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Research Article

On the Solution of Stochastic Generalized Burgers Equation

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Abstract. We are interested in one dimensional nonlinear stochastic partial differential equation: the generalized Burgers equation with homogeneous Dirichlet boundary conditions, perturbed by additive space-time white noise. We establish a result of existence and uniqueness of the local solution to the viscous equation using fixed point argument, then if we impose a condition to the viscosity coefficient we can prove that this solution is global.

Keywords. Stochastic Burgers equation; Space-time white noise; Fixed point argument; Viscosity coefficient

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1. Introduction

It is well known that the Burgers equation is not a good model for turbulence. It does not display any chaos; even when a force is added to the right hand side all solutions converge to a unique stationary solution as time goes to infinity.

However the situation is totally different when the force is a random one. Several authors have, indeed, suggested to use the stochastic Burgers equation as a simple model for turbulence ([1], [2], [3], [9]). The equation has also been proposed in ([10]) to study the dynamics of interfaces.

Here we consider the generalized Burgers equation with a random force which is a space-time white noise

$$\frac{\partial u(t,x)}{\partial t} = \rho \frac{\partial^2 u(t,x)}{\partial x^2} - \partial_x f(u(t,x)) + \frac{\partial^2 \widetilde{W}}{\partial t \partial x}, \tag{1}$$

where ρ is the viscosity coefficient and, $\widetilde{W}(t,x)$, $t \geq 0$, $x \in \mathbb{R}$ is a zero mean Gaussian process whose covariance function is given by

$$E\left[\widetilde{W}(t,x)\widetilde{W}(s,y)\right] = (t \wedge s)(x \wedge y), \quad t,s \geq 0,\ x,y \in \mathbb{R}.$$

Alternatively, we can consider a cylindrical Wiener process W by setting

$$W(t) = \frac{\partial \widetilde{W}}{\partial x} = \sum_{h=1}^{\infty} \beta_h e_h, \tag{2}$$

where $\{e_h\}$ is an orthonormal basis of $L^2(0,2\pi)$ and $\{\beta_h\}$ is a sequence of mutually independent real Brownian motions in a fixed probability space $(\Omega, \mathcal{F}, \mathbb{P})$ adapted to a filtration $\{\mathcal{F}_t\}_{t\geq 0}$. The series (2) defining W does not converge in $L^2(0,2\pi)$ but it is convergent in any Hilbert space U such that the embedding

$$L^2(0,2\pi) \subset U$$

is Hilbert-Schmidt ([5]).

In the following we shall write (1) as follows:

$$du(t,x) = \left(\rho \frac{\partial^2 u(t,x)}{\partial x^2} - \partial_x f(u(t,x))\right) dt + dW, \quad x \in [0,2\pi],\ t > 0, \tag{3}$$

where W is defined by (2). We assume that $f : \mathbb{R} \to \mathbb{R}$ is a locally Lipschitz continuous function.

Equation (3) is supplemented with Dirichlet boundary conditions

$$u(0,t) = u(2\pi,t) = 0, \tag{4}$$

and the initial condition

$$u(x,0) = u_0(x),\ x \in [0,2\pi]. \tag{5}$$

Our aim in this paper is to prove problem (3) with boundary and initial conditions (4), (5) has a unique global solution.

The next section, we set the notations, introduce the stochastic convolution and prove local existence in time.

2. Local Existence in Time

Define the unbounded self-adjoint operator A on $L^2(0,2\pi)$ by

$$Au = \rho \frac{\partial^2 u}{\partial x^2},$$

for u on the domain

$$D(A) = \{u \in H^2(0,2\pi) : u(0) = u(2\pi) = 0\}.$$

Denote e^{tA}, $t \geq 0$ the semigroup on $L^2(0,2\pi)$ generated by A. It is well known that e^{tA}, $t \geq 0$, has a natural extension, that we still denote by e^{tA}, $t \geq 0$, as a contraction semigroup on $L^2(0,2\pi)$

