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Common Fixed Point Results for ϕ-ψ-Weak Contraction Mappings via f-α-Admissible Mappings in Intuitionistic Fuzzy Metric Spaces

Research Article

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Abstract. In this paper, by using the concept of f-α-admissible mappings, we prove common fixed point in intuitionistic fuzzy metric spaces. We also introduce the notion of f-α-ϕ-ψ-weak contraction mappings in intuitionistic fuzzy metric spaces. The presented theorems extend, generalize and improve the corresponding results which given in the literature.

Keywords. Common fixed point; Intuitionistic fuzzy metric space; α-admissible; ϕ-ψ-weak contractions

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1. Introduction

In 1965, the concept of fuzzy set was introduced by Zadeh [20] in domain X and $[0,1]$. In 1986, Atanasov [2] introduced the notion of an intuitionistic fuzzy metric space. Afterward, Park [14] gave the notion of an intuitionistic fuzzy metric space and generalized the notion of a fuzzy metric space due to George and Veeramani [6]. In 2008, Saadati *et al.* [17] modified the idea of an intuitionistic fuzzy metric space and presented the new notion of an intuitionistic fuzzy metric space.

On the other hand, in 1922, Banach [3] proved a theorem, which is well known as Banach's Fixed Point Theorem to establish the existence of solutions for integral equations. In 1981, Heilpern [7] developed fixed point theory in fuzzy metric spaces, introduced the concept of fuzzy contraction mappings and proved some fixed point theorems for fuzzy contraction mappings. Afterward, in 2006, Rafi and Noorani [15] introduced the concept of intuitionistic fuzzy contraction mappings and proved the existence fixed point in intuitionistic fuzzy metric spaces for an intuitionistic fuzzy contraction mapping.

In this paper, we introduced the concept of f-α-ϕ-ψ-weak contraction mappings in intuitionistic fuzzy metric spaces and prove some common fixed point results. In particular, the presented theorems extend, generalize and improve the results given in Beg *et al.* [4].

2. Preliminaries

First, we give some definitions, examples and remarks in intuitionistic fuzzy metric spaces.

Definition 2.1 ([18])**.** A binary operation $*:[0,1]\times[0,1]\to[0,1]$ is a continuous t-norm if it satisfies the following conditions:

(i) $*$ is associative and commutative;

(ii) $*$ is continuous;

(iii) $a*1=a$ for every $a\in[0,1]$;

(iv) $a*b\le c*d$ if $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

Example of continuous t-norm are $a*b=ab$ and $a*b=\min\{a,b\}$ (minimum t-norm).

Definition 2.2 ([18])**.** A binary operation $\diamond:[0,1]\times[0,1]\to[0,1]$ is a continuous t-conorm if it satisfies the following conditions:

(i) $\diamond$ is associative and commutative;

(ii) $\diamond$ is continuous;

(iii) $a\diamond 0=a$ for every $a\in[0,1]$;

(iv) $a\diamond b\le c\diamond d$ if $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

Example of continuous t-conorm are $a\diamond b=\min\{a+b,1\}$ and $a\diamond b=\max\{a,b\}$(maximum t-conorm).

Definition 2.3 ([14])**.** The 5-tuple $(X,M,N,*,\diamond)$ is an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets in $X \times X \times (0,\infty)$ satisfying the following conditions:

(i) $M(x,y,t) > 0$;

(ii) $M(x,y,t) = 1$ iff $x = y$;

(iii) $M(x,y,t) = M(y,x,t)$;

(iv) $M(x,y,t) * M(y,z,s) \le M(x,z,t+s)$;

(v) $M(x,y,\cdot) : (0,\infty) \to (0,1]$ is continuous;

(vi) $N(x,y,t) < 1$;

(vii) $N(x,y,t) = 0$ iff $x = y$;

(viii) $N(x,y,t) = N(y,x,t)$;

(ix) $N(x,y,t) \diamond N(y,z,s) \ge N(x,z,t+s)$;

(x) $N(x,y,\cdot) : (0,\infty) \to (0,1]$ is continuous;

(xi) $M(x,y,t) + N(x,y,t) \le 1$,

for all $x,y,z \in X$ and $s,t > 0$.

