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Research Article

Portfolio Optimization using the Optimized α in Renyi Entropy

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Abstract. In this paper, a new approach of portfolio optimization using Renyi entropy is presented. In the proposed method, we use Renyi Entropy as a measurement of risk by determining the minimum return. We also attempt to examine the relationship between Renyi Entropy's α-level and risk. Then, a single-objective function with penalty function approach based on Renyi Entropy-mean-semi-variance models is developed. As a result, we find the optimized α denoting the most appropriate portfolio related to the risk level denoted by the investor. Finally by providing an illustrative example, the validity of this method is checked and the conclusion is drawn.

Keywords. Entropy; Portfolio optimization; Utility function; Intelligent optimization algorithm

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1. Introduction

Markowitz [9] was the first who introduced the modern portfolio selection theory. The first measure to study the returns of the investment considered by Markowitz was variance, relying on this logic that the bigger the variance, the higher is the risk. He also introduced the concept of portfolio efficient frontier as a way to determine the whole possible portfolios in which the expected returns are maximized while minimizing the variance. Since then, the Markowitz mean-variance and the mean-semi variance models have been used to obtain the effective frontier. Many researchers such as Merton [10] and Green and Hollifield [5] used these models in portfolio optimization problems.

Shannon [13] introduced the quantitative and qualitative communications model, involving $H(X)$ as the entropy of the random variable X, as a statistical process, which led to the foundation of the information theory. Since then, the risk is related to uncertainty; the concept of entropy should be applicable in calculating the optimum portfolio and the efficient frontier. Entropy was first used by Philippatos and Wilson [11] in portfolio selection. Ke and Zhang [7] used the Shannon entropy to modify the mean-variance model. Then, Bugar and Uzsoki [2] examined the portfolio diversification methodology. Besides, Bera and Park [1] argued that although the Markowitz mean-semi variance method is being a most commonly applicable approach in solving portfolio diversification problems, comparing to the mean variance method, it sometimes results in highly limiting the variety of assets in the portfolio. Usta and Kantar [15] also used the Mean-Variance-Skewness-Entropy model for portfolio diversification. They relied on multi-objective approach in selecting portfolio. Xu [16] compared the mean-variance efficiency of the models incorporating different entropy measures by applying multiple criteria method and Yari et al. [17] employed entropy optimization measures to determine weights of each criterion of growth and reduction to better predict the future worth of each share.

This paper tries to improve portfolio selection models using Renyi entropy measures. We used Shannon entropy [13] and Renyi entropy [12] as the objective functions and we also compare them with each other. Adopting a penalty function approach, we convert the objective functions with side conditions into single-objective functions without side conditions. In the other words, the entropy measures were added to the traditional portfolio optimizations models. Then, the outcome is suggested as a way for developing investment strategies. Our empirical work is running the models and determining the profit and loss of investment which are tested with original data of six big companies, using the PSO [8] intelligent optimization algorithm.

The paper is organized as follows: In Section 2, we provide the definitions used in the study and describe the traditional models for portfolio optimization. Section 3 describes the proposed models for portfolio optimization and diversification, incorporating Shannon and Renyi entropy. Section 4 is dedicated to our empirical work, which helps in comparing the performance of these models. We have the conclusions in the last chapter.

2. Traditional portfolio selection models

Portfolio theory deals with selecting the optimum way of investing in a given set of assets [4]. Each possible strategy is considered as a portfolio selection model. In this section, we present a major traditional portfolio selection model (the mean-semi variance model). Before presenting the models, the definitions used in the study are provided as follow:

2.1 Preliminaries

Definition 2.1. Assume $P_i(t)$ be the ith price of the stock in time t, where $t = 1,2,\dots,T$ and $i = 1,2,\dots,n$. Then the return of ith stock is calculated as:

$$r_i(t) = \frac{P_i(t+1) - P_i(t)}{P_i(t)} = \frac{P_i(t+1)}{P_i(t)} - 1. \tag{1}$$

If x_i is considered as the capital needed to buy ith stock, then the proportion of the capital to be invested in stock i, shown as w_i, will be defined as:

$$w_i = \frac{x_i}{\sum_{i=1}^{n} x_i}, \quad 0 \le w_i \le 1, \ \sum_{i=1}^{n} w_i = 1. \tag{2}$$

