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Research Article

Relation-Theoretic Common Fixed Point Theorems for a Pair of Implicit Contractive Maps in Metric Spaces

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Abstract. In this manuscript, we demonstrate some common fixed point theorems for a pair of weakly compatibleness of the operators under implicit contractive properties in metric spaces endowed with binary relation. An illustration is establish to emphaize the cogency of our results. The outcome of our proofs are unification of several relation theorems existing in the literature.

Keywords. Common fixed points; Weakly compatible mappings; Binary relation; Implicit function

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1. Introduction

The metrical fixed point results in different abstract spaces equipped with partial ordering have gained rigorous attentions in the application of fixed point theory to mathematics and applied mathematics. The existence and uniqueness of common fixed point for pairs of contractive mappings for both self and non-self mappings are being studied by many authors (for instance Eke *et al*. [8], Eke and Akewe [9], and Eke [10]). The idea of partial ordering was first initiated by Turinici [19] unknownly to Ran and Reurings [18] in 1986. In [18] fixed point theorem for Banach contraction operators in metric spaces equipped with partial ordered is proved. In this same reference, the solution of schematic linear equations established under certain conditions exists and is unique. Several fixed point results arose out of this idea. As a generalization of partial ordering, many authors employed several versions of binary relations to established their various fixed point theorems (see [7], [11]) and others in literature.

Alam and Imdad [2] proved fixed point theorem for the classical Banach contraction mappings in a completed metric space equipped with binary relation. In this result, it was discovered that the contraction condition hold only for those elements linked with the binary relation not for every pair of elements. Further, Alam and Imdad [3] generalized the result in [2] by employing some topological properties such as; completeness, closedness, continuity, compatibility in the context of relation-theoretic to prove the existence and uniqueness of common fixed point for a pair of Banach contraction principle in metric spaces equipped with arbitrarily binary relation.

The work of Popa [17] in 1997 introduced the concept of implicit functions whose strength lie in the unification power of more general contractive conditions. Many researchers found it interesting to work in this area. In particular, Ali and Imdad [11] proved common fixed point theorem for two pairs of weakly compatible operators by employing the implicit function without continuity, thereby generalizing the work of Popa [17].

Ahmadullah *et al*. [1] proved that the fixed point of an implicit contraction operators in a metric space with respect to binary relation exists and is unique. Based on Ahmadullah *et al*. [1], we demonstrate the existence and uniqueness of the common fixed point of two nonlinear contractive operators employing an implicit function in a metric space associated with arbitrarily binary relation in this paper.

2. Preliminaries

Definition 2.1 ([12–14])**.** Let X be a non empty set and (f,g) a pair of self-mappings. Then

(i) $a \in X$ is known as a coincidence point of (f,g) if $fa = ga$.

(ii) $b \in X$ is known as a point of coincidence of (f,g) assuming $a \in X$ gives $b = fa = ga$.

(iii) (f,g) has a common fixed point at the coincidence point $a \in X$ if $a = fa = ga$.

(iv) (f,g) is said to commute if $f(ga) = g(fa)$, $\forall\ a \in X$.

(v) (f,g) is said to be weakly compatible if they commute at their coincidence point, that is, for any $a \in X$,

$$fa = ga \Longrightarrow f(ga) = g(fa).$$

(vi) f is called g-continuous at $a \in X$ if for every sequence $\{a_n\} \subset X$,

$$g(a_n) \to g(a) \Longrightarrow f(a_n) \to f(a).$$

Note that f is g-continuity if is g-continuity at every point of X.

Lemma 2.2 ([4])**.** *Suppose X is a nonempty set with f and g a self-map. Let (f,g) be weakly compatible, then each point of coincidence for the pair (f,g) is also a coincidence point for the pair (f,g).*

2.1 Implicit Functions

This subsection deals with the definition of implicit mapping along with various examples that could be found in some works in the existing literature (e.g. see [15], [16]). According to Ahmadullah *et al*. [1] and for the purpose of our results we give the following definitions of the implicit mapping:

Let E be the accumulation of every continuous real valued mappings $E : \mathbb{R}_+^6 \to \mathbb{R}$ that fulfill the following properties:

(E_1) E is decreasing in the fifth variable and $E(q,p,p,q,q+p,0) \leq 0$, for $q,p \geq 0$ means that there is $0 \leq h < 1$ such that $q \leq hp$;

(E_2) $E(q,0,q,0,q,0) > 0$, for every $q > 0$.

