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Some Stability Charts of A Neural Field Model of Two Neural Populations

Research Article

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Abstract. In this paper we study on the neural field model of two neuron populations. We make the stability analysis of the linearized model by considering the effect of the synaptic connectivity function. We separate the plane into regions on which we find the number of roots with positive real parts. Hence we find the asymptotic stability region. To separate the plane we use the D-curves and we determine some properties of these curves.

Keywords. Neural field model; Asymptotic stability

MSC. 37C75; 34K20; 45J05

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1. Introduction

The neural field models are considered very frequently in neuroscience. They model the activity of a large neuron populations in the brain in a continuous space. Hence they are described by using the integral equations or integro differential equations. The most important studies for these models are made by Wilson and Cowan, Amari [13], [1]. These models describe the mean activity of neural populations. For some biological reasons such as the finite speed of propagation of an action potential and the release of neurotransmitter, a delay term is added to these models. These delay terms may affect the location of the characteristic roots on the plane and hence the stability of the systems. Hence these effects on the stability and the existence of the solutions of the system is investigated in many papers [3], [4], [6], [9], [10], [12].

The stability of the neural field models is also considered in many articles. Some scientists used numerical techniques as in [2], [5], a center manifold result is given by Veltz and Faugeras [11], a study for the existence and uniqueness of the solution for these models is made by Faye and Faugeras [5].

In this study we focus on the stability regions of a neural field model for two neuron populations by using D-subdivision method. Also we study on some properties of D-curves. In Section 2.1, we consider the case that the synaptic connectivity function $J_{21}(x,y)=0$ and we study on the stability properties by determining the D-curves. In Section 2.2 we consider that $J_{21}(x,y)\neq 0$ and we show the stability regions by sketching the D-curves.

2. Stability Analysis of the Model

Consider the neural field model for p neural population on the space $\Omega\subset R^d$ which presenting the dynamics of mean membran potential

$$\left.\begin{aligned}&\left(\frac{d}{dt}+l_i\right)V_i(t,r)=\sum_{j=1}^{p}\int_{\Omega}J_{ij}(r,\bar{r})S[\sigma_j(V_j(t-\tau_{ij}(r,\bar{r}),\bar{r})-h_j)]d\bar{r}+I_i^{ext}(r,t),\ t\geq 0, 1\leq i\leq p\\&V_i(t,r)=\phi_i(t,r),\quad t\in[-T,0]\end{aligned}\right\}\tag{2.1}$$

given in [11], [12].

In this study we study on the linear neural field model for two neural population ($p=2$). Here $V_1(x,t)$ and $V_2(x,t)$ describe the synaptic inputs for a large group of neurons at position x and time t, and $\frac{d}{dt}V_1(x,t)$ and $\frac{d}{dt}V_2(x,t)$ describe time derivative. The synaptic connectivity function $J_{ij}(x,y)$ which is π periodic even function describes how neurons in the jth population at position y influence the neurons in the ith population at position x, i.e., determines the coupling between the neurons. The stability of these solutions can be determined by examining the linearized system and using the D-partition method. We consider that the delay term is constant, hence we take the maximum delay as $\tau(x-y)=\tau$. We assume that $x,y\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ and the boundary conditions are periodic. Hence the model we considered is the following

$$\begin{aligned}&\frac{d}{dt}U_1(x,t)+l_1U_1(x,t)\\&\quad=\sigma_1 s_1\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}J_{11}(x,y)U_1(y,t-\tau(x-y))dy+\sigma_2 s_1\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}J_{12}(x,y)U_2(y,t-\tau(x-y))dy,\end{aligned}\tag{2.2}$$

$$\begin{aligned}&\frac{d}{dt}U_2(x,t)+l_2U_2(x,t)\\&\quad=\sigma_1 s_1\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}J_{21}(x,y)U_1(y,t-\tau(x-y))dy+\sigma_2 s_1\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}J_{22}(x,y)U_2(y,t-\tau(x-y))dy.\end{aligned}\tag{2.3}$$

2.1 The Case: $J_{21}(x,y)=0$

In this section, we study the case where neurons in the first population excite each other but inhibit the neurons in the second one. Moreover, neurons in the second neuron population excite each other and the neurons in the first population.