Definition 2.2. The weighted average of stock returns in time t, called the portfolio returns, is defined as:

$$R_p(t) = \sum_{i=1}^{n} w_i r_i(t). \tag{3}$$

Definition 2.3. The average portfolio returns, called the expected returns, is defined as:

$$\mu_p = E\{R_p(t)\} = \frac{1}{T}\sum_{t=1}^{T} R_p(t) = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{n} w_i r_i(t) = \sum_{i=1}^{n} w_i \mu_i = w^T \cdot \mu, \tag{4}$$

where

$$\mu_i = E\{r_i(t)\} = \frac{1}{T}\sum_{t=1}^{T} r_i(t).$$

Definition 2.4. The variance of the average expected returns, called the portfolio variance, is defined as:

$$\sigma_p^2 = \text{var}(R_p(t)) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}, \quad \sigma_{ij} = \text{cov}(r_i(t), r_j(t)) = \rho_{ij}\sigma_i\sigma_j, \tag{5}$$

where ρ_{ij} shows the correlation between two stocks. The portfolio variance may be defined as:

$$\sigma_p^2 = w^T \Sigma w, \tag{6}$$

where S is the variance-covariance matrix and $w = (w_1, w_2, \dots, w_n)$.

Definition 2.5. The portfolio variance for returns below the expected value, called the portfolio semi-variance, is defined as:

$$\sigma_{p-}^2 = E\{(R_p(t) - \mu_p)^2 | (R_p(t) - \mu_p)^2 R_p(t) < \mu_p\} = \sum_i \sum_j w_i w_j \sigma_{i-}\sigma_{j-}\rho_{ij} = w^T \sum_{-} w, \tag{7}$$

where $\sigma_{i-}^2 = E\{(r_i(t) - \mu_i)^2 | r_i(t) < \mu_i\}$.

But since, many researchers believe that in variance calculation, the positive distances of values from the average, not only is not bad, but it shows a better yield of a share, they use semi-variance instead of variance in their calculation. The definition of mean-semi variance model is as follows:

2.2 Mean-semi Variance Model

The basic assumption of the Markowitz's Mean-semi Variance is that higher expected returns can be obtained by taking more risk. Thus the optimum portfolio can be selected by choosing w_is in a way that the following equation is minimized against w_is:

$$\begin{aligned} &\text{Min}\,\sigma_p^2 = w^T S_{-} w, \\ &\text{s.t. Max}\,\mu_p = \sum_{i=1}^{n} w_i \mu_i \geq \mu_0, \quad i = 1,2,3,\ldots,n, \\ &\sum_{i=1}^{n} w_i = 1. \end{aligned} \tag{8}$$

where μ_0 is the pre-determined expected return for the portfolio.

Definition 2.6. The violation function for the above problem's side condition is as:

$$V(w) = \begin{cases} 0 & \mu_p \geq \mu_0 \\ 1 - \frac{\mu_p}{\mu_0} & \mu_p < \mu_0 \end{cases} \tag{9}$$

Using Definition 2.6, the problem can be converted into a single-objective problem, without any constraint. This problem adopts the multiplicative penalty function [3, 14]:

$$\begin{aligned} &\text{Min}\, w^T \Sigma_{-} w + \text{Penalty}, \\ &\text{Min}(w^T \Sigma_{-} w)(1 + \lambda V(w)) = (w^T \Sigma_{-} w)\left(1 + \lambda\, \text{Max}\left(0, 1 - \frac{\mu_p}{\mu_0}\right)\right) \end{aligned} \tag{10}$$

where μ_0 is the pre-determined expected return for the portfolio, and λ a positive real number.

3. Entropy Models for Portfolio Selection

In this section, an overview of the Shannon entropy and Renyi entropy is provided.

3.1 Shannon Entropy-mean-semi Variance Model

It is accepted that our understanding of a signal (i.e. a variant or quantified information) depends on its logarithm [13]. In the other words:

$$\log(\text{signal}) \sim \text{Perception(or understanding)}.$$

Given this fact, the state can be summarized as follows:

$$= \log\left(\frac{1}{w_i}\right) \sim \text{uncertainty in decisions related to Stock } i$$

Since we are dealing with n number of stocks, the empiric quantities $\frac{1}{n}\sum -\log\left(\frac{1}{w_i}\right)$ are interesting for us. However, it would be more realistic to consider weighted mean $H(w) = -\sum_{i=1}^{n} w_i \log w_i$. This formula can be defined as the portfolio entropy where w_i is the proportion of capital investment in stock i, and n is the number of stocks [6]. In entropy, we have also these conditions that H is maximum when $w_i = \frac{1}{n}$; and if $w_i = 1$ (for one i) and $w_i = 0$ (for the rest), $H = 0$.

