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Research Article

Characterisation of KrishSupra-P Distribution and Its Biomedical Application

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Abstract. In this article, an innovative generalization of Sauleh distribution is considered and named it as *KrishSupra-P Distribution* (KSPD). Its several statistical properties were discussed and the model parameters are predicted using the *Maximum Likelihood Estimation* (MLE) approach. The model's biomedical application using a real data set is discussed and contrasted with some other famous distributions.

Keywords. Weighted distribution, Sauleh distribution, Survival analysis, Estimates, Parameters

Mathematics Subject Classification (2020). 62E10

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1. Introduction

Statistical Distributions (SD) have a significant role in data science. A distribution indicates the values that the variable is most likely to be taken. The weighted distribution offers a method for handling issues with data interpretation and model formulation. Fisher [7] proposed the notion of *Weighted Distribution* (WD) to explain the ascertainment bias, and Rao [14] formalized the ascertainment approach for modelling statistical data where standard distributions have not been suited to capture such observations with equivalent probability. WDs are utilized in various research fields connected to ecology biomedicine, branching, as well as reliability processes. This concept of weighted distributions offers a novel method for the distribution. The

WDs offer a method for fitting models to the unknown weight function when samples could be drawn from original and developed distributions. The WD decreases to length biased when the weight function takes into account only the unit length of interest. Numerous scholars have examined the different weighted probability models and provided examples of how they are used in various contexts.

Kilany [10] examined the *Weighted Lomax Distribution* (WLD) and its uses. Ganaie *et al.* [8] examined the WLD and its applications. Praseeja *et al.* [12] detailed the characteristics of SRIMIN-H distribution and its biomedical application as a generalised application of WD. Mohiuddin *et al.* [11] explored the features and uses of the weighted Nwikpe distribution. Hassan *et al.* [9] examined the properties and estimate of the length-biased form of the power WLD using censored sample data. Asgharzadeh *et al.* [4] presented a novel *Lindley Distribution* (LD), that is also by WD and applies to survival analysis. Shakhatreh [17] suggested a weighted exponential distribution with two parameters. Alsmairan and Al-Omari [3] suggested weighted Suja distribution with application to ball bearings data. Saghir *et al.* [16] provided a brief overview and descriptions of the WD. Ahmad *et al.* [1] examine the distributions in the weighted exponential family. Castillo and Pérez-Casany [5] created a weighted Poisson distribution for solutions with over- and under-dispersion. Rather and Subramanian [15] examines the uses of the weighted Akshaya distribution in engineering science. Afterwards, Dar *et al.* [6] explored the use of the weighted Gamma-Pareto distribution.

Sauleh Distribution is a combination of two-distributions, gamma and exponential distribution. This distribution is a newly introduced one-parametric lifetime distribution suggested by Aijaz *et al.* [2].

2. KrishSupra-P Distribution (KSPD)

The *probability density function* (pdf) of the Sauleh distribution is presented as,

$$f(x;\theta)=\frac{\theta^4}{\theta^4+2\theta+6}(\theta+x^2+x^3)e^{-\theta x}, \quad x>0,\ \theta>0 \tag{2.1}$$

and the *cumulative distribution function* (cdf) of Sauleh distribution is presented as,

$$F(x;\theta)=1-\left(1+\frac{\theta^2x(\theta^2x^2+\theta x+3x+2\theta+6)}{\theta^4+2\theta+6}\right)e^{-\theta x}, \quad x>0,\ \theta>0. \tag{2.2}$$

Let X be a non-negative random variable with pdf $f(x)$, then the pdf of a weighted random variable X_w is presented as,

$$f_w(x)=\frac{w(x)f(x)}{E(w(x))}, \quad x>0,$$

where $w(x)$ be the non-negative weight function and $E(w(x))=\int w(x)f(x)dx<\infty$.