To get the characteristic equation, we take $U_1(x,t) = u_1(t)e^{ikx}$, $U_2(x,t) = u_2(t)e^{ikx}$ as in Fourier method and then writing $u_1(t) = c_1 e^{\lambda t}$ and $u_2(t) = c_2 e^{\lambda t}$ we get the following equations

$$\lambda e^{ikx}u_1(t) + l_1 e^{ikx}u_1(t) - K_1 e^{-\lambda\tau}u_1(t)\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{11}(x,y)e^{iky}dy - K_2 e^{-\lambda\tau}u_2(t)\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{12}(x,y)e^{iky}dy = 0,$$

$$\lambda e^{ikx}u_1(t) + l_2 e^{ikx}u_2(t) - K_2 e^{-\lambda\tau}u_2(t)\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{22}(x,y)e^{iky}dy = 0.$$

The solutions of this system are functions $\cos(2nx)$ and $\sin(2nx)$ [10]. Hence the equations for the characteristic values λ are

$$\lambda u_1(t) + l_1 u_1(t) - K_1 e^{-\lambda\tau}u_1(t)\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{11}(y)e^{iky}dy - K_2 e^{-\lambda\tau}u_2(t)\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{12}(y)e^{iky}dy = 0,$$

$$\lambda u_2(t) + l_2 u_2(t) - K_2 e^{-\lambda\tau}u_2(t)\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{22}(y)e^{iky}dy = 0. \tag{2.4}$$

Here we consider $F_1 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{11}(y)e^{iky}dy$, $F_2 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{12}(y)e^{iky}dy$ and $F_3 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{22}(y)e^{iky}dy$.

Considering this system of equations for functions $u_1(t)$ and $u_2(t)$ we get the following equation

$$\lambda^2 + \lambda l_2 - \lambda K_2 e^{-\lambda\tau}F_3 + \lambda l_1 + l_1 l_2 - l_1 K_2 e^{-\lambda\tau}F_3 - K_1 e^{-\lambda\tau}F_1\lambda - K_1 e^{-\lambda\tau}F_1 l_2 + K_1 K_2 e^{-2\lambda\tau}F_1 F_3 = 0. \tag{2.5}$$

We choose the parameter space as (l_1, K_1). Writing $\lambda = \mu + i\nu$ and taking $\mu = 0$ we split the real and imaginary parts of (2.5) we have

$$P: -\nu^2 - \nu K_2\sin(\tau\nu) + l_1 l_2 - l_1 K_2\cos(\tau\nu)F_3 - K_1\nu\sin(\tau\nu)F_1$$
$$-K_1 l_2\cos(\tau\nu)F_1 + K_1 K_2\cos(2\tau\nu)F_1 F_3 = 0, \tag{2.6}$$

$$R: -\nu K_2 F_3\cos(\tau\nu) + \nu l_2 + \nu l_1 + l_1 K_2\sin(\tau\nu)F_3 - K_1\nu\cos(\tau\nu)F_1$$
$$+K_1 l_2\sin(\tau\nu)F_1 - K_1 K_2\sin(2\tau\nu)F_1 F_3 = 0. \tag{2.7}$$

To sketch the boundaries of subregions on the parameter space we have the expressions for l_1 and K_1 depending on the parameter ν.

$$l_1 = \frac{-\nu^3\cos(\tau\nu)F_1 - \nu^2 K_2\sin(2\tau\nu)F_1F_3 + \nu K_2 l_2 F_1 F_3}{\left(\begin{array}{l} l_2^2 F_1\sin(\tau\nu) - l_2 K_2\sin(2\tau\nu)F_1F_3 + K_2\nu F_1F_3 - K_2^2\sin(\tau\nu)F_3^2F_1 \\ +\nu^2 F_1\sin(\tau\nu) - \nu K_2\cos(2\tau\nu)F_1F_3 \end{array}\right)}$$
$$+\frac{-\nu K_2^2\cos(\tau\nu)F_3^2F_1 - \nu l_2^2\cos(\tau\nu)F_1 + \nu K_2 l_2\cos(2\tau\nu)F_1F_3}{\left(\begin{array}{l} l_2^2 F_1\sin(\tau\nu) - l_2 K_2\sin(2\tau\nu)F_1F_3 + K_2\nu F_1F_3 - K_2^2\sin(\tau\nu)F_3^2F_1 \\ +\nu^2 F_1\sin(\tau\nu) - \nu K_2\cos(2\tau\nu)F_1F_3 \end{array}\right)}, \tag{2.8}$$

