Restrained Regular Domination on a Litact Graph

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Abstract. We present the first research on restrained regular domination, which is a variant of standard domination. Assume that $G = (V, E)$ is a graph. If every vertex in $V - D_{rer}$ has at least one neighbour in both D_{rer} and $V - D_{rer}$, and every vertex in $\langle D_{rer} \rangle$ has an identical degree, then a set $D_{rer} \subseteq V$ is a *restrained regular dominating set*, abbreviated RRDS. The least cardinality of all G's RRDS is the *RRD number of G*, represented by $\gamma_{rer}(G)$. We ascertain the optimal bounds that can be applied to $\gamma_{rer}[m(G)]$, and we identify the most optimal lower bounds for $\gamma_{rer}[m(G)] + \gamma_{rer}[m(\bar{G})]$ and $\gamma_{rer}[m(G)] \cdot \gamma_{rer}[m(\bar{G})]$, both G and $\bar{G}$ are connected. We also characterize those graphs satisfying these bounds.

Keywords. Graph, Litact graph, Regular domination number, Restrained domination number, Restrained regular domination number

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1. Introduction

Every *graph* taken into consideration here is *finite*, *simple*, *connected*, *undirected*, *non-trivial graph*, and *without isolated vertices*. Graph theoretical techniques can be used to study a variety of network issues. Now, after almost two decades of growth, domination theory became one of the main fields in graph theory. The variety of applications this field has to both theoretical and practical issues-line facility location issues-may be cause of its steady and quick growth. The *regular domination number*, and the *restrained domination number* are two most fundamental *domination-type* metrics that have been researched. Combining these two RRD numbers yielded the result.

The researchers created a new type of domination in litact graphs called restrained regular domination, which was inspired by the ideas of regular and restrained domination in litact graphs.

Numerous authors have provided domination-type parameters in recent years. For example, Alrikabi *et al*. [2] introduced the idea of the *Restrained Captive Domination Number*, and Hemalatha *et al*. [12] studied the *Restrained and Total Restrained Domination of Ladder Graphs*. Recently, Muddebihal and Swati [19] looked into lower and upper bounds on *Restrained Lict Domination in Graphs*, while Ibrahim and Omran [14] provided some lower and upper bounds on *Restrained Whole Domination in Graphs*. Additionally, the graphs have been described by Monsanto and Rara [18] using *Resolving Restrained Domination*. Furthermore, the graphs with *Restrained Italian Domination* were described by Samadi *et al*. [22]. Moreover, *Outer-restrained Domination in the Join and Corona of Graphs* was characterized by Tuble and Enriquez [25]. In the present study, some recent articles were also considered to establish our results, such as Abiad *et al*. [1], Amjadi *et al*. [3], Barnköpf *et al*. [4], Barack *et al*. [5], Borg [6], Brešar and Henning [7], Burdett *et al*. [8], Buvaneswari and Umamaheswari [9], Consistente and Cabahug [10], Hayat *et al*. [11], Hussain *et al*. [13], Jayasekaran and Binoja [15], Mojdeh and Abdallah [17], Nair and Sunitha [20], Rajan *et al*. [21], Sarmitha *et al*. [23], Shi *et al*. [24], Volkmann [27], Xia [28], Zerovnik [29], and Zhang and Zhu [30].

This study attempts to identify and characterize the RRD number in the litact graphs, drawing inspiration from the works mentioned above. We discover the optimal bounds that are feasible for $\gamma_{rer}[m(G)]$, $\gamma_{rer}[m(G)] + \gamma_{rer}[m(\bar{G})]$ and $\gamma_{rer}[m(G)] \cdot \gamma_{rer}[m(\bar{G})]$, where both G and $\bar{G}$ are connected, and calculate the RRD number for the *litact graph* in the form of various *parameters* and *domination parameters* of a graph G.

2. Preliminaries

Using each of the definitions listed below, we were able to get the following results.

Definition 2.1. If all points in $V - D$ are connected to a point in D, then D is considered a *dominating set* in a *graph* $G(V, E)$. *Domination number* of G, represented by $\gamma(G)$, is set's smallest *cardinality*.

Definition 2.2. A *dominating set* D_{re} in a *graph* G is *restrained dominating set*, if each point in $V - D_{re}$ is connected to another point in D_{re} also to the point in $V - D_{re}$. *Restrained domination number*, $\gamma_{re}(G)$, is the set D_{re}'s minimal cardinality.