For various selections of weight function $w(x)$ weighted models are of many forms when $w(x)=x^c$, the resulting distribution is termed as WD. Here, we examine the weighted version of the Sauleh distribution using $w(x)=x^c$ to get the KSPD. The WD's pdf is provided by

$$f_w(x)=\frac{x^cf(x)}{E(x^c)}, \tag{2.3}$$

where

$$E(x^c) = \int_0^\infty x^c f(x,\theta)dx$$
$$= \frac{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}{\theta^c(\theta^4+2\theta+6)}. \tag{2.4}$$

Substituting eqs. (2.1) and (2.4) in eq. (2.3), we will find the KSPD's pdf as

$$f_w(x) = \frac{x^c\theta^{c+4}}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}(\theta+x^2+x^3)e^{-\theta x} \tag{2.5}$$

and the cdf of KSPD could be attained as

$$F_w(x) = \int_0^x f_w(x)dx$$
$$= \int_0^x \frac{x^c\theta^{c+4}}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}(\theta+x^2+x^3)e^{-\theta x}dx$$
$$= \frac{1}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}\int_0^x x^c\theta^{c+4}(\theta+x^2+x^3)e^{-\theta x}dx$$
$$= \frac{1}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)} \cdot\left(\theta^{c+5}\int_0^x x^c e^{-\theta x}dx+\theta^{c+4}\int_0^x e^{-\theta x}dx+\theta^{c+4}\int_0^x x^{c+3}e^{-\theta x}dx\right). \tag{2.6}$$

Simplifying eq. (2.6), we get the cdf of KSPD,

$$F_w(x) = \frac{1}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}(\theta^4\gamma(c+1,\theta x)+\theta\gamma(c+3,\theta x)+\gamma(c+4,\theta x)). \tag{2.7}$$

Here $\gamma(c+1,\theta x)$, $\gamma(c+3,\theta x)$ and $\gamma(c+4,\theta x)$ represents the incomplete Gamma function.

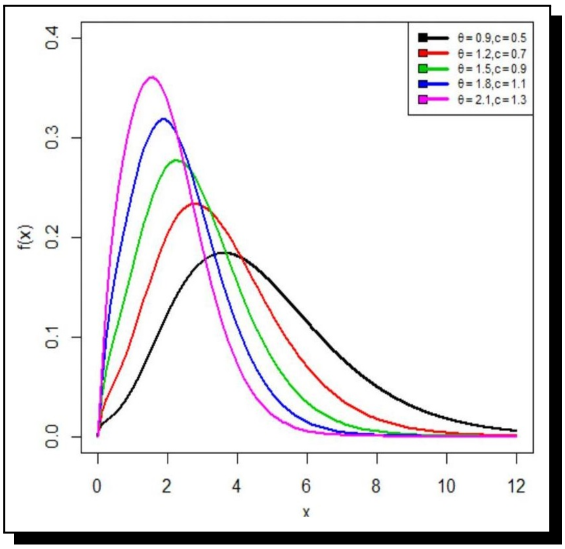

Figure 1. The pdf of KSPD

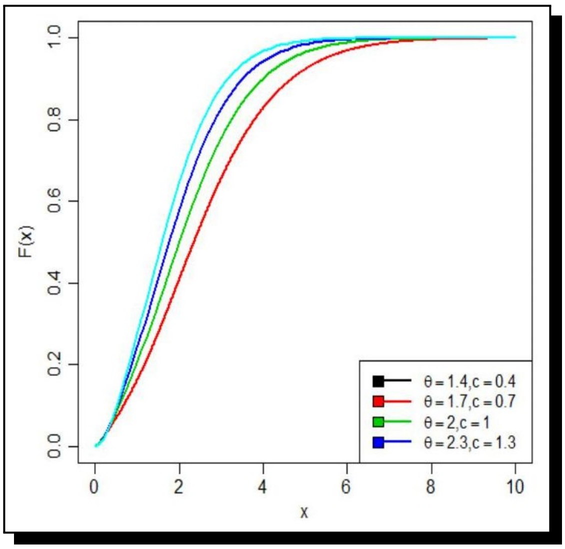

Figure 2. The cdf of KSPD

From Figures 1 and 2, the nature of pdf and cdf of the KSPD is observed with different α and θ.