$$K_1 = \frac{2\nu l_2 K_2 F_3\cos(\tau\nu) - 2\nu^2 K_2\sin(\tau\nu)F_3 - \nu K_2^2F_3^2 - \nu l_2^2 - \nu^3}{\left(\begin{array}{l} l_2^2 F_1\sin(\tau\nu) - l_2 K_2\sin(2\tau\nu)F_1F_3 + K_2\nu F_1F_3 - K_2^2\sin(\tau\nu)F_3^2F_1 \\ +\nu^2 F_1\sin(\tau\nu) - \nu K_2\cos(2\tau\nu)F_1F_3 \end{array}\right)} \tag{2.9}$$

and the line

$$(l_2 - K_2F_3)l_1 + (K_2F_1F_3 - F_1l_2)K_1 = 0 \tag{2.10}$$

for $\nu = 0$ is the singular line.

Now we give two theorems about the D-curves. The D-curves sketched for values $\theta = \tau\nu$ and chosen in the $J_n = (n\pi, (n+1)\pi)$, $n = 0, 1, 2, \ldots$ are called by $C_n(F_1, F_3, K_2, l_2)$ in the parameter space (l_1, K_1).

Theorem 1. *The curves $C_n(F_1, F_3, K_2, l_2)$ do not intersect each other.*

Proof. Assume that $l_1(\theta_1) = l_1(\theta_2)$ and $K_1(\theta_1) = K_1(\theta_2)$ for $\theta_1 \in J_a$, $\theta_2 \in J_b$, $a \neq b$. This yield $\cos(\theta_1) = \cos(\theta_2)$ and $\sin(\theta_1) = \sin(\theta_2)$ and for the choose of $\theta_1 \in J_a$ and $\theta_2 \in J_b$, $a \neq b$ we get $\theta_1 = \theta_2$. Hence we conclude that the curves $C_n(F_1, F_3, K_2, l_2)$ do not intersect each other. □

Theorem 2. *The curves $C_n(F_1, F_3, K_2, l_2)$ intersect the line $l_1 = 0$ only once.*

Proof. Taking $\theta = \tau\nu$ we get

$$l_1(\theta) = \frac{\begin{pmatrix} -\frac{\theta^3}{\tau^3}\cos(\theta)F_1 - \frac{\theta^2}{\tau^2}K_2\sin(2\theta)F_1F_3 + \frac{\theta}{\tau}K_2l_2F_1F_3 \\ -\frac{\theta}{\tau}K_2^2\cos(\theta)F_3^2F_1 - \frac{\theta}{\tau}l_2^2\cos(\theta)F_1 + \frac{\theta}{\tau}K_2l_2\cos(2\theta)F_1F_3 \end{pmatrix}}{\begin{pmatrix} l_2^2F_1\sin(\theta) - l_2K_2\sin(2\theta)F_1F_3 + K_2\frac{\theta}{\tau}F_1F_3 - K_2^2\sin(\theta)F_3^2F_1 \\ +\frac{\theta^2}{\tau^2}F_1\sin(\theta) - \frac{\theta}{\tau}K_2\cos(2\theta)F_1F_3 \end{pmatrix}} \tag{2.11}$$

The roots for $l_1(\theta) = 0$ are $\theta = \frac{\pi}{2} + n\pi$, $n = 0, 1, 2, \ldots$. For the values $\theta = \tau\nu$ chosen in the regions $J_n = (n\pi, (n+1)\pi)$ the coordinates $K_1(\theta)$ are determined uniquely. Hence the curves $C_n(F_1, F_3, K_2, l_2)$ intersect the line $l_1 = 0$ only once. □