Suppose G is a different collection of all continuous real valued function $E : \mathbb{R}_+^6 \to \mathbb{R}$ that gratify (E_1) and (E_2) together with the below condition;

(E_3) E is decreasing in the sixth variable, and $E(q,q,0,0,q,q) > 0$, for every $q > 0$.

2.2 Relation-Theoretic Concepts

This part of the work consider some basic definitions, propositions and some results that will be needed in proofing our main results. Note that in this work, λ represents a non-empty binary relation whereas $\mathbb{N}_0$ represent the positive whole numbers, that is, $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

Definition 2.3 ([14])**.** Let λ be a binary relation and is a subset of non-empty set X. Then we write that "a related to b" under λ is equivalent to $(a,b) \in \lambda$.

Definition 2.4 ([18])**.** Suppose λ is a binary relation defined on a non-empty set X. Then each pair of points $a,b \in X$ is known as λ-comparative if either $(a,b) \in \lambda$ or $(b,a) \in \lambda$, which resulted to $[a,b] \in \lambda$.

Definition 2.5 ([11])**.** A binary relation is complete if its elements are comparable (i.e., $[a,b] \in \lambda$ for each $a,b \in X$).

Definition 2.6 ([11])**.** Suppose X is a non-empty set with a binary relation λ defined on it. Then

(i) the inverse of λ, represented by λ^{-1} is defined as $\lambda^{-1} = \{(a,b) \in X^2 : (b,a) \in \lambda\}$;

(ii) the least symmetric relation having λ (i.e., $\lambda^s := \lambda \cup \lambda^{-1}$) is called symmetric closure of λ. Usually, it is represented by λ^s.

Proposition 2.7 ([2])**.** *If λ is a binary relation defined on a non-empty set X, then $(a,b) \in \lambda^s$ is equivalent to $[a,b] \in \lambda$.*

Definition 2.8 ([2])**.** If λ is a binary relation defined on a non-empty set X. Then $\{a_n\} \subset X$ is known as λ-upholding if $(a_n,a_{n+1}) \in \lambda$, for every $n \in \mathbb{N}_0$.

Definition 2.9 ([2])**.** Suppose X is a nonempty set and $f : X \to X$. Then λ is said to be f-closed if

$$(a,b) \in \lambda \Longrightarrow (fa,fb) \in \lambda, \text{ for each } a,b \in X.$$

Definition 2.10 ([3]). Suppose (X,d,λ) is a metric space equppied with a binary relation λ. We say that X is λ-complete if each λ-upholding Cauchy sequence in X has a limit in X.

Definition 2.11 ([3]). Suppose (X,d,λ) is a metric space equipped with a binary relation λ. An operator $f : X \to X$ is known as λ-continuous at a if every λ-upholding sequence $\{a_n\}$ converges to a implies fa_n converges to fa. Usually, f is λ-continuous if the whole space is λ-continuous.

Definition 2.12 ([2]). If (X,d,λ) is a metric space equppied with a binary relation λ. We say λ is d-self-closed if each λ-upholding sequence $\{a_n\}$ converges to a, there is a subsequence $\{a_{n_k}\}$ of $\{a_n\}$ such that $[a_{n_k},a] \in \lambda$, for each $k \in \mathbb{N}_0$.

Definition 2.13 ([14]). Suppose X is a non-empty set with a binary relation defined on it with $a,b \in X$. Suppose there exists a finite sequence $\{c_0,c_1,c_2,\cdots,c_l\} \subset X$ such that $c_0 = a$, $c_l = b$ and (c_i,c_{i+1}) in λ for every $i \in \{0,1,2,\cdots,l-1\}$, we have the finite sequence to be the track of length l (where $l \in \mathbb{N}$) that connect a to b.

3. Common Fixed Point Results

This part of the work establish several common fixed point theorems for two weakly compatibleness of the operators gratifying certain implicit contraction properties in metric spaces equipped with binary relation. We equally provide an illustration to validate our results.