Then (M,N) is called an intuitionistic fuzzy metric on X. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2.4. Every fuzzy metric space $(X,M,*)$ is an intuitionistic fuzzy metric space of the form $(X,M,1-M,*,\diamond)$ such that continuous t-norm $*$ and continuous t-conorm $\diamond$ are associated [13], that is, $x \diamond y = 1-((1-x)*(1-y))$ for all $x,y \in X$.

Remark 2.5. In an intuitionistic fuzzy metric space $(X,M,N,*,\diamond)$, $M(x,y,\cdot)$ is non-decreasing and $N(x,y,\cdot)$ is non-increasing for all $x,y \in X$.

Definition 2.6 ([14])**.** A sequence $\{x_n\}$ in an intuitionistic fuzzy metric space is said to be a Cauchy sequence if and only if for each $r \in (0,1)$ and $t > 0$ there exists $n_0 \in \mathbf{N}$ such that $M(x_n,x_m,t) > 1-r$ and $N(x_n,x_m,t) < r$ for all $n,m \ge n_0$.

Definition 2.7 ([14])**.** A sequence $\{x_n\}$ in an intuitionistic fuzzy metric space is called convergent to $x \in X$ if, for each $t > 0$, we have $\lim_{n\to\infty} M(x_n,x,t) = 1$ and $\lim_{n\to\infty} N(x_n,x,t) = 0$.

Definition 2.8 ([14])**.** An intuitionistic fuzzy metric space is complete if and only if every Cauchy sequence is convergent. An intuitionistic fuzzy metric space is compact if every sequence contains a convergent subsequence.

Definition 2.9 ([15]). Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space. A mapping $f:X\to X$ is intuitionistic fuzzy contractive if there exists $k\in(0,1)$ such that $\frac{1}{M(fx,fy,t)}-1\le k\left(\frac{1}{M(x,y,t)}-1\right)$ and $N(fx,fy,t)\le kN(x,y,t)$, for all $x,y\in X$ and $t>0$.

Definition 2.10 ([9]). Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space. A mapping $f:X\to X$ is intuitionistic fuzzy contractive if there exists $k\in(0,1)$ such that $\frac{1}{M(fx,fy,t)}-1\le k\left(\frac{1}{M(x,y,t)}-1\right)$ and $\frac{1}{N(fx,fy,t)}-1\ge\frac{1}{k}\left(\frac{1}{N(x,y,t)}-1\right)$, for all $x,y\in X$ and $t>0$.

Definition 2.11. Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space. We will say that the sequence $\{x_n\}$ in X is intuitionistic fuzzy contractive if there exists $k\in(0,1)$ such that, for each $n\ge 0$ and $t>0$, $\frac{1}{M(x_{n+1},x_{n+2},t)}-1\le k\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)$ and $N(x_{n+1},x_{n+2},t)\le kN(x_n,x_{n+1},t)$.

Definition 2.12. Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space and $f,T:X\to X$ be two mappings. A point z in X is called coincidence point (common fixed point) of f and T if $fz=Tz$ ($z=fz=Tz$).

Definition 2.13 ([8]). Two finite families $\{f_i\}$ and $\{T_j\}$ of self mappings on X are said to be pairwise commuting if:

(i) $f_if_j=f_jf_i$, where $i,j\in\{1,2,\dots,m\}$;

(ii) $T_iT_j=T_jT_i$, where $i,j\in\{1,2,\dots,n\}$;

(iii) $f_iT_j=T_jf_i$, where $i\in\{1,2,\dots,m\}$ and $j\in\{1,2,\dots,n\}$.

Definition 2.14 ([11]). Let X be a nonempty set. Two mappings $f,T:X\to X$ are said to be weakly compatible if $fTx=Tfx$ for all $x\in X$ which $fx=Tx$.