3. Survival Analysis, Hazard Function, and Reverse Hazard Function of KSPD

3.1 Survival Function

The weighted Sauleh distribution's survival function might be found as

$$S(x) = 1 - F_w(x)$$
$$= 1 - \frac{1}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}(\theta^4\gamma(c+1,\theta x)+\theta\gamma(c+3,\theta x)+\gamma(c+4,\theta x)). \quad (3.1)$$

From Figure 3, it is clear that the Survival function of KrishSupra-P distribution is decreases while X increases with respect to different α and θ.

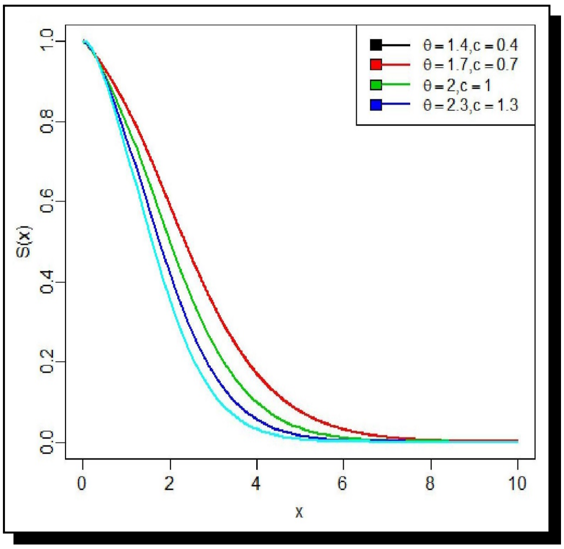

Figure 3. Survival function of KrishSupra-P distribution

3.2 Hazard Function

It is expressed by

$$h(x) = \frac{f_w(x)}{1-F_w(x)}$$
$$= \frac{x^c\theta^{c+4}(\theta+x^2+x^3)e^{-\theta x}}{(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))-(\theta^4\gamma(c+1,\theta x)+\theta\gamma(c+3,\theta x)+\gamma(c+4,\theta x))}. \quad (3.2)$$

3.3 Reverse Hazard Function

It is represented by

$$h_r(x) = \frac{f_w(x)}{F_w(x)}$$
$$= \frac{x^c\theta^{c+4}(\theta+x^2+x^3)e^{-\theta x}}{(\theta^4\gamma(c+1,\theta x)+\theta\gamma(c+3,\theta x)+\gamma(c+4,\theta x))}. \quad (3.3)$$

4. Statistical Properties

4.1 Moments

Let X be the random variable that follows KSPD with c and θ parameters, thus, rth order moment $E(X^r)$ is expressed as,

$$\begin{aligned}
E(X^r) &= \mu_r' \\
&= \int_0^\infty x^r f_w(x)dx \\
&= \int_0^\infty \frac{c^{c+r}\theta^{c+4}}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\theta\Gamma(c+4)}(\theta+x^2+x^3)e^{-\theta x}dx \\
&= \frac{\theta^{c+4}}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\theta\Gamma(c+4)}\int_0^\infty x^{c+r}(\theta+x^2+x^3)e^{-\theta x}dx \\
&= \frac{\theta^{c+4}}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)} \\
&\quad \times\left(\theta\int_0^\infty x^{(c+r+1)-1}e^{-\theta x}dx+\int_0^\infty x^{(c+r+3)-1}e^{-\theta x}dx+\int_0^\infty x^{(c+r+4)-1}e^{-\theta x}dx\right).
\end{aligned}$$

Simplifying this we get,

$$E(X^r) = \mu_r' = \frac{\theta^4\Gamma(c+r+1)+\theta\Gamma(c+r+3)+\Gamma(c+r+4)}{\theta^r(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))}. \tag{4.1}$$