Definition 2.3. G is known as a *regular graph* if each point in it holds the equal degree. A *dominating set*, D_r is a *regular dominating set* of G if $\langle D_r \rangle$ is regular. The *regular domination number* of G, represented by $\gamma_r(G)$, is minimal *cardinality* of such D_r.

Definition 2.4. Point set of the *litact graph*, $m(G)$ is made up of the G's edges and cut vertices. If *edges and cut vertices* are incident or adjacent in G, then the two vertices in $m(G)$ are connected.

Definition 2.5. If each point in $V[m(G)] - D_{rer}$ is adjacent to the point in D_{rer} and to the point in $V[m(G)] - D_{rer}$, and $\langle D_{rer} \rangle$ is regular, then a *dominating set*, $D_{rer} \subseteq V[m(G)]$ is a *restrained regular dominating set* in a *litact graph* $m(G)$. Within this collection, *restrained regular domination number*, $\gamma_{rer}[m(G)]$ represents the lowest *cardinality* of vertices in D_{rer}.

3. Results

First, we start with the standard graph $K_{1,p}$ observation below, which leads directly to the following conclusion.

Observation 3.1. *For each Star graph $K_{1,p}$, with a minimum of three vertices, $\gamma_{rer}[m(K_{1,p})] = 1$.*

The upper limit for any RRD-number of the *litact graph m(G)* in the form of *order n of G* is established in the following theorem.

Theorem 3.2. *If G is a graph, then $\gamma_{rer}[m(G)] \leq n$. Equality holds if $G \cong C_n$, for $n \geq 5$.*

Proof. If G is a *cycle graph* of *order $n \geq 5$*, it is simple to confirm that $\gamma_{rer}[m(G)] = n$. Otherwise, for any *graph G of order n*, it is obvious that $\gamma_{re}(G) \leq n$. Only *connected graph of order n* in which $\gamma_{re}(K_{1,n-1}) = n$ is the star graph $K_{1,n-1}$. Let $D \subseteq V[m(G)]$ is a *dominating set* such that $\gamma[m(G)] = |D|$. Suppose $A = \{v_1, v_2, \ldots, v_i\} \subseteq V[m(G)]$ is the set of non-end points in $m(G)$, then $A \cup B$, where $B \subseteq D$ forms a RRDS in $m(G)$. Clearly, $\gamma_{rer}[m(G)] = |A \cup B| \leq n \Rightarrow \gamma_{rer}[m(G)] \leq n$. □

The characterization of RRD number in *litact graph* of *graph G* is shown below.

Theorem 3.3. *In a (p,q)-graph* G, $\gamma_{rer}[m(G)] + p > \left\lfloor \frac{q-1}{\Delta(G)} \right\rfloor$.

Proof. Suppose G is a graph. Assume $S \subseteq V(m(G))$ is the *dominating set* in $m(G)$ and that each point in $V(m(G)) - S$ has at least one neighbour in both S and $V(m(G)) - S$ along with each vertex of $\langle S \rangle$ holds the equal degree. As a result, $\gamma_{rer}[m(G)] = |S|$. Suppose Δ is the maximum degree of G, and since $\frac{q-1}{\Delta}$ is smaller than $|V(m(G)) - S|$ in value, it is evident that,

$$\left\lfloor \frac{q-1}{\Delta} \right\rfloor < |V(m(G)) - S| \quad \Rightarrow \quad |V(m(G)) - S| > \left\lfloor \frac{q-1}{\Delta} \right\rfloor. \tag{3.1}$$

It is also clear that,

$$|D| + p > |V(m(G)) - S|. \tag{3.2}$$

From inequalities (3.1) and (3.2), we get

$$|D| + p > \left\lfloor \frac{q-1}{\Delta} \right\rfloor \quad \Rightarrow \quad \gamma_{rer}[m(G)] + p > \left\lfloor \frac{q-1}{\Delta(G)} \right\rfloor. \qquad \square$$

Next, we derive the relationship between the RRD number of $m(G)$, $p(G)$, and $l(G)$.

Theorem 3.4. *In a graph G, $\gamma_{rer}[m(G)] \leq p - l$. Here l is number of end vertices of G.*

Proof. In the following two cases, this result can be demonstrated.