Beg *et al*. [4] introduced the concept of ψ-waek contraction in intuitionistic fuzzy metric spaces as follows:

Definition 2.15 ([4]). Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space and $f,T:X\to X$ be two mappings. The mapping T is called intuitionistic ψ-weak contraction with respect to f if there exists a function $\psi:[0,\infty)\to[0,\infty)$ with $\psi(r)>0$ for $r>0$ and $\psi(0)=0$, such that

$$\frac{1}{M(Tx,Ty,t)}-1\le\left(\frac{1}{M(fx,fy,t)}-1\right)-\psi\left(\frac{1}{M(fx,fy,t)}-1\right)$$

and

$$N(Tx,Ty,t)\le N(fx,fy,t)-\psi(N(fx,fy,t)),$$

hold for all $x,y\in X$ and $t>0$. If f is the identity mapping, then T is called intuitionistic ψ-weak contraction.

Khan *et al*. [12] introduced the following concept of an altering distance in metric fixed point theory.

Definition 2.16 ([12]). A function $\phi:[0,\infty)\to[0,\infty)$ is an altering distance function if $\phi(t)$ is monotone non-decreasing and continuous and $\phi(t)=0$ if and only if $t=0$.

Definition 2.17 ([4]). Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space and $f,T:X\to X$ be two mappings. The mapping T is called intuitionistic (ϕ,ψ)-weak contraction with respect to f if there exist a function $\psi:[0,\infty)\to[0,\infty)$ with $\psi(r)>0$ for $r>0$ and $\psi(0)=0$ and an altering distance function ϕ such that

$$\phi\left(\frac{1}{M(Tx,Ty,t)}-1\right)\le\phi\left(\frac{1}{M(fx,fy,t)}-1\right)-\psi\left(\frac{1}{M(fx,fy,t)}-1\right)$$

and

$$\phi(N(Tx,Ty,t))\le\phi(N(fx,fy,t))-\psi(N(fx,fy,t)) \tag{2.1}$$

hold for all $x,y\in X$ and $t>0$. If f is the identity mapping, then T is called intuitionistic (ϕ,ψ)-weak contraction.

3. Main Results

In this section, we will introduce the concept of generalized α-admissible as follow:

Definition 3.1. ([16]) Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space and let $T,f:X\to X$ be two mappings. We say that T is f-α-admissible if there exist three function $\alpha:X\times X\times(0,\infty)\to[0,\infty)$ such that, for all $t>0$ and $x,y\in X$, we have

$$\alpha(fx,fy,t)\ge1 \quad\Longrightarrow\quad \alpha(Tx,Ty,t)\ge1. \tag{3.1}$$

Definition 3.2. Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space and let $T,f:X\to X$ be two mappings. The mappings T is called intuitionistic f-α-ψ-weak contraction with respect to f, if there exist two functions $\alpha:X\times X\times(0,\infty)\to[0,\infty)$ and $\psi:[0,\infty)\to[0,\infty)$ with $\psi(r)>0$ for $r>0$ and $\psi(0)=0$ such that

$$\alpha(fx,fy,t)\left(\frac{1}{M(Tx,Ty,t)}-1\right)\le\left(\frac{1}{M(fx,fy,t)}-1\right)-\psi\left(\frac{1}{M(fx,fy,t)}-1\right) \tag{3.2}$$

and

$$\alpha(fx,fy,t)(N(Tx,Ty,t))\le N(fx,fy,t)-\psi(N(fx,fy,t)) \tag{3.3}$$

for all $x,y\in X$ and $t>0$.