To sketch the D-curves in the parameter space (l_1, K_1) we choose the parameters as $K_2 = F_1 = l_2 = \tau = 1$ and $F_3 = 2$. Hence we have

$$l_1 = \frac{-\nu^3\cos(\nu) - 2\nu^2\sin(2\nu) + 2\nu - 5\nu\cos(\nu) + 2\nu\cos(2\nu)}{5\sin(\nu) - 2\sin(2\nu) + 2\nu + 2\nu^2\sin(\nu) - 2\nu\cos(2\nu)}, \tag{2.12}$$

$$K_1 = \frac{4\nu\cos(\nu) - 4\nu^2\sin(\nu) - 5\nu - \nu^3}{5\sin(\nu) - 2\sin(2\nu) + 2\nu + 2\nu^2\sin(\nu) - 2\nu\cos(2\nu)}. \tag{2.13}$$

The limit point for the intersection of the singular line and the D-curve is $\left(\lim_{\nu\to 0} l_1, \lim_{\nu\to 0} K_1\right) = (-1, -1)$.

To determine the asymptotic stability and unstability regions we choose any point $B(l_0, K_0)$ in any subregion separated by the D-curves. Then we use the following Stépán's formula [8], [7]

$$k = m + (-1)^m \sum_{j=1}^{s} (-1)^{j+1}\operatorname{sgn}(R(\rho_j, l_0, K_0)) \tag{2.14}$$

where $d = 2m$, $m \in Z^+$, and ρ_js, $j = 1, \ldots, s$ are the positive real roots of $P(\nu, l_0, K_0)$ such $\rho_1 \geq \ldots \geq \rho_s$. Here k denotes the number of characteristic roots with positive real parts.

Now we have the graph representing the regions of asymptotic stability (where $k = 0$) and unstability (where $k > 0$).

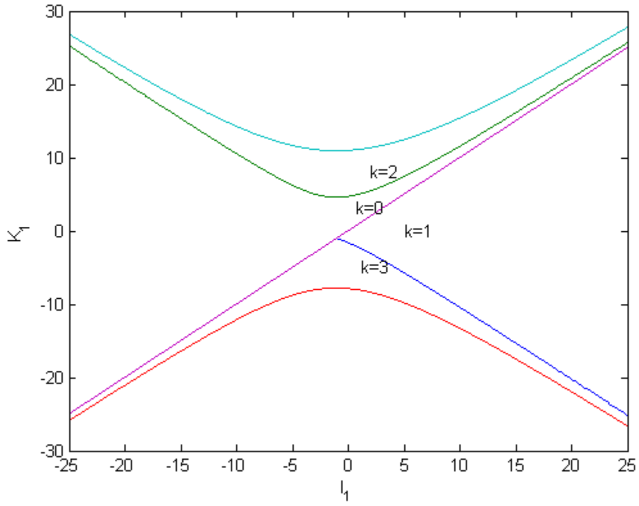

Figure 1. Stability chart for the system

2.2 The Case: $J_{21}(x,y) \neq 0$

In this section, we study the case where neurons in each population excite nearby neurons and more distant neurons.

We have the following characteristic equations after using the Fourier transformations

$$\lambda e^{ikx}u_1(t) + l_1 e^{ikx}u_1(t) - K_1 e^{-\lambda\tau}u_1(t)\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{11}(x,y)e^{iky}dy - K_2 e^{-\lambda\tau}u_2(t)\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{12}(x,y)e^{iky}dy = 0, \tag{2.15}$$

$$\lambda e^{ikx}u_2(t) + l_2 e^{ikx}u_2(t) - K_1 e^{-\lambda\tau}u_1(t)\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{21}(x,y)e^{iky}dy - K_2 e^{-\lambda\tau}u_2(t)\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{22}(y)e^{iky}dy = 0. \tag{2.16}$$