Given the entropy measure as the objective function, we have:

$$\begin{aligned} &\operatorname{Min} H(w) = -\sum_{i=1}^{n} w_i \log w_i, \\ &\text{s.t. } \operatorname{Max} \mu_p = \sum_{i=1}^{n} w_i \mu_i \geq \mu_0, \quad i = 1,2,3,\dots,n, \\ &\qquad \operatorname{Min} \sigma_p^2 = \operatorname{var}(R_p(t)) = w^T \Sigma_{-} w = \sigma_0^2, \\ &\qquad \sum_{i=1}^{n} w_i = 1. \end{aligned} \tag{11}$$

This can also be dealt with in the following way:

$$\begin{aligned} &\operatorname{Min} H(w) = -\sum_{i=1}^{n} w_i \log w_i, \\ &\text{s.t. } \operatorname{Min} -\mu_p \leq -\mu_0, \quad i = 1,2,3,\dots,n, \\ &\qquad \operatorname{Min} \sigma_p^2 = \operatorname{var}(R_p(t)) = w^T \Sigma_{-} w = \sigma_0^2, \\ &\qquad \sum_{i=1}^{n} w_i = 1. \end{aligned} \tag{12}$$

Definition 3.1. The violation functions for side conditions are as follows:

$$Vret(w) = \begin{cases} 0 & \mu_p \geq \mu_0 \\ 1 - \frac{\mu_p}{\mu_0} & \mu_p < \mu_0 \end{cases} \tag{13}$$

$$Vrisk(w) = \begin{cases} 0 & \sigma_p^2 < \sigma_0^2 \\ \frac{\sigma_p^2}{\sigma_0^2} - 1 & \sigma_p^2 \geq \sigma_0^2 \end{cases} \tag{14}$$

Using (13) and (14), the equation (12) can be rewritten as a single-objective function without any constraints, using a multiplicative penalty function approach.

$$\operatorname{Min} \sum_{i=1}^{n} w_i \log w_i + \text{Penalty}, \tag{15}$$

$$\begin{aligned} &\operatorname{Min} \left(-\sum_{i=1}^{n} w_i \log w_i \right) (1 + \lambda_1 Vret(w) + \lambda_2 Vrisk(w)) \\ &\quad = \left(\sum_{i=1}^{n} w_i \log w_i \right) \left(1 + \lambda_1 \operatorname{Max}\left(0, 1 - \frac{\mu_p}{\mu_0}\right) + \lambda_2 \operatorname{Max}\left(0, \frac{\sigma_p^2}{\sigma_0^2} - 1\right) \right), \end{aligned}$$

where μ_0 is the pre-determined expected return and σ_0^2 is the pre-determined risk for the portfolio and λ_1, λ_2 are positive real numbers.

3.2 Renyi Entropy-mean-semi Variance Model

In information theory, the Renyi entropy is characterized by determining different quantities of uncertainty and irregularity in a system:

$$H_\alpha(X) = \frac{1}{1-\alpha} \log \sum_{i=1}^{n} w_i^\alpha, \quad \alpha > 0 (\neq 1).$$

The Renyi entropy is a generalized form of the Shannon entropy

$$\lim_{\alpha\to 1} H_\alpha(x) = H(x).$$

Taking the Renyi entropy as objective function, we have:

$$\begin{aligned} &\operatorname{Min} H(w) = \frac{1}{1-\alpha}\log\sum_{i=1}^{n} w_i^\alpha, \\ &\text{s.t. } \operatorname{Max}\mu_p = \sum_{i=1}^{n} w_i\mu_i \ge \mu_0, \quad i = 1,2,3,\dots,n, \\ &\qquad \operatorname{Min}\sigma_p^2 = \operatorname{var}(R_p(t)) = w^T S_{-} w = \sigma_0^2. \end{aligned} \tag{16}$$