for any $p \geq 1$. Finally, we denote by $\{e_k\}$ the complete orthonormal system on $L^2(0,2\pi)$ which diagonalizes A and $\{\lambda_k\}$ the corresponding eigenvalues. We have

$$e_k(x) = \sqrt{\frac{2}{\pi}} \sin k\pi x, \quad k = 1,2,\dots$$

and

$$\lambda_k = -\pi^2 k^2, \quad k = 1,2,\dots.$$

Now, we rewrite (3), (4), (5) as the abstract differential stochastic equation

$$\begin{cases} du = (Au - \partial_x f(u))dt + dW, \\ u(0) = u_0. \end{cases} \tag{6}$$

Recall that the solution to the linear problem

$$\begin{cases} du = Au\,dt + dW, \\ u(0) = u_0 \end{cases} \tag{7}$$

is unique and given by the so-called stochastic convolution

$$W_A(t) = \int_0^t e^{(t-s)A} dW(s). \tag{8}$$

It can be shown that W_A is a Gaussian process and it is mean square continuous with values in $L^2(0,2\pi)$. Moreover, W_A has a version which is, a.s. for $\omega \in \Omega$, α-Hölder continuous with respect to (t,x) for any $\alpha \in [0,1/4[$ ([5]).

We set

$$v(t) = u(t) - W_A(t), \quad t \geq 0,$$

then u satisfies (6) if and only if v is a solution of

$$\begin{cases} \frac{dv}{dt} = Av - \partial_x f(v + W_A), \\ v(0) = u_0. \end{cases} \tag{9}$$

From now we will study equation (9) a.s. $\omega \in \Omega$ and consider for the moment that W_A is an α-Hölder continuous function with respect to (t,x) for any $\alpha \in [0,1/4[$. We will return to the stochastic point of view (and to equation (6)) at the end of § 3.

Let us write (9) as

$$v(t) = e^{tA}u_0 - \int_0^t e^{(t-s)A} \partial_x f(v + W_A) ds; \tag{10}$$

then if v satisfies (10) we say that it is a mild solution of (9).

We are going to solve equation (10) by a fixed point argument in the space $C([0,T^*];L^p(0,2\pi))$ for $p > 1$ and for some $T^* > 0$. We set

$$\Sigma_p(m,T^*) = \{v \in C([0,T^*];L^p(0,2\pi)) : |v(t)|_{L^p(0,2\pi)} \leq m, \text{ for all } t \in [0,T^*]\},$$

and consider an initial datum u_0 $\mathcal{F}_0$-measurable and belonging to $L^p(0,2\pi)$, $\omega \in \Omega$ a.s. We will see, in the proof of the Lemma 2.1 below that if $z(t)$ is, a bounded function from $[0,T]$ into $L^p(0,2\pi)$, then, for $t > 0$, the function $e^{tA}\frac{\partial}{\partial x} f(z)$ is also in $L^p(0,2\pi)$. Hence the integral in (10) is convergent in $L^p(0,2\pi)$ a.s. Thus (10) has a meaning as an equality in $L^p(0,2\pi)$.

Lemma 2.1. *For any $p \geq 2$ and $m > |u_0|_{L^p(0,2\pi)}$, there exists a stopping time $T^* > 0$ such that* (10) *has a unique solution in $\Sigma_p(m,T^*)$.*

Proof. Take any v in $\Sigma_p(m,T^*)$ and define $z = Gv$ by

$$z(t) = e^{tA}u_0 - \int_0^t e^{(t-s)A}\partial_x f(v+W_A)ds,$$

where $G : C([0,T^*];L^p(0,2\pi)) \to C([0,T^*];L^p(0,2\pi))$ is a non-linear operator. Then

$$|z(t)|_{L^p(0,2\pi)} \leq |e^{tA}u_0|_{L^p(0,2\pi)} + \int_0^t |e^{(t-s)A}\partial_x f(v+W_A)|_{L^p(0,2\pi)}ds.$$