Theorem 3.1. *Suppose (X,d,λ) is a metric space endowed with a binary relation. If (f,g) be a pair of selfmapping on X gratifying the below properties:*

(i) *$(Y,d) \subseteq X$ and X is λ-complete.*

(ii) *$f(X) \subseteq g(X) \cap Y$*

(iii) *there exists $a_0 \in X$ such that $(ga_0,fa_0) \in \lambda$.*

(iv) *λ to be (f,g)-closed.*

(v) *$Y \subset g(X)$*

(vi) *either f is (g,λ)-continuous or $\lambda/_Y$ is d-self closed.*

(vii) *there is an implicit mapping $E \in \psi$ with*

$$E(d(fa,fb),d(ga,gb),d(ga,fa),d(gb,fb),d(ga,fb),d(gb,fa)) \le 0, \tag{3.1}$$

for every $a,b \in X$ with $(a,b) \in \lambda$.

Then (f,g) has a point of coincidence.

Proof. Take an arbitrary $a_0 \in X$ such that $(ga_0,fa_0) \in \lambda$. Construct an iterative sequence $\{ga_n\}$ that gives

$$ga_{n+1} = fa_n, \quad \text{for every } n \in \mathbb{N}_0.$$

Since $(ga_0,fa_0) \in \lambda$ and λ is (f,g)-closed then we obtain,

$$(fa_0,fa_1),(fa_2,fa_3),\cdots,(fa_n,fa_{n+1}) \in \lambda.$$

Therefore,

$$(ga_n, ga_{n+1}) \in \lambda, \quad \forall\ n \in \mathbb{N}_0.$$

This implies that $\{ga_n\}$ is λ-upholding. Using (vii) we have

$$E(d(fa_n, fa_{n+1}), d(ga_n, ga_{n+1}), d(ga_n, fa_n), d(ga_{n+1}, fa_{n+1}), d(ga_n, fa_{n+1}),$$
$$d(ga_{n+1}, fa_n)) \leq 0$$

or

$$E(d(ga_{n+1}, ga_{n+2}), d(ga_n, ga_{n+1}), d(ga_n, ga_{n+1}), d(ga_{n+1}, ga_{n+2}),$$
$$d(ga_n, ga_{n+2}), d(ga_{n+1}, ga_{n+1})) \leq 0$$

or

$$E(d(ga_{n+1}, ga_{n+2}), d(ga_n, ga_{n+1}), d(ga_n, ga_{n+1}), d(ga_{n+1}, ga_{n+2}), d(ga_n, ga_{n+1})$$
$$+ d(ga_{n+1}, ga_{n+2}), d(ga_{n+1}, ga_{n+1})) \leq 0. \tag{3.2}$$

Let $q = d(ga_{n+1}, ga_{n+2})$ and $p = d(ga_n, ga_{n+1})$, and substitute in (3.2) to get

$$E(q, p, p, q, p+q, 0) \leq 0.$$

Using (E_1) implies that there is $h \in [0,1)$ such that $q \leq hp$ yields

$$d(ga_{n+1}, ga_{n+2}) \leq h d(ga_n, ga_{n+1}).$$

Consequently, we have

$$d(ga_{n+1}, ga_{n+2}) \leq h^{n+1} d(ga_0, ga_1), \quad \forall\ n \in \mathbb{N}_0.$$

For $n, m \in \mathbb{N}_0$ with $n > m$ and utilizing triangle inequality, we have

$$\begin{aligned} d(ga_m, ga_n) &\leq d(ga_m, ga_{m+1}) + d(ga_{m+1}, ga_{m+2}) + \cdots + d(ga_{n-1}, ga_n) \\ &\leq (h^m + h^{m+1} + \cdots + h^{n-1+m}) d(ga_0, ga_1) \leq \frac{h^m}{1-h} d(ga_0, ga_1) \\ &\to 0 \ \text{ as } \ m \to \infty \end{aligned}$$

This shows that $\{ga_n\}$ is λ-upholding Cauchy sequence. Afterward, $\{ga_n\} \subseteq g(X)$ and $\{ga_n\} \subseteq g(X) \cap Y$ with respect to (ii) and (v), we obtain $\{ga_n\}$ is λ-upholding Cauchy sequence in Y. Since (Y, d) is λ-complete, there is $b \in Y$ so that $\{ga_n\}$ converges to b. As $Y \subseteq g(X)$, there is $a \in X$ so that

$$\lim_{n\to\infty} ga_n = b = ga.$$

With the (g,λ)-continuity of f, and in view of (i) and (vii), we get

$$\lim_{n\to\infty} ga_{n+1} = \lim_{n\to\infty} fa_n = fa.$$

Since the limit of a sequence is unique then we obtain $fa = ga$. Therefore, the coincidence point of f and g is a.