Case(i): The proof is obvious if the number of end vertices, $l = 0$, according to Theorem 3.2.

Case (ii): Suppose that $l \neq 0$, then there is at least one end vertex in G. Assume that $L \subseteq V(G)$ is a set of G's end vertices, so that $|L| = l$. The point set of the *litact graph m(G)* is formed by the union of *edge set* and *cut vertex set* of G, so that $V'[m(G)] = E(G) \cup C(G)$. Suppose a *dominating set $S \subseteq V'[m(G)]$* creates a RRDS in a litact graph $m(G)$. Then, it is clear that

$$\gamma_{rer}[m(G)] = |S| \leq |V(G) \cup L(G)| = p - l \quad \Rightarrow \quad \gamma_{rer}[m(G)] \leq p - l. \qquad \square$$

We then determine the relationship between $\gamma_{rer}[m(G)]$, $p(G)$ and $q(G)$.

Theorem 3.5. *If G is any (p,q)-connected graph, then $p - q \leq \gamma_{rer}[m(G)]$.*

Proof. The following cases make it simple to confirm this result.

Case (i): If G is any cycle graph, the proof is obvious since $p - q = 0$ and $\gamma_{rer}[m(G)] \geq 1$.

Case (ii): If G is any path graph, the proof is obvious since $p - q = 1$ and $\gamma_{rer}[m(G)] \geq 1$.

Case (iii): If G is any star graph $K_{1,n}$, the proof is obvious since $p - q = 1$ and $\gamma_{rer}[m(G)] \geq 1$.

Case (iv): If G is any other graph, let $E(G)$ is an *edge set of G*, so that $|E(G)| = q$, and let $V(G)$ be the collection of points of G, $|V(G)| = p$. Let $S \subseteq V[m(G)]$ be the collection of points in the *litact graph $m(G)$* such that *each vertex that is not in S is adjacent to at least one vertex in S* and points that remain of $m(G)$. Furthermore, every vertex in $\langle S \rangle$ has the same degree. Thus, we conclude S constitutes an RRD-set in $m(G)$. Therefore, $\gamma_{rer}[m(G)] = |S|$. It is evident from the ideas of fundamental definitions that,

$$|V(G)| + |E(G)| - |S| \leq 2|E(G)|$$
$$\Rightarrow \quad p + q - \gamma_{rer}[m(G)] \leq 2q$$
$$\Rightarrow \quad p - q \leq \gamma_{rer}[m(G)]. \qquad \square$$

The following theorem demonstrates the relationship between $\gamma_{rer}[m(G)]$, $\alpha_0(G)$ and $\operatorname{diam}(G)$.

Theorem 3.6. *If G is a graph, $\gamma_{rer}[m(G)] \leq \alpha_0(G) + \operatorname{diam}(G)$.*

Proof. Assume that $U = \{u_1, u_2, \ldots, u_i\} \subseteq V(G)$; $1 \leq i < n$ is the smallest vertex set that covers all the lines of G, so that $|U| = \alpha_0(G)$. Similarly, let's say that another set of vertices, $X = \{u_1, u_2, \ldots, u_j\} \subseteq V(G)$; $1 \leq j \leq n$, creates the *longest path between any two vertices* in G, ensuring that $|X| = \operatorname{diam}(G)$. Let a dominating set $S \subseteq V[m(G)]$ is a point set in *litact graph $m(G)$* such that *each vertex that is not in S is adjacent to at least one vertex in S* and other points of $m(G)$. Moreover, assume that every vertex in $\langle S \rangle$ has the same degree. Therefore, $|S| = \gamma_{rer}[m(G)]$. Using the concepts of diameter, vertex covering, and RRD number, it is simple to verify that $|S| \leq |U \cup X| = |U| + |X|$. Hence, as a result $\gamma_{rer}[m(G)] \leq \alpha_0(G) + \operatorname{diam}(G)$. $\qquad \square$

Next theorem explains $\gamma_{rer}[m(G)]$, $\alpha_1(G)$, q, and $\gamma(G)$'s relationship.