As we noticed before, e^{tA}, $t \geq 0$ is a contraction semigroup on $L^p(0,2\pi)$ which has a regularizing effect and, for any $s_1 \leq s_2$ in $\mathbb{R}$, and $r \geq 1$, e^{tA} maps $W^{s_1,r}(0,2\pi)$ into $W^{s_2,r}(0,2\pi)$, for all $t > 0$. Moreover, the following estimate holds

$$|e^{tA}z|_{W^{s_2,r}(0,2\pi)} \leq C_1\big(t^{\frac{s_1-s_2}{2}}+1\big)|z|_{W^{s_1,r}(0,2\pi)} \tag{11}$$

for all $z \in W^{s_1,r}(0,2\pi)$. The constant C_1 depends only on s_1, s_2 and r, see for instance ([11]).

Using the Sobolev embedding theorem we have

$$|e^{(t-s)A}\partial_x f(v+W_A)|_{L^p(0,2\pi)} \leq C_2|e^{(t-s)A}\partial_x f(v+W_A)|_{W^{\frac{1}{p},\frac{p}{2}}(0,2\pi)}$$

and, thanks to (11) with $s_1 = -1$, $s_2 = 1/p$, $r = p/2$

$$\begin{aligned}|e^{(t-s)A}\partial_x f(v+W_A)|_{L^p(0,2\pi)} &\leq C_1C_2\big((t-s)^{-\frac{1}{2}-\frac{1}{2p}}+1\big)|\partial_x f(v+W_A)|_{W^{-1,\frac{p}{2}}(0,2\pi)}\\ &\leq C_1C_2\big((t-s)^{-\frac{1}{2}-\frac{1}{2p}}+1\big)|f(v+W_A)|_{L^{\frac{p}{2}}(0,2\pi)}.\end{aligned}$$

Therefore,

$$\begin{aligned}|z(t)|_{L^p(0,2\pi)} \leq& |u_0|_{L^p(0,2\pi)} + C_1C_2\int_0^t \big((t-s)^{-\frac{1}{2}-\frac{1}{2p}}+1\big)|f(v+W_A)|_{L^{\frac{p}{2}}(0,2\pi)}ds\\
\leq& |u_0|_{L^p(0,2\pi)} + C_1C_2Lip_1\int_0^t \big((t-s)^{-\frac{1}{2}-\frac{1}{2p}}+1\big)\big(1+|v+W_A|_{L^{\frac{p}{2}}(0,2\pi)}\big)ds\\
\leq& |u_0|_{L^p(0,2\pi)} + C_1C_2Lip_1\int_0^t \big((t-s)^{-\frac{1}{2}-\frac{1}{2p}}+1\big)\big(1+|v|_{L^{\frac{p}{2}}(0,2\pi)}+|W_A|_{L^{\frac{p}{2}}(0,2\pi)}\big)ds\\
\leq& |u_0|_{L^p(0,2\pi)} + C_1C_2Lip_1\big(1+(2\pi)^{\frac{1}{p}}m+\mu_p\big)\int_0^t \big((t-s)^{-\frac{1}{2}-\frac{1}{2p}}+1\big)ds\\
\leq& |u_0|_{L^p(0,2\pi)} + C_1C_2Lip_1\big(1+(2\pi)^{\frac{1}{p}}m+\mu_p\big)\left(\frac{2p}{p-1}t^{\frac{1}{2}-\frac{1}{2p}}+t\right),\end{aligned}$$

where Lip_1 is the Lipschitz constant of f which depend on $m+\mu_p$, and

$$\mu_p = \sup_{t\in[0,T]}|W_A(t)|_{L^{\frac{p}{2}}(0,2\pi)}.$$

Hence $|z(t)|_{L^p(0,2\pi)} \leq m$ for all $t \in [0,T^*]$ provided

$$|u_0|_{L^p(0,2\pi)} + C_1C_2Lip_1\big(1+(2\pi)^{\frac{1}{p}}m+\mu_p\big)\left(\frac{2p}{p-1}(T^*)^{\frac{1}{2}-\frac{1}{2p}}+T^*\right) \leq m. \tag{12}$$

It is clear that for any $m > |u_0|_{L^p(0,2\pi)}$ there exists a T^* satisfying (12). Now consider $v_1, v_2 \in \Sigma_p(m,T^*)$ and set $z_i = Gv_i$, $i = 1,2$ and $z = z_1 - z_2$.

