subset Y_n.$$

Thus by using Mathematical Induction, we have

$$Y_{n+1} \subset Y_n, \quad \text{for } n = 1,2,3,\cdots.$$

Now for any $i \in \mathbb{N}$,

$$T_i(Y_1) = T_i(\operatorname{conv}(S(Y_0))) \subset \operatorname{conv}(T_i(S(Y_0))) \subset \operatorname{conv}(S(Y_0)) = Y_1.$$

Next assume that

$$T_i(Y_n) \subset Y_n$$

holds and consider

$$T_i(Y_{n+1}) = T_i(\operatorname{conv}(S(Y_n))) \subset \operatorname{conv}(T_i(S(Y_n))) \subset \operatorname{conv}(S(Y_n)) = Y_{n+1}.$$

We get

$$T_i(Y_{n+1}) \subset Y_{n+1}.$$

Thus by Mathematical Induction

$$T_i(Y_{n+1}) \subset Y_{n+1}, \quad \text{for } n = 1,2,3,\cdots.$$

Consider

$$\mu(Y_n) = \mu(\operatorname{conv}(S(Y_{n-1}))) = \mu(S(Y_{n-1})).$$

Therefore

$$F\left(\mu(Y_n)\right) = F\left(\mu(T_i(Y_{n-1}))\right) - \tau \le F\left(\mu(T_i(Y_{n-2}))\right) - 2\tau \le \cdots \le F\left(\mu(T_i(Y_0))\right) - n\tau,$$

which implies

$$\lim_{n\to\infty} F\left(\mu(Y_n)\right) = -\infty. \tag{2}$$

From eq. (2) and ($\mathrm{D_{F_2}}$), we conclude

$$\lim_{n\to\infty} \mu(Y_n) = 0.$$

We conclude that

$$Y_\infty \subset Y.$$

is a non-empty, compact and invariant under S. Thus S has a fixed point. The set

$$S_{\text{fix}} = \{x \in Y : S(x) = x\}.$$

is closed, as S is continuous.
Now for any $x \in S_{\text{fix}}$, $T_i(x)$ is a fixed point of S

$$S(T_i(x)) = T_i(Sx) = T_i(x).$$

Thus $T_i(S_{\text{fix}}) \subset S_{\text{fix}}$.
Clearly,

$$\mu(S_{\text{fix}}) \neq 0.$$

Now consider

$$\tau + F\left(\mu(S_{\text{fix}})\right) = \tau + F\left(\mu(S(S_{\text{fix}}))\right) \leq F\left(\mu(T_i(S_{\text{fix}}))\right). \tag{3}$$

From ($\mathrm{D}_{\mathrm{F}_1}$) and (3)

$$\mu(S_{\text{fix}}) < \mu(T_i(S_{\text{fix}})). \tag{4}$$

But

$$\mu(T_i(S_{\text{fix}})) \leq \mu(S_{\text{fix}}). \tag{5}$$

From eqs. (4) and (5)

$$\mu(S_{\text{fix}}) < \mu(T_i(S_{\text{fix}})) \leq \mu(S_{\text{fix}}).$$

A contradiction, thus

$$\mu(S_{\text{fix}}) = 0.$$

Therefore, S_{fix} is compact.
(b): As Y_∞ is compact and invariant under T_i, so T_i has a fixed point and by continuity of T_i the set

$$T_{\text{fix}} = \{x \in Y : T_i(x) = x\},$$

is closed. Note that $\overline{\mathbf{Sx}}$ is a fixed point of T_i therefore T_{fix} is invariant by S.
(c): As T_i is affine for each i, then T_{fix} is convex. Note that $S(T_{\text{fix}}) \subset T_{\text{fix}}$ and $T_i(T_{\text{fix}}) \subset T_{\text{fix}}$. By similar argument as we used in part (a), we conclude that T_{fix} is compact. Thus S has a fixed point in T_{fix}. Which means S and T_i have a common fixed point. Consequently, the set

$$S_{\text{com}} = \{x \in Y : T_i(x) = S(x) = x\},$$

is compact. □

4. Applications

Recently, a new definition of fractional derivative has been defined by Khalil *et al.* [14], we consider a system of an infinite conformable fractional order differential equations:

$$y_i^{(\alpha_i)} = \frac{e^{-\tau}}{b(t)} b_i(t) y_i + f_i(t, y_1, y_2, \cdots); \ \alpha_i \in (0,1) \tag{6}$$

with the initial conditions

$$y_i(0) = y_i^0. \tag{7}$$

Where $t \in J = [0, I]$, J is fixed real interval, $i = 1, 2, \cdots$, and $\tau > 0$.
First assume that the following hypothesis hold:

(i) $y_0 = y_i^0 \in c$.