Hence for characteristic values λ we have the following equations

$$\lambda u_1(t) + l_1 u_1(t) - K_1 e^{-\lambda\tau}u_1(t)\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{11}(y)e^{iky}dy - K_2 e^{-\lambda\tau}u_2(t)\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{12}(y)e^{iky}dy = 0, \tag{2.17}$$

$$\lambda u_2(t) + l_2 u_2(t) - K_1 e^{-\lambda\tau}u_1(t)\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{21}(y)e^{iky}dy - K_2 e^{-\lambda\tau}u_2(t)\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{22}(y)e^{iky}dy = 0. \tag{2.18}$$

Then we have the following condition

$$\begin{aligned}&\lambda^2 + \lambda l_2 - \lambda K_2 e^{-\lambda\tau}F_4 + \lambda l_1 + l_1 l_2 - l_1 K_2 e^{-\lambda\tau}F_4 - K_1 e^{-\lambda\tau}F_1\lambda - K_1 e^{-\lambda\tau}F_1 l_2\\ &+ K_1K_2 e^{-2\lambda\tau}F_1F_4 - K_1K_2e^{-2\lambda\tau}F_2F_3 = 0\end{aligned} \tag{2.19}$$

where $F_1 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{11}(y)e^{iky}dy$, $F_2 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{12}(y)e^{iky}dy$, $F_3 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{21}(y)e^{iky}dy$ and $F_4 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} J_{22}(y)e^{iky}dy$.

As in the previous case we study on the stability on the parameter space (l_1, K_1). Writing $\lambda = \mu + i\nu$ and taking $\mu = 0$ we write the real and imaginary parts of (2.19) and we have

$$P : -\nu^2 - \nu K_2 F_4 \sin(\tau\nu) + l_1 l_2 - l_1 K_2 \cos(\tau\nu) F_4 - K_1 \nu \sin(\tau\nu) F_1 \\ -K_1 l_2 \cos(\tau\nu) F_1 + K_1 K_2 \cos(2\tau\nu) F_1 F_4 - K_1 K_2 \cos(2\tau\nu) F_2 F_3 = 0, \tag{2.20}$$

$$R : -\nu K_2 F_4 \cos(\tau\nu) + \nu l_2 + \nu l_1 + l_1 K_2 \sin(\tau\nu) F_4 - K_1 \nu \cos(\tau\nu) F_1 \\ +K_1 l_2 \sin(\tau\nu) F_1 - K_1 K_2 \sin(2\tau\nu) F_1 F_4 + K_1 K_2 \sin(2\tau\nu) F_2 F_3 = 0. \tag{2.21}$$

To sketch the D-curves on the parameter space we have the expressions for l_1 and K_1 depending on the parameter ν.

$$l_1 = \frac{\left(\begin{array}{l} -\nu^3 \cos(\tau\nu) F_1 + (F_1F_4 - F_2F_3)(-\nu^2 K_2 \sin(2\tau\nu) - \nu K_2^2 \sin(\tau\nu)\sin(2\tau\nu) F_4 \\ +\nu K_2 l_2 \cos(2\tau\nu) - \nu K_2^2 \cos(\tau\nu)\cos(2\tau\nu)) \end{array} \right)}{\left(\begin{array}{l} (l_2^2 - \nu^2) F_1 \sin(\tau\nu) + (F_1F_4 - F_2F_3)(-l_2 K_2 \sin(2\tau\nu) + K_2^2 \sin(\tau\nu) F_4 \\ -\nu K_2 \cos(2\tau\nu)) + K_2 \nu F_1 F_4 \end{array} \right)} \\ + \frac{-\nu l_2^2 \cos(\tau\nu) F_1 + \nu K_2 l_2 F_1 F_4}{\left(\begin{array}{l} (l_2^2 - \nu^2) F_1 \sin(\tau\nu) + (F_1F_4 - F_2F_3)(-l_2 K_2 \sin(2\tau\nu) \\ +K_2^2 \sin(\tau\nu) F_4 - \nu K_2 \cos(2\tau\nu)) + K_2 \nu F_1 F_4 \end{array} \right)}, \tag{2.22}$$