Theorem 3.7. *If G is a graph, $\gamma_{rer}[m(G)] + \alpha_1(G) \leq q + 2\gamma(G)$.*

Proof. G is a *graph*, so that $q = |E(G)|$. Assume that $A \subseteq E(G)$ be the set of end edges in G. Then, $|A \cup B| = \alpha_1(G)$ is union of $B \subseteq E(G) - A$ and A, which is a set with the fewest edges that covers all of G's vertices. Consider the set of vertices $U = \{v_i : 1 \leq i < n\} \subseteq V(G)$ to be the set and each point in $V - U$ is connected to at least one point in $U(G)$, such that $\gamma(G) = |U|$. Without loss of generality, consider $D \subseteq V[m(G)]$ forms a RRD-set in a *litact graph $m(G)$*, so that $|D| = \gamma_{rer}[m(G)]$. From the aforementioned concepts, it is very clear that, for any G, $|D \cup A \cup B| \leq |E| + 2|U|$. Therefore,

$$\gamma_{rer}[m(G)] + \alpha_1(G) \leq q + 2\gamma(G).$$

The upper bound for $\gamma_{rer}[m(G)]$ in terms of $p(G)$ and $\beta_1(G)$ is now obtained. $\qquad \square$

Theorem 3.8. *For any (p,q)-graph G, $\left\lfloor \frac{\gamma_{rer}[m(G)]}{2} \right\rfloor \leq p - \beta_1(G)$, where $\beta_1(G)$ is edge independence number of G.*

Proof. Assume $E_1 \subseteq E(G)$ is the edge set containing the greatest number of edges that do not adjacent one another. Thus, $|E_1| = \beta_1(G)$. Assume, without losing generality, that a dominating set $S \subseteq V[m(G)]$ forms an RRD set in $m(G)$, such that $|S| = \gamma_{rer}[m(G)]$.

From Theorem 3.2, we have

$$\gamma_{rer}[m(G)] \leq n = |V(G)| \quad \Rightarrow \quad |S| \leq |V(G)|. \tag{3.3}$$

It is fact that,

$$|V(G)| - |E_1| \leq |V(G)|. \tag{3.4}$$

Operating (3.3)-(3.4), we get

$$|S| - |V(G)| + |E_1| \leq 0 \quad \Rightarrow \quad |S| \leq |V(G)| - |E_1|$$

It is clear that,

$$\left\lfloor \frac{|S|}{2} \right\rfloor \leq |S| \leq |V(G)| - |E_1| = p - \beta_1(G).$$

Therefore,

$$\left\lfloor \frac{\gamma_{rer}[m(G)]}{2} \right\rfloor \leq p - \beta_1(G).$$

Subsequent upper bound for $\gamma_{rer}[m(G)]$ determined in the form of $\gamma_t(G)$ and $\beta_0(G)$. ☐

Theorem 3.9. *If G is a graph, $\gamma_{rer}[m(G)] \leq \gamma_t(G) + \beta_0(G)$.*

Proof. Let $X \subseteq V(G)$ be the largest collection of nonadjacent points in G, so that $|X| = \beta_0(G)$. Assume $S \subseteq V(G)$ is a *total dominating set* in G with *no isolated vertices* and $|S| = \gamma_t(G)$. Without losing generality, let $Y = \{v_i; 1 \leq i < n\} \subseteq V[m(G)]$ be the *set of all end vertices* in $m(G)$.
Let $Z = \{v_i; i = 1, 2, \ldots, n\} \subseteq V[m(G)]$ represents collection of vertices that are not adjacent to the vertices of Y. Next, the set $D \cup Y$, where $D \subseteq Z$, creates a restrained regular dominating set in $m(G)$, whenever each vertex of $\langle D \rangle$ has equal degree. Therefore, $|D| = \gamma_{rer}[m(G)]$. From the above concepts, it follows that

$$|D| \leq |X \cup S| = |X| + |S| \quad \Rightarrow \quad \gamma_{rer}[m(G)] \leq \gamma_t(G) + \beta_0(G). \qquad ☐$$

According to the following theorem, $\gamma_{rer}[m(T)]$, $\Delta(T)$, and $s(T)$ are related in a tree.