Definition 3.3. Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space and let $T,f:X\to X$ be two mappings. The mapping T is called intuitionistic f-α-ϕ-ψ-weak contraction with respect to f, if there exist three function $\alpha:X\times X\times(0,\infty)\to[0,\infty)$, $\phi:[0,\infty)\to[0,\infty)$ and $\psi:[0,\infty)\to[0,\infty)$ with $\psi(r)>0$ for $r>0$ and $\psi(0)=0$ such that

$$\alpha(fx,fy,t)\phi\left(\frac{1}{M(Tx,Ty,t)}-1\right)\le\phi\left(\frac{1}{M(fx,fy,t)}-1\right)-\psi\left(\frac{1}{M(fx,fy,t)}-1\right) \tag{3.4}$$

and

$$\alpha(fx,fy,t)\phi(N(Tx,Ty,t))\le\phi(N(fx,fy,t))-\psi(N(fx,fy,t)) \tag{3.5}$$

for all $x,y\in X$ and $t>0$.

Next example guarantee the Definition 3.3

Example 3.4. Let $X = \left\{\frac{1}{n}, n \in \mathbf{N}\right\} \cup \{0,4\}$ and $*$ be a minimum t-norm and $\diamond$ be a maximum t-conorm. Let M,N be defined by

$$M(x,y,t) = \begin{cases} \dfrac{t}{t+|x-y|}, & \text{if } t>0 \\ 0, & \text{if } t=0 \end{cases}$$

and

$$N(x,y,t) = \begin{cases} \dfrac{|x-y|}{t+|x-y|}, & \text{if } t>0 \\ 1, & \text{if } t=0. \end{cases}$$

Define the mapping $T: X \to X$ by

$$T(x) = \begin{cases} \dfrac{x}{4}, & \text{if } x \neq 0 \\ 1, & \text{if } x=4 \end{cases}$$

and define the function $\alpha : X \times X \times (0,\infty) \to [0,\infty)$ by

$$\alpha(fx,fy,t) = \begin{cases} 1, & \text{if } x,y \in X \setminus \{4\} \\ 0, & \text{otherwise.} \end{cases}$$

Also, defined $\phi,\psi : [0,\infty) \to [0,\infty)$ by $\phi(t) = \frac{t}{2}$, $\psi(t) = \frac{t}{8}$ and let $f(x) = \frac{x}{2}$. In fact, if at least one between x and y is equal to 4, then $\alpha(fx,fy,t) = 0$ and so (3.4), (3.5) holds trivially. Otherwise, if both x and y are in $X \setminus \{4\}$, then $\alpha(fx,fy,t) = 1$ and so (3.1). Then, we have

$$\begin{aligned}
\phi\left(\frac{1}{M(fx,fy,t)}-1\right) - \psi\left(\frac{1}{M(fx,fy,t)}-1\right) &= \phi\left(\frac{1}{M(\frac{x}{2},\frac{y}{2},t)}-1\right) - \psi\left(\frac{1}{M(\frac{x}{2},\frac{y}{2},t)}-1\right) \\
&= \phi\left(\frac{|\frac{x}{2}-\frac{y}{2}|}{t}\right) - \psi\left(\frac{|\frac{x}{2}-\frac{y}{2}|}{t}\right) \\
&= \frac{3|\frac{x}{2}-\frac{y}{2}|}{8t} \\
&\geq \frac{2|\frac{x}{2}-\frac{y}{2}|}{8t} \\
&= \frac{|\frac{x}{4}-\frac{y}{4}|}{2t} \\
&= \phi\left(\frac{|\frac{x}{4}-\frac{y}{4}|}{t}\right) \\
&= \phi\left(\frac{1}{M(\frac{x}{4},\frac{y}{4},t)}-1\right) \\
&= 1\cdot\phi\left(\frac{1}{M(Tx,Ty,t)}-1\right) \\
&= \alpha(fx,fy,t)\phi\left(\frac{1}{M(Tx,Ty,t)}-1\right).
\end{aligned}$$

From the last inequality and the fact that $N = 1-M$, we conclude that the conditions (3.4) and (3.5) are satisfied. Therefore T is intuitionistic f-α-ϕ-ψ-weak contraction with respect to f.

Now, we are ready to state and prove our the first main result.