Put $r=1,2,3$ and 4 in eq. (4.1), we will find the 1st four moments of KSPD,

$$E(X) = \mu_1' = \frac{\theta^4\Gamma(c+2)+\theta\Gamma(c+4)+\Gamma(c+5)}{\theta(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))},$$

$$E(X^2) = \mu_2' = \frac{\theta^4\Gamma(c+3)+\theta\Gamma(c+5)+\Gamma(c+6)}{\theta^2(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))},$$

$$E(X^3) = \mu_3' = \frac{\theta^4\Gamma(c+4)+\theta\Gamma(c+6)+\Gamma(c+7)}{\theta^3(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))},$$

$$E(X^4) = \mu_4' = \frac{\theta^4\Gamma(c+5)+\theta\Gamma(c+7)+\Gamma(c+8)}{\theta^4(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))},$$

$$\text{Variance} = \frac{\theta^4\Gamma(c+3)+\theta\Gamma(c+5)+\Gamma(c+6)}{\theta^2(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))} - \left(\frac{\theta^4\Gamma(c+2)+\theta\Gamma(c+4)+\Gamma(c+5)}{\theta(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))}\right)^2, \tag{4.2}$$

$$\text{SD}(\sigma) = \sqrt{\left(\frac{\theta^4\Gamma(c+3)+\theta\Gamma(c+5)+\Gamma(c+6)}{\theta^2(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))} - \left(\frac{\theta^4\Gamma(c+2)+\theta\Gamma(c+4)+\Gamma(c+5)}{\theta(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))}\right)^2\right)}. \tag{4.3}$$

4.2 Harmonic Mean (HM)

The KSPD's harmonic mean is

$$\begin{aligned}
\text{HM} = E\left(\frac{1}{x}\right) &= \int_0^\infty \frac{1}{x} f_w(x)dx \\
&= \int_0^\infty \frac{x^{c-1}\theta^{c+4}}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}(\theta+x^2+x^3)e^{-\theta x}dx \\
&= \frac{x^{c+4}}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}\int_0^\infty x^{c-1}(\theta+x^2+x^3)e^{-\theta x}dx
\end{aligned}$$

$$= \frac{\theta^{c+4}}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)} \times \left(\theta\int_0^\infty x^{(c+1)-2}e^{-\theta x}dx + \int_0^\infty x^{(c+2)-1}e^{-\theta x}dx + \int_0^\infty x^{(c+3)-1}e^{-\theta x}dx\right).$$

Simplifying this we get,

$$\text{HM} = \frac{\theta(\theta^3\Gamma(c+1)+\theta\Gamma(c+2)+\Gamma(c+3))}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}. \tag{4.4}$$

4.3 Moment Generating Function and Characteristic Function

Let X denote the random variable following KSPD with parameters θ and c, then the moment generating function of the suggested model may be found as

$$M_X(t) = E(e^{tx}) = \int_0^\infty e^{tx} f_w(x)dx$$

with Taylor's series, obtain

$$\begin{aligned} M_X(t) &= \int_0^\infty \left(1+tx+\frac{(tx)^2}{2!}+\dots\right) f_w(x)dx \\ &= \int_0^\infty \sum_{j=0}^\infty \frac{t^j}{j!} x^j f_w(x)dx \\ &= \sum_{j=0}^\infty \frac{t^j}{j!}\mu'_j, \end{aligned}$$

that is,

$$\begin{aligned} M_X(t) &= \sum_{j=0}^\infty \frac{t^j}{j!}\left(\frac{\theta^4\Gamma(c+j+1)+\theta\Gamma(c+j+3)+\Gamma(c+j+4)}{\theta^j(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))}\right) \\ &= \frac{1}{(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))}\sum_{j=0}^\infty \frac{t^j}{j!\theta^j}(\theta^4\Gamma(c+j+1)+\theta\Gamma(c+j+3)+\Gamma(c+j+4)). \end{aligned} \tag{4.5}$$