By defining violation functions, and converting (16) into a single-objective function, we have

$$\begin{aligned} &\operatorname{Min}\left(\frac{1}{1-\alpha}\log\sum_{i=1}^{n} w_i^\alpha\right)(1+\lambda_1 Vret(w)+\lambda_2 Vrisk(w)) \\ &\quad = \left(\frac{1}{1-\alpha}\log\sum_{i=1}^{n} w_i^\alpha\right)\left(1+\lambda_1\operatorname{Max}\left(0,1-\frac{\mu_p}{\mu_0}\right)+\lambda_2\operatorname{Max}\left(0,\frac{\sigma_p^2}{\sigma_0^2}-1\right)\right). \end{aligned} \tag{17}$$

4. Measure of Risk using Renyi Entropy

Suppose that $u(w)$ is a utility function; then if an individual be risk-averse, we have $u''(w) < 0$ and $u(w)$ as a concave function; and if an individual be risk-prone, then we have $u''(w) > 0$ and $u(w)$ as a convex function.

In our method, the renyi measure is proposed as the alternative to variance to measure the risk in portfolio optimization. If we use renyi measure with α variety, we get several measures for risk; then we have to run our program with a simulation program to see the outcomes for different αs. Considering the use of renyi measure for risk, we get the measure discussed below:

$$\begin{aligned} F &= \frac{1}{1-\alpha}\log\sum_{i=1}^{n} w_i^\alpha = \frac{1}{1-\alpha}\log\sum_{i=1}^{n}\left(\frac{x_i}{\sum_{i=1}^{n} x_i}\right)^\alpha \\ &= \frac{1}{1-\alpha}\log\frac{1}{A^\alpha}\sum_{i=1}^{n} {x_i}^\alpha \\ &= \frac{\alpha}{\alpha-1}\log A + \frac{1}{1-\alpha}\log\sum_{i=1}^{n} {x_i}^\alpha \\ &= \frac{\alpha}{\alpha-1}\log A + \frac{1}{1-\alpha}\log n + \frac{1}{1-\alpha}\log E(X^\alpha). \end{aligned} \tag{18}$$

Analysis

For $\alpha < 1$, maximization of the measure means the minimization of the expected utility of a person whose utility function is given by $u(w) = w^\alpha$. In this case the person is risk-averse.

For $\alpha > 1$, maximization of the measure means the maximization of the expected utility of a person whose utility function is given by $u(w) = w^\alpha$. In this case the person is risk-prone.

Therefore, maximization of this measure also implies minimization of the expected utility of a risk-averse person and maximization of the expected utility of a risk-prone person. In this case as $\alpha \to 1$, this again implies maximizing of the expected utility of a risk-prone person.

The steps of the proposed method are as follows:

(1) In the first step, daily, weekly or monthly T stock returns of the selected companies must be considered. For better accuracy, daily data are preferred; however in long term investments, weekly and monthly data can be used, if we have enough data.
Daily Profit/Loss is calculated by the following formula.

$$\text{Profit/Loss} = C_{T+j+1} \times S_{T+j} - C_{T+j} \times S_{T+j}, \quad j = 0, 1, 2, \ldots, m$$

$$C_a = \text{Optimum capital allocation until the time } a$$

$$S_a = \text{The stock price at the time } a$$

(2) The values, μ_0 (pre-determined expected return) and σ_0^2 (pre-determined risk) are determined to minimize the objective function. These values can be chosen by many statistical methods.

(3) The PSO algorithm is run for the mentioned T returns. The output will constitute the appropriate capital allocation strategy for investment in these n companies and again T+s (stock returns of the next time unit for the same n companies) are selected.

(4) In a simulation method, all the procedure is performed by the entire positive αs less than 5. We know that when α is equal to 1, we have Shannon entropy.

5. The Empirical Work

In order to test the applications of the models presented in Section 2 and 3, the one-year span (1st January-31th December 2016) daily data on the stocks of six companies, namely IBM, Google, Microsoft, John Wiley & Sons, Yahoo, and Facebook was inserted to models. Then the problems were converted into a single-objective one and were solved using a tailor-made PSO optimization algorithm in MATLAB.

We performed the program introduced in section 4 for 50 different α from 0.1 to 5 and the results for three of them ($\alpha = 1$ which is Shannon entropy, one bigger and one less than it) is shown in the following figures.

Tables 1, 2 and 3 are obtained with setting the α level at 5, 0.05 and 1, respectively. Selection using the Shanon Mean-Semi variance leads to a more diverse and decentralized portfolio for a limited amount of assets in comparison to the Renyi's ($\alpha = 5, 0.05$) Mean-Semi variance models. Also, when α is equal to 0.05, the portfolio is more centralized on a limited number of assets in comparison to other models.