$$K_1 = \frac{2\nu l_2 K_2 F_4 \cos(\tau\nu) - 2\nu^2 K_2 \sin(\tau\nu) F_4 - \nu K_2^2 F_4^2 - \nu l_2^2 - \nu^3}{\left(\begin{array}{l} (l_2^2 - \nu^2) F_1 \sin(\tau\nu) + (F_1F_4 - F_2F_3)(-l_2 K_2 \sin(2\tau\nu) \\ +K_2^2 \sin(\tau\nu) F_4 - \nu K_2 \cos(2\tau\nu)) + K_2 \nu F_1 F_4 \end{array} \right)} \tag{2.23}$$

and the line

$$(l_2 - K_2 F_4) l_1 + (K_2 F_1 F_4 - l_2 F_1 - K_2 F_2 F_3) K_1 = 0 \tag{2.24}$$

for $\nu = 0$ is the singular line.

For the D-curves we consider the parameters as $K_2 = F_1 = F_2 = l_2 = \tau = 1$ and $F_3 = F_4 = 2$. Hence we have the expressions depending on the parameter ν as given below:

$$l_1 = \frac{-\nu^3 \cos(\nu) - \nu \cos(\nu) + 2\nu}{\sin(\nu) - \nu^2 \sin(\nu) + 2\nu}, \tag{2.25}$$

$$K_1 = \frac{4\nu \cos(\nu) - 4\nu^2 \sin(\nu) - 5\nu - \nu^3}{\sin(\nu) - \nu^2 \sin(\nu) + 2\nu} \tag{2.26}$$

and the singular line as $-l_1 - K_1 = 0$.

The limit point for the intersection of the singular line and the D-curve is $\left(\lim_{\nu\to 0} l_1, \lim_{\nu\to 0} K_1\right) = \left(\frac{1}{3}, -\frac{1}{3}\right)$. In this case we have the following

$$P : -\nu^2 - 2\nu \sin(\nu) + l_1 - 2l_1 \cos(\nu) - K_1 \nu \sin(\nu) - K_1 \cos(\nu) = 0,$$

$$R : -2\nu \cos(\nu) + \nu l_1 + \nu + 2l_1 \sin(\nu) - K_1 \nu \cos(\nu) + K_1 \sin(\nu) = 0.$$

Now we have the following graph representing the regions of asymptotic stability (where $k = 0$) and unstability (where $k > 0$).

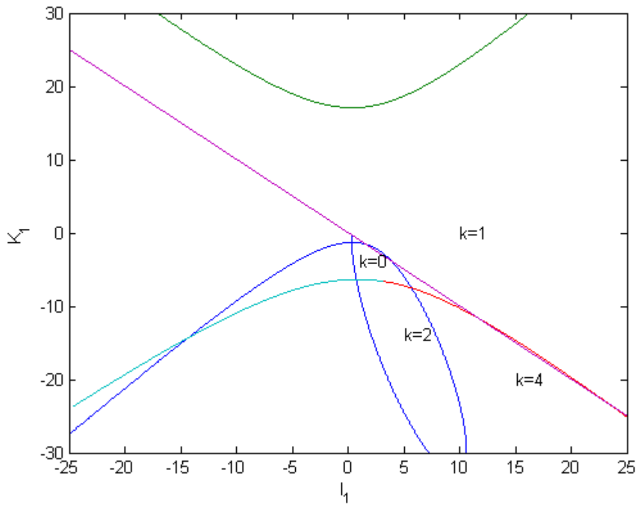

Figure 2. Stability chart for the system

3. Conclusion

In this study we consider the neural field model for two neural populations. First we assume that the term $J_{21}(x, y) = 0$, i.e. the neurons in the first population do not influence the ones in the second population. For the stability analysis we consider the linear system and by using the D-subdivision method we determine the D-regions. Then we find the asymptotic stability region via the Stépán's formula. Also we determine some important properties of D-curves. For a further analysis, we assume that $J_{21}(x, y) \neq 0$ and we use the same process to investigate the stability of the system. According to the graphs given in Figure 1 and Figure 2, we show how the change in the term $J_{21}(x, y)$ affects the stability of the system.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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