Likewise, the characteristic function of KSPD may be presented as

$$\begin{aligned} \varphi_x(t) &= M_X(it) \\ &= \frac{1}{(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))}\sum_{j=0}^\infty \frac{it^j}{j!\theta^j}(\theta^4\Gamma(c+j+1)+\theta\Gamma(c+j+3)+\Gamma(c+j+4)). \end{aligned} \tag{4.6}$$

5. Order Statistics

Suppose $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ represents the order statistics of the random sample $X_1, X_2, \dots, X_n$ drawn from a continuous population with pdf $f_x(x)$ and cdf $F_X(x)$, then the pdf of rth order statistics $X_{(r)}$ is presented as

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)(F_X(x))^{r-1}(1-F_X(x))^{n-r}. \tag{5.1}$$

Substitute the eqs. (2.5) and (2.7) in eq. (5.1), we will find the pdf of rth order statistics of KSPD as

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!}\left(\frac{x^c\theta^{c+4}}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}(\theta+x^2+x^3)e^{-\theta x}\right)$$

$$\times\left(\frac{1}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}(\theta^4\gamma(c+1,\theta x)+\theta\gamma(c+3,\theta x)+\gamma(c+4\theta x))\right)^{r-1}$$
$$\times\left(1-\frac{1}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}(\theta^4\gamma(c+1,\theta x)+\theta\gamma(c+3,\theta x)+\gamma(c+4,\theta x))\right)^{n-r}.$$

Thus, the pdf of higher-order statistic $X_{(n)}$ of KSPD may be presented as

$$f_{x(n)}(x)=\frac{nx^c\theta^{c+4}}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}(\theta+x^2+x^3)e^{-\theta x}$$
$$\times\left(\frac{1}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}(\theta^4\gamma(c+1,\theta x)+\theta\gamma(c+3,\theta x)+\gamma(c+4,\theta x))\right)^{n-1}.$$

Pdf of 1st order statistic $X_{(1)}$ of KSPD may be expressed as

$$f_{x(1)}(x)=\frac{nx^c\theta^{c+4}}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}(\theta+x^2+x^3)e^{-\theta x}$$
$$\times\left(1-\frac{1}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}(\theta^4\gamma(c+1,\theta x)+\theta\gamma(c+3,\theta x)+\gamma(c+4,\theta x))\right)^{n-1}.$$

6. Likelihood Ratio Test

Let $X_1,X_2,\dots,X_n$ be a random sample of size n from KSPD. To examine the *null hypothesis*,

$$H_0: f(x)=f(x;\theta) \quad \text{against} \quad H_1: f(x)=f_w(x;\theta).$$

The statistic utilized to find whether the size-n random sample is obtained from the KSPD or the Sauleh distribution is,

$$\begin{aligned}\Delta&=\frac{L_1}{L_0}\\&=\prod_{i=1}^{n}\frac{f_w(x;\theta)}{f(x;\theta)}\\&=\prod_{i=1}^{n}\left(\frac{x_i^c\theta^c(\theta^4+2\theta+6)}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}\right)\\&=\left(\frac{\theta^c(\theta^4+2\theta+6)}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}\right)^n\prod_{i=1}^{n}x_i^c.\end{aligned} \tag{6.1}$$

We should reject the *null hypothesis*, if

$$\Delta=\left(\frac{\theta^c(\theta^4+2\theta+6)}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}\right)^n\prod_{i=1}^{n}x_i^c>k$$

or

$$\begin{aligned}\Delta^*&=\prod_{i=1}^{n}x_i^c\\&>k\left(\frac{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}{\theta^c(\theta^4+2\theta+6)}\right)^n\\&>k^*,\end{aligned}$$

where $k^*=k\left(\frac{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}{\theta^c(\theta^4+2\theta+6)}\right)^n$.

For large samples of size n, the value of p may alternatively be found using the chi-square distribution, where, $2\log\Delta$ is given as a chi-square distribution with degree of freedom. Therefore, we reject the null hypothesis when the value of probability is provided by, we reject the null hypothesis.