We calculate the daily profit and loss using formula, based on the optimum capital allocated weights, given in Tables 1, 2 and 3.

Table 1. Optimum capital allocation from the beginning of the year 2016 until the T time, based on Renyi($\alpha = 5$)-Mean-Semi variance

Date	Investment strategy					
	JW-A	FB	YHOO	MSFT	IBM	GOOGLE
10/17/2016	0	0.165151	0.550741	0.059251	0.172978	0.051879
10/18/2016	0	0.191775	0.550813	0.067032	0.184277	0.006103
10/19/2016	0	0.188117	0.556622	0.060725	0.159961	0.034575
10/20/2016	0	0.181256	0.557172	0.063223	0.156716	0.041633
10/21/2016	0	0.214508	0.558453	0.060608	0.166432	0
10/24/2016	0	0.193232	0.565918	0.055238	0.139183	0.046429
10/25/2016	0	0.194385	0.57481	0.086357	0.132561	0.011887
10/26/2016	0	0.194346	0.580781	0.064692	0.127793	0.032388
10/27/2016	0.001533	0.19734	0.58589	0.063339	0.130645	0.021253
10/28/2016	0	0.187774	0.591915	0.061311	0.148938	0.010061
10/31/2016	0	0.148296	0.597922	0.05091	0.146295	0.056577
11/1/2016	0.592338	0.250372	0.154994	0	0.001826	0.00047
11/2/2016	0	0.16803	0.610196	0.053444	0.115214	0.053117
11/3/2016	0.001037	0.19273	0.613254	0.051876	0.132901	0.008203
11/4/2016	0	0.166224	0.608925	0.039707	0.169517	1.56E-02
11/7/2016	0	0.149154	0.616417	0.070499	0.148812	0.015119
11/8/2016	0	0.110307	0.611003	0.08615	0.151632	0.040907
11/9/2016	0	0.127425	0.611477	0.061225	0.148152	0.051721
11/10/2016	0	0.116477	0.617555	0.076205	0.164291	0.025472
11/11/2016	0.000821	0.112653	0.619464	0.04614	0.20033	0.020592
11/14/2016	0	0.137416	0.619075	0.051294	0.192214	0
11/15/2016	0.026403	0.10437	0.600605	0.063746	0.195505	0.00937
11/16/2016	0.631488	0.143907	0.053637	0.028929	0.142039	0
11/17/2016	0.001535	0.112288	0.603211	0.075408	0.185729	0.021829
11/18/2016	0.006604	0.143147	0.600194	0.067582	0.181812	0.00066
11/21/2016	0.002577	0.138539	0.606398	0.061257	0.19123	0
11/22/2016	0	0.121771	0.612549	0.06462	0.201059	0
11/23/2016	0.017045	0.125089	0.618107	0.04518	0.182488	0.01209
11/25/2016	0.006151	0.137671	0.625539	0.062081	0.168558	0
11/28/2016	0	0.102021	0.631012	0.072493	0.177396	0.017078
11/29/2016	0.01094	0.134636	0.629136	0.050151	0.175138	0
11/30/2016	0	0.115759	0.629482	0.074584	0.180174	0
12/1/2016	0	0.098282	0.631985	0.046235	0.190319	0.03318
12/2/2016	0.672906	0.112696	0	0.057409	0.10538	5.16E-02
12/5/2016	0.600749	0.068075	0.203656	0	0.12752	0
12/6/2016	0.007151	0.095367	0.604897	0.057863	0.18897	0.045752
12/7/2016	0.055713	0.119586	0.610271	0.035428	0.179002	0
12/8/2016	0.030715	0.137677	0.61008	0.011629	0.209614	0.000286
12/9/2016	0.657847	0.091135	0	0.049713	0.098567	0.102738
12/12/2016	0.00729	0.107323	0.606237	0.072121	0.207029	0
12/13/2016	0.006005	0.110206	0.601502	0.067994	0.201948	0.012345
12/14/2016	0.019143	0.121248	0.610245	0.061477	0.166694	0.021193
12/15/2016	0	0.102381	0.615503	0.071865	0.197805	0.012445
12/16/2016	0.629078	0.071672	0	0.072359	0.099434	0.127456
12/19/2016	0.633217	0.115386	0	0.073766	0.105541	0.07209
12/20/2016	0.633425	0.085313	0.00083	0.063929	0.112852	0.103651
12/21/2016	0.022534	0.119251	0.535429	0.078119	0.203431	0.041237
12/22/2016	0.014584	0.126087	0.540948	0.065793	0.2179	3.47E-02
12/23/2016	0.03026	0.069774	0.5296	0.047812	0.32161	0.000944
12/27/2016	0.106265	0.068917	0.533644	0.056695	0.234479	0
12/28/2016	0.626108	0.110403	0.024559	0.057277	0.181652	0
12/29/2016	0.628028	0.099421	0.027235	0.040424	0.204891	0