(ii) A mapping $f = (f_1, f_2, ...) : J \times c \to c$ is uniformly continuous.

(iii) There exists a sequence (d_i) such that $(d_i) \to 0$ and

$$|f_i(t, y_1, y_2, \cdots)| \le d_i$$

for any $t \in J = [0, I]$ and $y = (y_i) \in c$.

(iv) $b_i(t)$ denotes the functions, which are continuous on J. Moreover the sequence $(b_i(t))$ converges uniformly on $J = [0, I]$.
Denote by

$$B = \sup\{b(t) : t \in J\}$$

and

$$b(t) = \sup\{b_i(t) : i = 1, 2, \ldots\}.$$

4.1 Theorem. *Suppose* (i)-(iv) *assumptions are satisfied. If $I < \frac{1}{B}$, then the system* (6) *together with the initial conditions* (7) *has a solution $y(t) = (y_i(t))$ on $[0, I]$ such that $y(t) \in c$ for any $t \in J$.*

Proof. First, for $t \in J = [0, I]$ and $y = (y_i) \in c$, we denote here

$$h_i(t, y) = \frac{e^{-\tau}}{b(t)} b_i(t) y_i + f_i(t, y)$$

and

$$h(t, y) = (h_1(t, y), h_2(t, y), \ldots) = (h_i(t, y)).$$

For arbitrary fixed natural numbers p and q

$$\begin{aligned}
\left|h_p(t, y) - h_q(t, y)\right| &= \left|\frac{e^{-\tau}}{b(t)} b_p(t) y_p + f_p(t, y) - \frac{e^{-\tau}}{b(t)} b_q(t) y_q - f_q(t, y)\right| \\
&\le \left|\frac{e^{-\tau}}{b(t)}\right| \cdot \left[\left|b_p(t) y_p - b_q(t) y_q\right|\right] + \left|f_p(t, y) - f_q(t, y)\right| \\
&= \left|\frac{e^{-\tau}}{b(t)}\right| \cdot \left[\left|b_p(t) y_p - b_p(t) y_q + b_p(t) y_q - b_q(t) y_q\right|\right] + \left|f_p(t, y)\right| + \left|f_q(t, y)\right| \\
&\le \left|\frac{e^{-\tau}}{b(t)}\right| \cdot \left[\left|b_p(t)\right| \cdot \left|y_p - y_q\right| + \|y\| \cdot \left|b_p(t) - b_q(t)\right|\right] + d_p + d_q.
\end{aligned}$$

From the assumptions (iii) and (iv), $(h_i(t, y))$ is a real Cauchy sequence. Therefore, $(h_i)(t, y) \in c$.
Next

$$\begin{aligned}
|h_i(t, y)| &= \left|\frac{e^{-\tau}}{b(t)} b_i(t) y_i + f_i(t, y)\right| \\
&\le \frac{e^{-\tau}}{b(t)} \cdot b(t) \cdot |y_i| + d_i
\end{aligned}$$

$$\le e^{-\tau}|y_i| + d_i$$
$$\le e^{-\tau}\|y\| + D,$$

where $D = \sup\{d_i : i = 1, 2, \cdot\}$. Hence

$$\|h(t, y)\| \le e^{-\tau}\|y\| + D.$$

Suppose a mapping $h(t, y)$ on $[0, I] \times B(y_0, r)$, we choose

$$r = \frac{DI_1 + BI_1\|y_0\|}{1 - BI_1}.$$

Now x, $y \in B(y_0, r)$ and for fixed but arbitrary t, s from $[0, I]$. Consider for any i

$$\begin{aligned}
|h_i(t,y) - h_i(s,x)| &= \left|\frac{e^{-\tau}}{b(t)} b_i(t) y_i + f_i(t,y) - \frac{e^{-\tau}}{b(s)} b_i(s) x_i + f_i(s,x)\right| \\
&\le \left|\frac{e^{-\tau}}{b(t)} b_i(t) y_i - \frac{e^{-\tau}}{b(s)} b_i(s) x_i\right| + |f_i(t,y) - f_i(s,x)| \\
&\le \left|\frac{e^{-\tau}}{b(t)} b_i(t) y_i + \frac{e^{-\tau}}{b(t)} b_i(s) y_i - \frac{e^{-\tau}}{b(t)} b_i(s) y_i - \frac{e^{-\tau}}{b(s)} b_i(s) x_i\right| + |f_i(t,y) - f_i(s,x)| \\
&\le \left|\frac{e^{-\tau}}{b(t)}\right| \cdot [(r + \|y_0\|) \cdot |b_i(t) - b_i(s)| + B\|y - x\|] + \|f_i(t,y) - f_i(s,x)\|.
\end{aligned}$$