$p(\Delta^* > \lambda^*)$, where $\lambda^* = \prod_{i=1}^{n} x_i^c$ is less than a specified level of significance and $\prod_{i=1}^{n} x_i^c$ is the observed value of the statistic Δ^*.

7. Bonferroni and Lorenz Curves

Economics uses the Bonferroni and Lorenz curves [13] to analyze how income inequality and poverty are distributed. The Lorenz and Bonferroni curves may be found by

$$B(p) = \frac{1}{p\mu_1'}\int_0^q x f(x)dx$$

and

$$L(p) = pB(p) = \frac{1}{\mu_1'}\int_0^q x f(x)dx,$$

where

$$\mu_1' = \frac{\theta^4\Gamma(c+2)+\theta\Gamma(c+4)+\Gamma(c+5)}{\theta(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))},$$

$$\begin{aligned}
B(p) &= \frac{\theta(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))}{p(\theta^4\Gamma(c+2)+\theta\Gamma(c+4)+\Gamma(c+5))}\int_0^q \frac{x^{c+1}\theta^{c+4}}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}(\theta+x^2+x^3)e^{-\theta x}dx \\
&= \frac{\theta^{c+5}}{p(\theta^4\Gamma(c+2)+\theta\Gamma(c+4)+\Gamma(c+5))}\int_0^q x^{c+1}(\theta+x^2+x^3)e^{-\theta x}dx \\
&= \frac{\theta^{c+5}}{p(\theta^4\Gamma(c+2)+\theta\Gamma(c+4)+\Gamma(c+5))} \\
&\quad \times \left(\theta\int_0^q x^{(c+2)-1}e^{-\theta x}dx + \int_0^q x^{(c+4)-1}e^{-\theta x}dx + \int_0^q x^{(c+5)-1}e^{-\theta x}dx\right).
\end{aligned}$$

Simplifying this we get,

$$B(p) = \frac{\theta^{c+5}}{p(\theta^4\Gamma(c+2)+\theta\Gamma(c+4)+\Gamma(c+5))}(\theta\gamma(c+2,\theta q)+\gamma(c+4,\theta q)+\gamma(c+5,\theta q)), \tag{7.1}$$

$$L(p) = \frac{\theta^{c+5}}{(\theta^4\Gamma(c+2)+\theta\Gamma(c+4)+\Gamma(c+5))}(\theta\gamma(c+2,\theta q)+\gamma(c+4,\theta q)+\gamma(c+5,\theta q)). \tag{7.2}$$

8. MLE and Fisher's Information Matrix

We predict the parameters of KSPD using the MLE approach. Let $X_1, X_2, \dots, X_n$ be a random sample size n obtained from the KSPD, then likelihood function could be presented as follows:

$$\begin{aligned}
L(x) &= \prod_{i=1}^{n} f_w(x) \\
&= \prod_{i=1}^{n}\left(\frac{x_i^c\theta^{c+4}}{(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))}(\theta+x_i^2+x_i^3)e^{-\theta x_i}\right)
\end{aligned}$$

$$= \frac{\theta^{n(c+4)}}{(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))^n}\prod_{i=1}^{n}(x_i^c(\theta+x_i^2+x_i^3)e^{-\theta x_i}).$$

Then

$$\log L = n(c+4)\log\theta - n\log(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)) + c\sum_{i=1}^{n}\log x_1 + \sum_{i=1}^{n}\log(\theta+x_i^2+x_1^3) - \theta\sum_{i=1}^{n}x_1. \tag{8.1}$$

Now differentiating the log-likelihood eq. (4.2) with respect to parameter θ and c. We get the following normal equation,

$$\frac{\partial \log L}{\partial\theta} = \frac{n(c+4)}{\theta} - n\left(\frac{4\theta^3\Gamma(c+1)+\Gamma(c+3)}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}\right) + \sum_{i=1}^{n}\left(\frac{1}{(\theta+x_i^2+x_i^3)}\right) - \sum_{i=1}^{n}x_i$$
$$= n\log\theta - n\psi(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)) + \sum_{i=1}^{n}\log x_i = 0.$$

It is very challenging to resolve the above nonlinear system of equations algebraically due to its complex form. Consequently, R and WOLFRAM MATH are the tools we use to estimate the suggested distribution's parameters.