Table 2. Optimum capital allocation from the beginning of the year 2016 until the T time, based on Renyi($\alpha = 0.05$)-Mean-Semi variance

Date	Investment strategy					
	JW-A	FB	YHOO	MSFT	IBM	GOOGLE
10/17/2016	0.359441	0.239498	0.401061	0	0	0
10/18/2016	0.258381	0.275491	0.466129	0	0	0
10/19/2016	0	0.315366	0.54132	0	0.143314	0
10/20/2016	0.375695	0.229595	0.39471	0	0	0
10/21/2016	0.101315	0.386277	0.512408	0	0	0
10/24/2016	0.268943	0	0.492735	0.238323	0	0
10/25/2016	0.237646	0.255313	0.507041	0	0	0
10/26/2016	0	0.168186	0.575026	0	0.256788	0
10/27/2016	0	0.222719	0.566205	0.211076	0	0
10/28/2016	0.483309	0.176953	0.339739	0	0	0
10/31/2016	0.407008	0.168917	0.424075	0	0	0
11/1/2016	0	0.174843	0.582454	0.242703	0	0
11/2/2016	0.328494	0.18463	0.486876	0	0	0
11/3/2016	0.097938	0.369887	0.532174	0	0	0
11/4/2016	0	0.144536	0.607741	0	0.247723	0
11/7/2016	0.085454	0.396674	0.517872	0	0	0
11/8/2016	0	0.363653	0.561331	0	0.075016	0
11/9/2016	0	0.144213	0.607465	0	0.248322	0
11/10/2016	0.269182	0	0.514462	0	0.216356	0
11/11/2016	0.427499	0.171207	0.401294	0	0	0
11/14/2016	0.423506	0	0.38236	0.194134	0	0
11/15/2016	0.428791	0	0.397638	0	0.173571	0
11/16/2016	0.139075	0	0.553707	0	0.307218	0
11/17/2016	0.437548	0	0.388327	0	0.174124	0
11/18/2016	0.22624	0.239162	0.534597	0	0	0
11/21/2016	0.225306	0	0.541162	0	0.233531	0
11/22/2016	0.382808	0.151841	0.465351	0	0	0
11/23/2016	0.325109	0.163216	0.511676	0	0	0
11/25/2016	0.428277	0	0.427498	0	0.144225	0
11/28/2016	0.440615	0	0.414609	0.144776	0	0
11/29/2016	0.34433	0	0.490241	0.165429	0	0
11/30/2016	0.364408	0	0.491822	0	0.14377	0
12/1/2016	0.094977	0	0.521201	0.383822	0	0
12/2/2016	0.496768	0	0.345375	0.157857	0	0
12/5/2016	0	0.391519	0.530533	0	0.077947	0
12/6/2016	0.089799	0	0.571606	0	0.338595	0
12/7/2016	0.42346	0	0.423237	0.153303	0	0
12/8/2016	0.191482	0	0.554516	0	0.254002	0
12/9/2016	0.511446	0	0.320115	0	0.168439	0
12/12/2016	0.108641	0	0.507192	0.384167	0	0
12/13/2016	0.441228	0	0.409169	0	0.149603	0
12/14/2016	0.366955	0	0.473858	0.159187	0	0
12/15/2016	0.322837	0	0.394263	0.282901	0	0
12/16/2016	0.336637	0	0.3974	0.265963	0	0
12/19/2016	0.249477	0.310611	0.439912	0	0	0
12/20/2016	0	0	0.475363	0.352644	0.171993	0
12/21/2016	0.20095	0	0.432903	0.366147	0	0
12/22/2016	0.219603	0	0.468379	0	0.312018	0
12/23/2016	0.449357	0.179983	0.346953	0	0.023707	0
12/27/2016	0.37986	0.224234	0.395906	0	0	0
12/28/2016	0.419925	0	0.355637	0.224438	0	0
12/29/2016	0.359441	0.239498	0.401061	0	0	0