Then

$$z(t) = \int_0^t e^{(t-s)A} \frac{\partial}{\partial x}[f(v_1 + W_A) - f(v_2 + W_A)]ds,$$

and we derive as above

$$|z(t)|_{L^p(0,2\pi)} \le C_1 C_2 \int_0^t \left((t-s)^{-\frac{1}{2}-\frac{1}{2p}} + 1\right)|f(v_1+W_A) - f(v_2+W_A)|_{L^{\frac{p}{2}}(0,2\pi)} ds.$$

According to the hypothesis on f, we have

$$\begin{aligned}|f(v_1+W_A) - f(v_2+W_A)|_{L^{\frac{p}{2}}(0,2\pi)} &\le Lip_2 |v_1 - v_2|_{L^{\frac{p}{2}}(0,2\pi)} \le Lip_2 (2\pi)^{\frac{1}{p}} |v_1 - v_2|_{L^p(0,2\pi)} \\ &= C_3 |v_1 - v_2|_{L^p(0,2\pi)},\end{aligned}$$

where Lip_2 is the Lipschitz constant of f which depend on $m + \mu_p$, and $C_3 = Lip_2 (2\pi)^{\frac{1}{p}}$, hence

$$\begin{aligned}|z(t)|_{L^p(0,\,2\pi)} &\le C_1 C_2 C_3 \int_0^t \left((t-s)^{-\frac{1}{2}-\frac{1}{2p}} + 1\right)|v_1 - v_2|_{L^p(0,\,2\pi)} ds \\ &\le C \max_{0\le s\le t} |v_1(s) - v_2(s)|_{L^p(0,\,2\pi)} \int_0^t \left((t-s)^{-\frac{1}{2}-\frac{1}{2p}} + 1\right) ds \\ &\le C\left(\frac{2p}{p-1}(T^*)^{\frac{1}{2}-\frac{1}{2p}} + T^*\right)|v_1 - v_2|_{C([0,\,T^*];\,L^p(0,\,2\pi))}\end{aligned}$$

for all $t \in [0, T^*]$ provided

$$|Gv_1 - Gv_2|_{C([0,\,T^*];\,L^p(0,\,2\pi))} \le C\left(\frac{2p}{p-1}(T^*)^{\frac{1}{2}-\frac{1}{2p}} + T^*\right)|v_1 - v_2|_{C([0,\,T^*];\,L^p(0,\,2\pi))}.$$

We take T^* such that

$$C\left(\frac{2p}{p-1}(T^*)^{\frac{1}{2}-\frac{1}{2p}} + T^*\right) < 1$$

and (12) holds so that G is a strict contraction on $\Sigma_p(m, T^*)$. □

Remark 2.1 ([4]). As mentioned before all the previous results are valid a.s. for $\omega \in \Omega$; in particular μ_p and T^* depend on ω. In the next section we will show that $T^* = T$ a.s. for $\omega \in \Omega$ and hence remove the dependence on ω for the time interval on which the solution exists.

3. Global Existence

We are still considering equation (10) as a deterministic one, working a.s. for $\omega \in \Omega$.

Theorem 3.1 (Global existence). *Let u_0 be given which is $\mathcal{F}_0$-measurable and such that for some $p \ge 2$, $u_0 \in L^p(0,2\pi)$ a.s. If $\rho \ge \frac{Lip_1 c}{2}$ then there exists a unique mild solution of equation* (6)*, which belongs a.s. to $C([0,T]; L^p(0,2\pi))$.*

In the following lemma, we derive a priori estimate which yields global existence.