Theorem 3.5. *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space. Let T and f be self-mappings on X such that the range of f contains the range of T ($TX \subseteq fX$) and $f(X)$ or $T(X)$ is a complete subset of X and $\alpha : X\times X\times(0,\infty)\to[0,\infty)$. Suppose that T is intuitionistic f-α-ϕ-ψ-weak contraction with respect to f and the following conditions hold:*

(i) *T is f-α-admissible;*

(ii) *there exists $x_0 \in X$ such that $\alpha(fx_0,Tx_0,t)\geq 1$ for all $t>0$;*

(iii) *T is continuous.*

Then, T and f have a coincidence point in X. If T and f are weakly compatible, then T and f have a unique common fixed point in X.

Proof. Let $x_0 \in X$ such that $\alpha(fx_0,Tx_0,t)\geq 1$ for all $t>0$ and choose a point x_1 in X such that $Tx_0 = fx_1$. Define the sequence $\{x_n\}$ and $\{y_n\}$ in X such that

$$y_n = Tx_n = fx_{n+1}, \quad n\in\mathbf{N}\cup\{0\}.$$

In particular, if $y_n = y_{n+1}$, then y_{n+1} is a point of coincidence of T and f. Consequently. We assume that $y_n \neq y_{n+1}$ for all $n\in\mathbf{N}$. By condition (ii), we have $\alpha(fx_0,Tx_0,t)=\alpha(fx_0,fx_1,t)\geq 1$. Since, hypothesis of T is f-α-admissible, we obtain

$$\alpha(Tx_0,Tx_1,t)=\alpha(fx_1,fx_2,t)\geq 1, \quad \alpha(Tx_1,Tx_2,t)=\alpha(fx_2,fx_3,t)\geq 1.$$

By induction, we get

$$\alpha(fx_n,fx_{n+1},t)\geq 1.$$

for all $n\in\mathbf{N}\cup\{0\}$. Now, by (3.4) and (3.5) with $x=x_n$, $y=x_{n+1}$, we have

$$\begin{aligned}
\phi\left(\frac{1}{M(y_n,y_{n+1},t)}-1\right) &= \phi\left(\frac{1}{M(Tx_n,Tx_{n+1},t)}-1\right)\\
&\leq \alpha(fx_n,fx_{n+1},t)\phi\left(\frac{1}{M(Tx_n,Tx_{n+1},t)}-1\right)\\
&\leq \phi\left(\frac{1}{M(fx_n,fx_{n+1},t)}-1\right)-\psi\left(\frac{1}{M(fx_n,fx_{n+1},t)}-1\right)\\
&= \phi\left(\frac{1}{M(y_{n-1},y_n,t)}-1\right)-\psi\left(\frac{1}{M(y_{n-1},y_n,t)}-1\right)\\
&< \phi\left(\frac{1}{M(y_{n-1},y_n,t)}-1\right),
\end{aligned}$$

which considering that the ϕ function is non-decreasing implies that $M(y_{n+1},y_n,t) > M(y_{n-1},y_n,t)$ for all $n\in\mathbf{N}$ and hence $M(y_{n-1},y_n,t)$ is an increasing sequence of positive real numbers in $(0,1]$.

Let $S(t) = \lim_{n\to\infty} M(y_{n-1}, y_n, t)$, we show that $S(t) = 1$ for all $t > 0$. If not, there exists $t > 0$ such that $S(t) < 1$, then from the above inequality on taking $n \to \infty$, we obtain

$$\phi\left(\frac{1}{S(t)} - 1\right) \le \phi\left(\frac{1}{S(t)} - 1\right) - \psi\left(\frac{1}{S(t)} - 1\right),$$

a contradiction. Therefore, $M(y_n, y_{n+1}, t) \to 1$ as $n \to \infty$. Now for each positive integer p,

$$M(y_n, y_{n+p}, t) \ge M(y_n, y_{n+1}, t/p) * M(y_{n+1}, y_{n+2}, t/p) * \cdots * M(y_{n+p-1}, y_{n+p}, t/p).$$