Table 3. Optimum capital allocation from the beginning of the year 2016 until the T time, based on Shannon-Mean-Semi variance

Date	Investment strategy					
	JW-A	FB	YHOO	MSFT	IBM	GOOGLE
10/17/2016	0.356338	0.244115	0.398773	0.000181	0.000186	0.000407
10/18/2016	0.216722	0.298072	0.48216	0.000627	0.002275	0.000144
10/19/2016	0.000788	0.043266	0.533739	0.001211	0.16556	0.255437
10/20/2016	0.000652	0.283338	0.52885	0.185669	0.000417	0.001075
10/21/2016	0.131398	0.360456	0.507203	0.000145	6.45E-05	7.35E-04
10/24/2016	0.449487	0.200512	0.347385	0.001558	0.000518	0.00054
10/25/2016	0.000285	0.259508	0.557551	0.179778	0.002616	0.000261
10/26/2016	0.000781	0.162266	0.574519	6.63E-04	2.61E-01	0.00081
10/27/2016	0.000628	0.146335	0.547706	0.298222	1.48E-03	0.005633
10/28/2016	0.188317	0.354426	0.453535	0.000139	0.002862	0.000721
10/31/2016	0.149897	0.289734	0.55806	0.000504	0.001647	0.000158
11/1/2016	0.357534	0.175136	0.465659	0.001289	0.00034	4.16E-05
11/2/2016	0.37462	0.165259	0.456359	0.000923	0.002601	2.39E-04
11/3/2016	0.000278	0.350453	0.561552	0.001547	0.079691	0.00648
11/4/2016	0.000842	0.148384	0.608379	0.003253	0.237579	0.001562
11/7/2016	0.003172	0.35154	0.50297	2.20E-06	0.142158	0.000157
11/8/2016	0.002428	0.130526	0.5539	0.311456	0.001652	3.79E-05
11/9/2016	0.218929	0.284519	0.493086	3.03E-04	0.001929	0.001234
11/10/2016	0.208036	0.00526	0.561311	0.001231	0.208669	0.015492
11/11/2016	0.357469	0.164569	0.476921	0.000373	0.00011	0.000557
11/14/2016	0.415901	0.000955	0.39534	5.88E-04	0.186748	0.000469
11/15/2016	0.399177	0.000322	0.359409	0.000574	2.39E-01	0.001723
11/16/2016	0.364839	0.001391	0.448022	0.000312	0.18509	0.000345
11/17/2016	0.373501	0.129187	0.488451	0.004921	0.003925	1.47E-05
11/18/2016	0.003148	0.000875	0.567393	2.23E-05	0.428151	0.00041
11/21/2016	0.4275	2.44E-01	0.326218	0.001412	1.05E-06	0.000618
11/22/2016	0.365697	3.36E-05	0.509093	0.121698	0.003013	0.000466
11/23/2016	0.138282	0.01719	0.558242	0.281726	0.004082	0.000479
11/25/2016	0.712818	0.002769	0.15288	0.001689	0.129816	2.88E-05
11/28/2016	0.359373	1.64E-06	0.518844	0.00068	0.119735	0.001368
11/29/2016	0.318095	0.000422	0.52195	0.071618	0.08755	0.000364
11/30/2016	0.470078	9.62E-05	0.382472	0.145693	0.001421	0.000239
12/1/2016	0.190099	0.314468	0.491785	0.000206	0.002106	0.001337
12/2/2016	0.289824	0.000183	0.39211	0.000926	0.316426	0.000532
12/5/2016	0.479627	0.000239	0.447447	0.068043	9.88E-05	0.004545
12/6/2016	0.604126	2.96E-05	0.33755	0.052564	0.003164	0.002567
12/7/2016	0.535025	0.000168	0.402311	0.002227	0.059699	0.00057
12/8/2016	0.570454	0.000508	0.365418	0.058423	1.58E-03	0.003619
12/9/2016	0.664118	0.000223	0.25957	0.002238	0.00272	0.07113
12/12/2016	0.407638	0.000388	0.480867	0.002489	3.52E-05	0.108583
12/13/2016	0.43702	0.002334	0.480622	0.001631	0.077833	0.00056
12/14/2016	0.364278	0.000158	0.518391	0.108715	0.00814	3.17E-04
12/15/2016	0.548633	0.002657	0.334186	0.110961	0.000814	0.002749
12/16/2016	0.470989	0.001273	0.405941	0.000574	0.119837	0.001386
12/19/2016	0.368244	0.001991	0.46852	0.159062	0.001839	0.000344
12/20/2016	0.000153	0.002061	0.549225	0.201672	0.246119	0.00077
12/21/2016	0.000128	0.138683	0.517013	0.245318	0.098396	0.000463
12/22/2016	0.00057	0.00109	0.521165	0.001764	0.473188	0.002223
12/23/2016	0.729611	0.00518	0.001652	0.177446	0.085853	0.000258
12/27/2016	0.463637	3.82E-03	0.419095	0.001195	0.109711	0.00254
12/28/2016	0.114499	0.177079	0.536517	2.71E-06	1.71E-01	1.06E-03
12/29/2016	0.356338	0.244115	0.398773	0.000181	0.000186	0.000407