Lemma 3.1. *If $v \in C([0,T]; L^p(0,2\pi))$ satisfies* (10) *and $\rho \ge \frac{Lip_1 c}{2}$, then*

$$|v(t)|_{L^p(0,2\pi)} \le e^{t Lip_1 c \frac{(p-1)}{2} \mu_\infty^2} |u_0|_{L^p(0,2\pi)},$$

where $c = (2\pi)^{\frac{2}{p(p-2)}}$ and $\mu_\infty = \sup_{t\in[0,T]} |W_A(t)|_{L^\infty(0,2\pi)}$.

Proof. Let $\{u_0^n\}$ be a sequence in $C^\infty(0,2\pi)$ such that

$$u_0^n \to u_0, \text{ in } L^p(0,2\pi),$$

and let $\{W^n\}$ be a sequence of regular processes such that

$$W_A^n(t) = \int_0^t e^{(t-s)A} dW^n(s) \to W_A(t)$$

in $C([0,T]\times[0,2\pi])$ a.s. for $\omega \in \Omega$.

Let v^n be the solution of

$$v^n(t) = e^{tA}u_0^n - \int_0^t e^{(t-s)A}\partial_x f\left(v^n + W_A^n\right) ds$$

given by Lemma 2.1. It is easy to see that v^n does exist on an interval of time $[0,T_n]$ such that $T_n \to T^*$ a.s. and that v^n converges to v in $C([0,T^*];L^p(0,2\pi))$ a.s. Moreover, v^n is regular a.s. and satisfies

$$\frac{\partial v^n}{\partial t} - \rho\frac{\partial^2 v^n}{\partial x^2} + \partial_x f(v^n + W_A^n) = 0. \tag{13}$$

Multiplying (13) by $|v^n|^{p-2}v^n$ and integrating over $[0,2\pi]$, we find

$$\frac{1}{p}\frac{\partial}{\partial t}|v^n|^p_{L^p(0,2\pi)} + \rho(p-1)\int_0^{2\pi}|v^n|^{p-2}\left(\frac{\partial v^n}{\partial x}\right)^2 dx + \int_0^{2\pi}\frac{\partial}{\partial x}f\left(v^n + W_A^n\right)|v^n|^{p-2}v^n dx = 0. \tag{14}$$

We integrate by parts the last integral

$$\int_0^{2\pi}\frac{\partial}{\partial x}f\left(v^n + W_A^n\right)|v^n|^{p-2}v^n dx = -(p-1)\int_0^{2\pi} f\left(v^n + W_A^n\right)|v^n|^{p-2}\frac{\partial}{\partial x}v^n dx,$$

then

$$\begin{aligned}
\left|\int_0^{2\pi}\frac{\partial}{\partial x}f\left(v^n + W_A^n\right)|v^n|^{p-2}v^n dx\right| &= (p-1)\left|\int_0^{2\pi} f\left(v^n + W_A^n\right)|v^n|^{p-2}\frac{\partial}{\partial x}v^n dx\right| \\
&\le (p-1)\int_0^{2\pi} |f\left(v^n + W_A^n\right)|v^n|^{p-2}\frac{\partial}{\partial x}v^n| dx \\
&\le (p-1)\int_0^{2\pi} Lip_1\left(1 + |v^n + W_A^n|\right)|v^n|^{p-2}\frac{\partial}{\partial x}v^n dx \\
&\le (p-1)\int_0^{2\pi} Lip_1 |v^n|^{p-2}\frac{\partial}{\partial x}v^n dx \\
&\quad + (p-1)\int_0^{2\pi} Lip_1 |v^n|^{p-1}\frac{\partial}{\partial x}v^n dx \\
&\quad + (p-1)\int_0^{2\pi} Lip_1 |W_A^n||v^n|^{p-2}\frac{\partial}{\partial x}v^n dx.
\end{aligned}$$

The first term is zero, indeed

$$\int_0^{2\pi}|v^n|^{p-2}\frac{\partial}{\partial x}v^n dx = -\int_0^{2\pi}(p-2)|v^n|^{p-2}\frac{\partial}{\partial x}v^n dx.$$

Hence

$$(p-1)\int_0^{2\pi}|v^n|^{p-2}\frac{\partial}{\partial x}v^n dx = 0.$$

In the same way, we can prove that the second term is also zero.