It follows that

$$\lim_{n\to\infty} M(y_n, y_{n+p}, t) \ge 1 * 1 * \cdots * 1 = 1.$$

Similarly, we have

$$\begin{aligned}
\phi(N(y_n, y_{n+1}, t)) &= \phi(N(Tx_n, Tx_{n+1}, t)) \\
&\le \alpha(fx_n, fx_{n+1}, t)\phi(N(Tx_n, Tx_{n+1}, t)) \\
&\le \phi(N(fx_n, fx_{n+1}, t)) - \psi(N(fx_n, fx_{n+1}, t)) \\
&= \phi(N(y_{n-1}, y_n, t)) - \psi(N(y_{n-1}, y_n, t)) \\
&< \phi(N(y_{n-1}, y_n, t)),
\end{aligned}$$

which considering that the ϕ function is non-decreasing implies that $N(y_n, y_{n+1}, t) < N(y_{n-1}, y_n, t)$ for all $n \in \mathbf{N}$ and hence $N(y_{n-1}, y_n, t)$ is a decreasing sequence of positive real number in $[0,1)$. Let $R(t) = \lim_{n\to\infty} N(y_{n-1}, y_n, t)$, we show that $R(t) = 0$ for all $t > 0$. If not, there exists $t > 0$ such that $R(t) > 0$, then from the above inequality on taking $n \to \infty$, we obtain

$$\phi(R(t)) \le \phi(R(t)) - \psi(R(t)),$$

a contradiction. Therefore, $N(y_n, y_{n+1}, t) \to 0$ as $n \to \infty$. Now for each positive integer p, by Definition 2.3(xi), must be

$$M(y_n, y_{n+p}, t) + N(y_n, y_{n+p}, t) \le 1$$

and then

$$\lim_{n\to\infty} (M(y_n, y_{n+p}, t) + N(y_n, y_{n+p}, t)) \le 1.$$

It follows that

$$\lim_{n\to\infty} N(y_n, y_{n+p}, t) = 0.$$

Hence y_n is a Cauchy sequence. If $f(X)$ is complete, then there exists $q \in f(X)$ such that$y_n \to q$ as $n \to \infty$. The same holds if $T(X)$ is complete with $q \in T(X)$. Let $p \in X$ be such that $fp = q$. Now, we show that p is a coincidence point of f and T. In fact, we have

$$\begin{aligned}
\phi\left(\frac{1}{M(Tp, fx_{n+1}, t)} - 1\right) &= \phi\left(\frac{1}{M(Tp, Tx_n, t)} - 1\right) \\
&\le \alpha(fp, fx_n, t)\phi\left(\frac{1}{M(Tp, Tx_n, t)} - 1\right) \\
&\le \phi\left(\frac{1}{M(fp, fx_n, t)} - 1\right) - \psi\left(\frac{1}{M(fp, fx_n, t)} - 1\right)
\end{aligned}$$

for every $t > 0$, which on taking $n \to \infty$ gives that

$$\begin{aligned}\lim_{n\to\infty} M(Tp, fx_{n+1}, t) = \lim_{n\to\infty} M(Tp, Tx_n, t) \\ = M(Tp, fp, t) = 1.\end{aligned}$$

That is $fp = Tp = q$ and so q is a point of coincidence of T and f. Now, we show that $fq = q$. Now, if q is a point of coincidence of T and f as T and f are weakly compatible, then we prove that q is common fixed point of T and f. Since $fp = Tp = q$ and f as T and f are weakly compatible, then $fq = Tq$. Using (3.3) and suppose that $fq \neq q$, then consider

$$\begin{aligned}\frac{1}{M(fq,q,t)} - 1 &= \frac{1}{M(Tq,Tp,t)} - 1 \\ &\le \alpha(fq,fp,t)\left(\frac{1}{M(Tq,Tp,t)} - 1\right) \\ &\le \left(\frac{1}{M(fq,fp,t)} - 1\right) - \psi\left(\frac{1}{M(fq,fp,t)} - 1\right) \\ &= \left(\frac{1}{M(fq,q,t)} - 1\right) - \psi\left(\frac{1}{M(fq,q,t)} - 1\right)\end{aligned}$$

a contradiction which leads to the result, that is $fq = q$ and so $fq = Tq = q$. Therefore T and f have a common fixed point in X.