Suppose that $\lambda/_Y$ is d-self-closed, then for every $\{ga_n\} \in Y$ with $\{ga_n\}$ converging to ga, there exists $\{ga_{n_k}\}$ of $\{ga_n\}$ such that $[ga_{n_k}, ga] \in \lambda/_Y \subseteq \lambda$, for every $k \in \mathbb{N}_0$.

Note that $[ga_{n_k}, ga] \in \lambda$, $\forall\ k \in \mathbb{N}_0$ implies that either $(ga_{n_k}, ga) \in \lambda$, $\forall\ k \in \mathbb{N}_0$ or $(ga, ga_{n_k}) \in \lambda$, $\forall\ k \in \mathbb{N}_0$.

Using condition (vii) and $[ga_{n_k}, ga] \in \lambda$, $\forall\ k \in \mathbb{N}_0$, we obtain

$$E(d(fa_{n_k}, fa), d(ga, ga_{n_k}), d(ga, fa), d(ga_{n_k}, fa_{n_k}), d(ga_{n_k}, fa), d(ga, fa_{n_k})) \le 0$$

or

$$E(d(fa, fa_{n_k}), d(ga_{n_k}, ga), d(fa, ga), d(fa_{n_k}, ga_{n_k}), d(fa, ga_{n_k}), d(fa_{n_k}, ga)) \le 0.$$

Taking $n \to \infty$ and using $ga_{n_k} \to ga$ with the fact that d is continuous and g is λ-continuous, we get

$$E(d(fa, ga), 0, d(fa, ga), 0, d(fa, ga), 0) \le 0.$$

Hence, using (E_2), we have

$$d(fa, ga) = 0, \text{ this implies that } fa = ga.$$

Similarly, employing $(ga, ga_{n_k}) \in \lambda$, $\forall\ k \in \mathbb{N}_0$ and owing to (E_2), we obtain

$$d(fa, ga) = 0 \Longrightarrow fa = ga. \qquad \square$$

The following theorem proves the existence and uniqueness of the common fixed point of f and g.

Theorem 3.2. *In view of the conditions stated in Theorem* 3.1 *and assuming the following axiom satisfies:*

(vii) $f(X)$ *is* $\lambda/^{s}_{gX}$ *connected.*

Then the point of coincidence for (f, g) *is unique. Furthermore, if* (f, g) *is weakly compatible, then their common fixed point is unique.*

Proof. It has been proved in Theorem 3.1 that f and g have a coincidence point. Suppose we consider two different points of coincidence for f and g say $\bar{a}$ and $\bar{b}$, with $a, b \in X$ so that

$$\bar{a} = ga = fa \text{ and } \bar{b} = gb = fb. \tag{3.3}$$

Then, we shall show that $\bar{a} = \bar{b}$.

Since $fa, fb \in f(X) \subseteq g(X)$, by (vii) there is a path (say$(gu_0, gu_1, \cdots, gu_k)$) of certain finite length $k \in \lambda/^{s}_{g(X)}$ from fa to fb (while $u_0, u_1, \cdots, u_k \in X$). Using (3.3) and without loss of generality, we choose $u_0 = a$ and $u_k = b$. Therefore, we obtain

$$[gu_i, gu_{i+1}] \in \lambda/_{g(X)} \quad \text{for all } i \text{ where } i \in [0, k-1]. \tag{3.4}$$

If we fixed the sequence $u_n^0 = \bar{a}$ and $u_n^k = \bar{b}$, then utilizing (3.3) gives

$$gu_{n+1}^0 = fu_n^0 = \bar{a} \text{ and } gu_{n+1}^k = fu_n^k = \bar{b} \quad \text{for all } n \in \mathbb{N}_0.$$