According to the Hölder's and Cauchy's inequalities we bound the third term as follows

$$Lip_1(p-1)\int_0^{2\pi}|W_A^n||v^n|^{p-2}\frac{\partial}{\partial x}v^n\,dx$$
$$\le Lip_1(p-1)|W_A^n|_{L^\infty(0,2\pi)}|v^n|_{L^{p-2}(0,2\pi)}^{\frac{p-2}{2}}\left(\int_0^{2\pi}|v^n|^{p-2}\left(\frac{\partial}{\partial x}v^n\right)^2dx\right)^{\frac{1}{2}}$$
$$\le Lip_1\,c(p-1)\mu_{n,\infty}|v^n|_{L^p(0,2\pi)}^{\frac{p-2}{2}}\left(\int_0^{2\pi}|v^n|^{p-2}\left(\frac{\partial}{\partial x}v^n\right)^2dx\right)^{\frac{1}{2}}$$
$$\le Lip_1\,c\frac{(p-1)}{2}\mu_{n,\infty}^2|v^n|_{L^p(0,2\pi)}^{p-2}+Lip_1\,c\frac{(p-1)}{2}\int_0^{2\pi}|v^n|^{p-2}\left(\frac{\partial}{\partial x}v^n\right)^2dx,$$

where $c=(2\pi)^{\frac{2}{p(p-2)}}$ and $\mu_{n,\infty}=\sup_{t\in[0,T]}|W_A^n(t)|_{L^\infty(0,2\pi)}$ for a.s. $\omega\in\Omega$.

Going back to (14) we obtain

$$\frac{1}{p}\frac{\partial}{\partial t}|v^n|_{L^p(0,2\pi)}^p+\rho(p-1)\int_0^{2\pi}|v^n|^{p-2}\left(\frac{\partial v^n}{\partial x}\right)^2dx$$
$$\le Lip_1\,c\frac{(p-1)}{2}\mu_{n,\infty}^2|v^n|_{L^p(0,2\pi)}^{p-2}+Lip_1\,c\frac{(p-1)}{2}\int_0^{2\pi}|v^n|^{p-2}\left(\frac{\partial v^n}{\partial x}\right)^2dx.$$

It follows

$$\frac{1}{p}\frac{\partial}{\partial t}|v^n|_{L^p(0,2\pi)}^p+(p-1)\left(\rho-\frac{Lip_1\,c}{2}\right)\int_0^{2\pi}|v^n|^{p-2}\left(\frac{\partial v^n}{\partial x}\right)^2dx\le Lip_1\,c\frac{(p-1)}{2}\mu_{n,\infty}^2|v^n|_{L^p(0,2\pi)}^{p-2}.$$

If we take ρ and Lip_1 such that

$$\rho\ge\frac{Lip_1c}{2}.$$

We obtain

$$\frac{\partial}{\partial t}|v^n|_{L^p(0,2\pi)}^p\le Lip_1\,c\frac{p(p-1)}{2}\mu_{n,\infty}^2|v^n|_{L^p(0,2\pi)}^p$$

and, according to Gronwall's lemma

$$|v^n|_{L^p(0,2\pi)}^p\le e^{tLip_1\,c\frac{p(p-1)}{2}\mu_{n,\infty}^2}|u_0^n|_{L^p(0,2\pi)}^p.$$

Taking the limit as $n\to\infty$, we see that a.s.

$$|v|_{L^p(0,2\pi)}^p\le e^{tLip_1\,c\frac{p(p-1)}{2}\mu_\infty^2}|u_0|_{L^p(0,2\pi)}^p.$$

It follows

$$|v|_{L^p(0,2\pi)}\le e^{tLip_1\,c\frac{(p-1)}{2}\mu_\infty^2}|u_0|_{L^p(0,2\pi)}$$

and the assertion of the lemma follows. □

Proof of Theorem 3.1. It is easily deduced from Lemma 2.1 and Lemma 3.1. □

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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