We will prove the uniqueness of a common fixed point of f and T. Let z be another common fixed point of f and T $(z \neq q)$.Then, there exists $t > 0$, such that

$$\begin{aligned}\phi\left(\frac{1}{M(q,z,t)-1}\right) &= \phi\left(\frac{1}{M(Tq,Tz,t)-1}\right) \\ &\le \alpha(fq,fz,t)\phi\left(\frac{1}{M(Tq,Tz,t)-1}\right) \\ &\le \phi\left(\frac{1}{M(fq,fz,t)-1}\right) - \psi\left(\frac{1}{M(fq,fz,t)-1}\right) \\ &< \phi\left(\frac{1}{M(q,z,t)-1}\right).\end{aligned}$$

a contradiction which leads to the result, that is $q = z$. Therefore q is a unique common fixed point of f and T. This completes the proof. □

Example 3.6. Set X, M, N, $*$, $\diamond$, α, ϕ, ψ, f, T as in Example 3.4. Obviously, $(X,M,N,*,\diamond)$ is a complete intuitionistic fuzzy metric space. Clearly, $T(X) \subset f(X)$. All the assumption of Theorem 3.5 are satisfied, then f and T have a unique common fixed point.

From Theorem 3.5, if the function $\alpha : X \times X \times (0,\infty) \to [0,\infty)$ such that $\alpha(fx, fy, t) = 1$ for all $x, y \in X$, we deduce the following corollary.

Corollary 3.7 ([4]). *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space and let T and f be self-mappings on X such that the range of f contains the range of T $(TX \subseteq fX)$ and fX or TX*

is a complete subset of X. Suppose that the following conditions hold:

$$\phi\left(\frac{1}{M(Tx,Ty,t)}-1\right) \le \phi\left(\frac{1}{M(fx,fy,t)}-1\right)-\psi\left(\frac{1}{M(fx,fy,t)}-1\right)$$

and

$$\phi(N(Tx,Ty,t)) \le \phi(N(fx,fy,t))-\psi(N(fx,fy,t))$$

for all $x,y \in X$ and $t>0$. If T and f are weakly compatible, then T and f have a unique common fixed point in X.

From Theorem 3.5, if we choose $f=I_X$ the identity mapping on X, we deduce the following corollary.

Corollary 3.8. *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space. Let T be self-mappings on X and $\alpha : X\times X\times(0,\infty)\to[0,\infty)$. Suppose that the following conditions hold:*

(i) *T is α-admissible;*

(ii) *there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0,t)\ge 1$ for all $t>0$;*

(iii) *T is continuous.*

Then, T has a unique fixed point.

Theorem 3.9. *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space and $\{f_i\}$, $\{T_k\}$, where $i\in\{1,\dots\}$ and $k\in\{1,\dots,n\}$, be two finite families of self mappings on X with $f=f_1f_2\cdots f_n$ and $T=T_1T_2\cdots T_m$. Let T be an intuitionistic f-α-ϕ-ψ-weak contraction with respect to f. If the range of f contains the range of T ($TX\subseteq fX$) and $f(X)$ or $T(X)$ is a complete subset of X and $\alpha : X\times X\times(0,\infty)\to[0,\infty)$ then T_k and f_i have a unique common fixed point in X.*

Proof. Using Theorem 3.5, we conclude that q is unique common fixed point of T and f.