Substitute $u_0^1 = u_1, u_0^2 = u_2, \cdots, u_0^{k-1} = u_{k-1}$. Recall that $f(X) \subseteq g(X)$. So, we construct sequences $\{u_n^1\}, \{u_n^2\}, \cdots, \{u_n^{k-1}\} \in X$ such that

$$gu_{n+1}^1 = fu_n^1, gu_{n+1}^2 = fu_n^2, \cdots, gu_{n+1}^{k-1} = fu_n^{k-1}, \quad \forall\ n \in \mathbb{N}_0.$$

Thus, we obtain

$$gu_{n+1}^i = fu_n^i, \quad \forall\ n \in \mathbb{N}_0 \text{ and for all } i \text{ where } i \in [0, k-1]. \tag{3.5}$$

Now we show that

$$[gu_n^i, gu_n^{i+1}] \in \lambda \ \ \forall\ n \in \mathbb{N}_0 \text{ and for all } i \text{ where } i \in [0, k-1].$$

Applying the mathematical induction principle, we prove that it is true for $n = 0$ by using (3.3) and (3.5).

If (3.5) holds for $n = r > 0$ that is,

$$[gu_r^i, gu_r^{i+1}] \in \lambda \ \ \forall\ r \in \mathbb{N}_0 \text{ and for all } i \text{ where } i \in [0, k-1].$$

Then, using λ is (f,g)-closed yields

$$[fu_r^i, fu_r^{i+1}] \in \lambda \ \ \forall\ r \in \mathbb{N}_0 \text{ and for all } i \text{ where } i \in [0, k-1],$$

which on using (3.5) yields

$$[gu_{r+1}^i, gu_{r+1}^{i+1}] \in \lambda \ \ \forall\ r \in \mathbb{N}_0 \text{ and for all } i \text{ where } i \in [0, k-1].$$

Hence it is true for $n = r+1$. Therefore (3.5) holds for every $n \in \mathbb{N}_0$. Define $q_n^i = d(gu_n^i, gu_n^{i+1})$ for every $n \in \mathbb{N}_0$ and for all i where $i \in [0, k-1]$. We claim that

$$\lim_{n\to\infty} q_n^i = 0.$$

Let $\lim_{n\to\infty} q_n^i = r > 0$. In view of (3.4) and (3.5) and hypothesis (iv) and (vii) for all i where $i \in [0, k-1]$ and for every $n \in \mathbb{N}_0$, we have

$$E(d(fu_n^i, fu_n^{i+1}), d(gu_n^i, gu_n^{i+1}), d(gu_n^i, fu_n^i), d(gu_n^{i+1}, fu_n^{i+1}), d(gu_n^{i+1}, fu_n^i),$$
$$d(gu_n^i, fu_n^{i+1})) \le 0$$

or

$$E(d(gu_{n+1}^i, gu_{n+1}^{i+1}), d(gu_n^i, gu_n^{i+1}), d(gu_n^i, gu_{n+1}^i), d(gu_n^{i+1}, gu_{n+1}^{i+1}), d(gu_n^i, gu_{n+1}^{i+1}),$$
$$d(gu_n^{i+1}, gu_{n+1}^i)) \le 0$$

Letting $n \to \infty$ and using $\lim_{n\to\infty} q_n^i = r$, we have

$$E(r, r, 0, 0, r, r) \le 0.$$

This contradicts (E_3), thus

$$\lim_{n\to\infty} q_n^i = r = 0.$$

Like-wisely, if $(gu_n^{i+1}, gu_n^i) \in \lambda$, then following the same process, we have

$$\lim_{n\to\infty} q_n^i = r = 0.$$

Therefore

$$\lim_{n\to\infty} q_n^i = \lim_{n\to\infty} d(gu_n^i, gu_n^{i+1}) = 0 \quad \text{for all } i \text{ where } i \in [0, k-1].$$

In view of $\lim_{n\to\infty} q_r^i = 0$ and the triangle inequality, we obtain

$$\begin{aligned} d(\bar{a}, \bar{b}) = d(u_n^0, u_n^k) &\le \sum_{i=0}^{k-1} d(gu_{n+1}^0, gu_{n+1}^k) \\ &= \sum_{i=0}^{k-1} q_n^i \to 0 \ \text{ as } \ n \to \infty \end{aligned}$$

Hence $d(\bar{a}, \bar{b}) = 0$. This leads to $\bar{a} = \bar{b}$. Therefore, the point of coincidence for f and g is unique.