Applying this formula, the trend of daily profit/loss is assessed as shown in Figure 1.

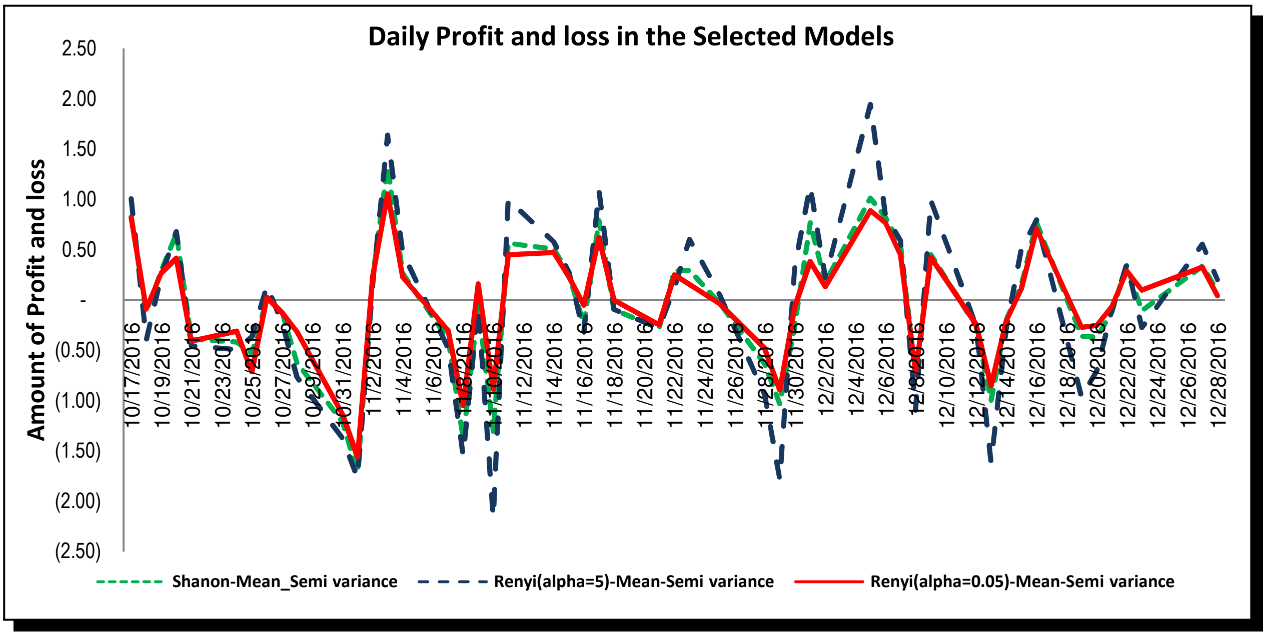

Figure 1. Daily profit and loss

6. Conclusions

The present study tried to examine the performance of Shannon Renyi entropy in portfolio analysis. A single-objective function, with penalty function approach, based on entropy-mean-semi variance models, were developed where for performance some simulation and PSO algorithms were employed.

We presented a model in which each investor due to his measure of risk, can choose a α in Renyi entropy and then concluded that despite the previous researches, we don't have to determine the measure of risk using other subsidiary tools beside a Shannon entropy, but we can have only a Renyi entropy and change the measure of Risk only by changing the α. In other words, one who choose the model Renyi($\alpha > 1$)-Mean-Semi variance for selecting his portfolio is risk-prone person and vice versa a person who choose the model Renyi($\alpha < 1$)-Mean-Semi variance is considered as risk-averse.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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