Now, we will show that q remains the fixed point of all the component mappings. We consider

$$\begin{aligned}
T(T_iq) &= ((T_1,T_2,\cdots,T_m)T_i)q \\
&= (T_1T_2\cdots T_{m-1})((T_mT_i)q) \\
&= (T_1\cdots T_{m-1})(T_iT_mq) \\
&\vdots \\
&= T_1T_i(T_2T_3T_4\cdots T_mq) \\
&= T_iT_1(T_2T_3\cdots T_mq) \\
&= T_i(T_q) \\
&= T_iq.
\end{aligned}$$

Similarly, we can show that $T(f_kq)=f_k(Tq)=f_kq, f(f_kq)=f_k(fq)=f_kq$ and $f(T_iq)=T_i(fq)=T_iq$, which implies that, for all i and k, T_iq and f_kq are other fixed point of the pair $\{T,f\}$. Now appealing to the uniqueness of a common fixed point of mappings T and f, we get $q=T_iq=f_kq$, which shows that q is a common fixed point of f_i and T_k for all i and k. □

Definition 3.10. Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space. Let $T,f:X\to X$ be two mappings. The mappings T is called intuitionistic f-α-ϕ-ψ-weak contraction of integral type with respect to f, if there exist three functions $\alpha: X\times X\times(0,\infty)\to[0,\infty)$, $\phi:[0,\infty)\to[0,\infty)$ and $\psi:[0,\infty)\to[0,\infty)$ with $\psi(r)>0$ for $r>0$ and $\psi(0)=0$, such that

$$\alpha(fx,fy,t)\phi\left(\int_0^{\frac{1}{M(Tx,Ty,t)}-1}\varphi(s)ds\right)\le\phi\left(\int_0^{\frac{1}{M(fx,fy,t)}-1}\varphi(s)ds\right)-\psi\left(\int_0^{\frac{1}{M(fx,fy,t)}-1}\varphi(s)ds\right), \tag{3.6}$$

and

$$\alpha(fx,fy,t)\phi\left(\int_0^{N(Tx,Ty,t)}\varphi(s)ds\right)\le\phi\left(\int_0^{N(fx,fy,t)}\varphi(s)ds\right)-\psi\left(\int_0^{N(fx,fy,t)}\varphi(s)ds\right), \tag{3.7}$$

for all $x,y\in X$ and $t>0$, where $\varphi:[0,\infty)\to[0,\infty)$ is a Lebesgue integrable function which is summable on each compact subset of $[0,\infty)$ and such that for all $\varepsilon>0$, $\int_0^{\epsilon}\varphi(s)ds>0$.

Theorem 3.11. *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space. Let T and f be self-mappings on X such that the range of f contains the range of T ($TX\subseteq fX$) and $f(X)$ or $T(X)$ is a complete subset of X and $\alpha:X\times X\times(0,\infty)\to[0,\infty)$. Suppose that T is intuitionistic f-α-ϕ-ψ-weak contraction of integral type with respect to f and the following conditions hold:*

(i) *T is f-α-admissible;*

(ii) *there exists $x_0\in X$ such that $\alpha(fx_0,Tx_0,t)\ge1$ for all $t>0$;*

(iii) *T is continuous.*

Then, T and f have a coincidence point in X. If T and f are weakly compatible, then T and f have a unique common fixed point in X.

Proof. Define $\Gamma:[0,\infty)\to[0,\infty)$ by $\Gamma=\int_0^x\varphi(s)ds$. So, condition (3.6) reduces to condition (3.4) and condition (3.7) reduces to condition (3.5) as $\phi\circ\Gamma$ is an altering distance function and $\psi\circ\Gamma:[0,\infty)\to[0,\infty)$ with $\psi(\Gamma(r))>0$ for $r>0$ and $\psi(\Gamma(0))=0$. Therefore, the conclusion follows immediately by Theorem 3.5. □

4. Conclusion

We introduced the new concept and the new notion of f-α-ϕ-ψ-weak contraction mappings in intuitionistic fuzzy metric spaces and also proved common fixed point in intuitionistic fuzzy metric spaces. The presented theorems extend and improve the corresponding results which given in the literature. In particular, Corollary 3.7 extend, generalize and improve the results given of Beg *et al.*, in [4].

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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