Theorem 3.10. *For any (p,q)-tree T, $\gamma_{rer}[m(T)] < \Delta(T) + s(T)$, where $\Delta(T)$ is the maximum degree of T and $s(T)$ is number of cut vertices of T.*

Proof. Assume that $S \subseteq V(T)$ is the *number of cut vertices in a tree T*, such that $|S| = s$, and that V_1 is the *maximum degree of the tree T*, such that $|V_1| = \Delta$. Without losing generality, assume that $D \subseteq V[m(T)]$ forms RRD-set in a *litact graph $m(T)$* of a tree T, so that $|D| = \gamma_{rer}[m(T)]$. It is evident that $|D| < |V_1 \cup S| = \Delta + s$ is obtained by applying the ideas of maximum degree, cut vertices and RRD set. Therefore,

$$\gamma_{rer}[m(T)] < \Delta(T) + s(T). \qquad ☐$$

We need the following theorems to establish our next results.

Theorem A ([26]). *In each graph G, $\frac{q}{\Delta'(G)+1} \leq \gamma'_m(G)$, where $\Delta'(G)$ is edge maximum degree.*

Theorem B ([16]). *If G has no isolated vertices, then*

$$\gamma_{2r}(G) \geq \frac{2p}{\Delta(G)+1}.$$

The aforementioned theorems immediately lead to the following corollaries.

Corollary 3.11. *In each graph G, $\gamma_{rer}[m(G)] + \frac{q}{\Delta'(G)+1} \le \gamma_t(G) + \beta_0(G) + \gamma'_m(G)$, where $\Delta'(G)$ is the edge maximum degree.*

Proof. Theorem A and Theorem 3.9 provide the proof. □

Corollary 3.12. *If G has no isolated vertices, then*

$$\gamma_{rer}[m(G)] + \frac{2p}{\Delta(G)+1} \le \alpha_0(G) + \operatorname{diam}(G) + \gamma_{2r}(G).$$

Proof. Theorem 3.6 and Theorem B provide the proof. □

A *Nordhaus-Gaddum-type* Result

Nordhaus and Gaddum provided the most accurate approximations for *sum and product* of the *chromatic numbers of a graph and its complement* in their 1956 study article *On complementary graphs*. Many writers later came to the same conclusions on different domination criteria.
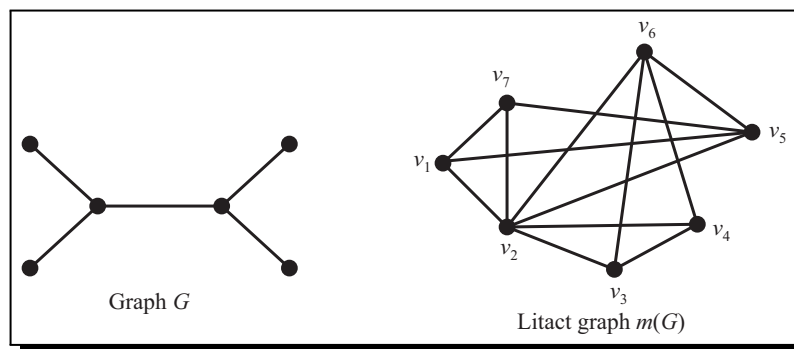
We now provide the best estimates for G and $\bar{G}$.

Theorem 3.13. *For any connected graphs G and $\bar{G}$ of order n,*
   (1) $\gamma_{rer}(m(G)) + \gamma_{rer}(m(\bar{G})) \le 2n$,
   (2) $\gamma_{rer}(m(G)) \cdot \gamma_{rer}(m(\bar{G})) \le n^2$.

Proof. The aforementioned results clearly apply to any connected graphs G and $\bar{G}$ of order n. □

4. Illustration

A *graph G* and it's *litact graph m(G)* with a regular domination number and a RRD number are described in Figure 1.

Figure 1. A graph G and it's litact graph $m(G)$ with $\gamma_r[m(G)] = 1$, $\gamma_{rer}[m(G)] = 1$

Example 4.1. Examine the graphs G and it's corresponding litact graph $m(G)$ in Figure 1. As can be observed, a RRDS of $m(G)$ is the set $D_{rer} = \{v_2\}$. According to this, $\gamma_{rer}[m(G)] = 1$.

5. Conclusion

This paper is the first to study the notion of restrained regular domination in litact graphs. We proved certain limitations of the RRD number of litact graphs and investigated the computational difficulty of this idea. We characterized *all trees* that reach the demonstrated bound. Additionally, we offered characterizations of litact graphs with various RRD values.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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