Thus, we can write

$$\begin{aligned}
\|h(t,y) - h(s,y)\| &= \sup\{|h_i(t,y) - h_i(t,x)| : i \in \mathbb{N}\} \\
&\le \left|\frac{e^{-\tau}}{b(t)}\right| \cdot (r + \|y_0\|) \sup\{b_i(t) - b_i(s) : i \in \mathbb{N}\} + B\|y - x\| + \|f(t,y) - f(s,x)\|.
\end{aligned}$$

As the sequence $(b_i(t))$ is equi-continuous on $[0, I]$ and f is uniformly continuous on $[0, I] \times c$. Therefore, we obtain that $h(t, y)$ is uniformly continuous on $J \times B(y_0, r)$. Moreover, take a non-empty $Y \subset B(y_0, r)$. Here we fix $t \in J$, $y \in Y$. Then for arbitrary fixed natural numbers p and q, we write

$$\begin{aligned}
\left|h_p(t,y) - h_q(t,y)\right| &= \left|\frac{e^{-\tau}}{b(t)} b_p(t) y_p + f_p(t,y) - \frac{e^{-\tau}}{b(t)} b_q(t) y_q - f_q(t,y)\right| \\
&\le \left|\frac{e^{-\tau}}{b(t)} b_p(t) y_p - \frac{e^{-\tau}}{b(t)} b_q(t) y_q\right| + \left|f_p(t,y) - f_q(t,y)\right| \\
&\le \left|\frac{e^{-\tau}}{b(t)}\right| \cdot \left[\left|b_p(t) y_p - b_p(t) y_q + b_p(t) y_q - b_q(t) y_q\right|\right] + \left|f_p(t,y)\right| + \left|f_q(t,y)\right| \\
&\le \left|\frac{e^{-\tau}}{b(t)}\right| \cdot \left[\sup_{t \in [0,I]} \left|b_p(t)\right| \cdot \left|y_p - y_q\right| + \|y\| \cdot \left|b_p(t) - b_q(t)\right|\right] + d_p + d_q \\
&\le \left|\frac{e^{-\tau}}{b(t)}\right| \cdot \left[b(t) \cdot \left|y_p - y_q\right| + (r + \|y_0\|) \cdot \left|b_p(t) - b_q(t)\right|\right] + d_p + d_q
\end{aligned}$$

From the above inequality, we get

$$\begin{aligned}
\mu(h(t,Y)) &= \lim_{k \to \infty}\left\{\sup_{y=(y_i)\in Y}\left\{\sup_{p,q \ge k}\left|h_p(t,y) - h_q(t,y)\right|\right\}\right\} \\
&\le \frac{e^{-\tau}}{b(t)} \cdot b(t)\left[\lim_{k \to \infty}\left\{\sup_{y=(y_i)\in Y}\left\{\sup_{p,q \ge k}\left\{\left|y_p - y_q\right|\right\}\right\}\right\}\right]
\end{aligned}$$

$$= e^{-\tau}\mu(Y).$$

Therefore

$$\ln\left(\mu(h(t,Y))\right) \le \ln\left(e^{-\tau}\mu(Y)\right),$$

or

$$\tau + \ln\left(\mu(h(t,Y))\right) \le \ln\left(\mu(Y)\right).$$

For $F(y) := \ln(y)$, we get

$$\tau + F\left(\mu(h(t,Y))\right) \le F\left(\mu(Y)\right).$$

From our fixed point theorem (Theorem 3.2), we conclude that system (6)-(7) has a solution in the space c. □

5. Conclusion

A new contractive condition has been introduced by relaxing the conditions of Wardowski's conditions on F-contractions. Fixed point theorem is presented to genralize many results present in the literature. The common fixed point theorem also generalize and extended many results. The application in Section 4, provides a usefulness of our main result to existence of solutions to a infinite system of fractional differential equations. This article would constitute a base for analysis of nonlinear operators.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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