Since the weakly compatibleness of f and g with their unique point of coincidence that is, $\bar{a} = \bar{b}$ together with Lemma 2.2 gives $\bar{a} = a$. Then $ga = g\bar{a}$ yields $\bar{a} = g\bar{a} = f\bar{a}$. Thus $\bar{a}$ is the common fixed point of f and g. Next, we demonstrate the uniqueness, let a^* be a different common fixed point of f and g. Following the same procedure as above, we have $a^* = \bar{a}$. Thus the proved. □

Example 3.3. Suppose $X = [0,4]$ is a metric space with the usual metric. Define a binary relation $\lambda = \{(a,b) \in X^2 : a \geq b, a,b \geq 0\} \cup \{(a,b) \in X^2 : a = b, a,b \geq 0\}$. Define $f,g : X \to X$ by $f(a) = 1 \ \forall \ a \in [0,4]$ and

$$g(a) = \begin{cases} \frac{a}{2}, & \text{if } a \in (1,2] \\ 1, & \text{if } a \in (2,4) \end{cases} \tag{3.6}$$

Let $Y = [\frac{2}{3},1]$, this implies that $f(X) = \{1\} \subset Y \subset g(X) = (\frac{1}{2},1]$ and Y is λ-complete. Clearly, λ is (f,g)-closed and f and g are λ-continuous. Choose $\{ga_n\} \in Y$ with ga_n converging to ga such that $\{ga_n, ga_{n+1}\} \in \lambda$, for every $n \in \mathbb{N}_0$.

Now we see that $\{ga_n, ga_{n+1}\} \in \lambda$ for every $n \in \mathbb{N}_0$, and there exists $n \in \mathbb{N}_0$ such that $ga_n = ga \in \{1,2,3,4\}$ for every $n \in \mathbb{N}$. Construct a subsequence $\{ga_{n_k}\}$ of $\{ga_n\}$ such that $\lim\limits_{k\to\infty} ga_{n_k} = ga$ for all $k \in \mathbb{N}_0$, this implies that $\{ga_{n_k}, ga\} \in \lambda$, for every $k \in \mathbb{N}_0$. Thus λ is $\lambda/_Y$ d-self-closed.

Define an implicit function $E : \lambda_+^6 \to \lambda$ by

$$E(f_1,f_2,f_3,f_4,f_5,f_6) = f_1 - \frac{4}{25} f_5 - \frac{2}{5} f_6.$$

This satisfies the requirement of our implicit function. f and g have a coincidence point. According to Theorem 3.2 the unique common fixed point of f and g is 1.

Remark 3.4. In Theorem 3.1 $f = g$ gives the result of Ahmadullah *et al*. [1, Theorem 1]. Theorem 3.1 generalized Alam and Imdad [3, Theorem 5] by replacing the contractive mappings with implicit contraction mappings.

Corollary 3.5. *Let (X,d,λ) be a metric space associated with a binary relation. If f is a selfmapping on X and fulfills below axioms:*

(i) *$(Y,d) \subseteq X$ and X is λ-complete.*

(ii) *there exist $x_0 \in X$ such that $fx_0 \in \lambda$.*

(iii) *λ is f-closed.*

(iv) *either f is λ-continuous or λ/Y is d-self closed.*

(v) *there is an implicit mapping $E \in \psi$ with*

$$E(d(fa,fb),d(a,b),d(a,fa),d(b,fb),d(a,fb),d(b,fa)) \leq 0, \tag{3.7}$$

for each $a,b \in X$ with $(a,b) \in \lambda$.

Then f has a fixed point.

4. Conclusion

This research work proves that the common fixed point for two weakly compatibleness of the operators gratifying certain contractive conditions associated with an implicit function in a metric space endowed with binary relation exist and is unique. This research suggests further generalization of the mappings of these results in future research works.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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