To find the confidence interval, we utilize the asymptotic normality findings. We have that if $\widehat{\beta}=(\widehat{\theta},\widehat{c})$ presents the MLE of $\beta=(\theta,c)$. We may present the outcomes as

$$\sqrt{n}(\widehat{\beta}-\beta)\to N_2(0,I^{-1}(\beta)),$$

where $I^{-1}(\beta)$ is Fisher's information matrix, i.e.,

$$I(\beta) = -\frac{1}{n}\begin{pmatrix} E\left(\frac{\partial^2\log L}{\partial\theta^2}\right) & E\left(\frac{\partial^2\log L}{\partial\theta\partial c}\right) \\ E\left(\frac{\partial^2\log L}{\partial c\partial\theta}\right) & E\left(\frac{\partial^2\log L}{\partial c^2}\right)\end{pmatrix}$$

$$E\left(\frac{\partial^2\log L}{\partial\theta^2}\right) = -\frac{n(c+4)}{\theta^2} - n\left(\frac{(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))12\theta^2\Gamma(c+1)-(4\theta^3\Gamma(c+1)+\Gamma(c+3))^2}{(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4))^2}\right) - \sum_{i=1}^{n}\left(\frac{1}{(\theta+x_i^2+x_i^3)^2}\right),$$

$$E\left(\frac{\partial^2\log L}{\partial c^2}\right) = -n\psi'(\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)),$$

$$E\left(\frac{\partial^2\log L}{\partial\theta\partial c}\right) = \frac{n}{\theta} - n\psi\left(\frac{4\theta^3\Gamma(c+1)+\Gamma(c+3)}{\theta^4\Gamma(c+1)+\theta\Gamma(c+3)+\Gamma(c+4)}\right).$$

As β being unknown, we predict $I^{-1}(\beta)$ by $I^{-1}(\widehat{\beta})$ and this could be applied to find asymptotic confidence interval for θ and c.

9. Analysis of Data

A data set of real-life in KSPD is deemed to examine its goodness of fit. Comparisons have been made between the fit for Lindley, exponential, and Sauleh distributions.

Table 1 represents the weight loss (kilograms) of 77 randomly selected patients (of age group 12 to 15 — Secondary School Students from Thrissur district of Kerala — (June 2021)) after being infected with COVID-19.

Table 1. The weight loss (kilograms) of patients from Thrissur district of Kerala

1.39	1.44	1.46	1.53	1.59	1.6	1.63	1.68	1.71	1.72
4.02	4.32	4.58	5.55	2.54	0.77	2.93	3.27	3.42	3.47
1.83	1.95	1.96	1.97	2.02	2.13	2.15	2.16	2.22	2.30
1.00	0.99	1.02	1.05	1.07	1.07	1.08	1.08	1.08	1.09
0.10	0.33	0.44	0.56	0.59	0.59	0.72	0.74	0.92	0.93
1.13	1.15	1.16	1.2	1.21	1.22	1.22	1.24	1.30	1.34
2.4	2.45	2.51	2.53	2.54	2.78	1.83	1.95	1.96	1.97
1.76	3.61	2.31	1.12	0.96	1.36	2.02			

The R software is used to forecast the unknown parameters in addition to the values of the model comparison criterion. To compare the KSPD with Sauleh distributions, exponential distributions, as well as Lindley distributions, we consider the parameters AICC (*Akaike Information Criterion Corrected*), AIC (*Akaike Information Criterion*), BIC (*Bayesian Information Criterion*), and $-2\log L$. The distribution with lower values of $-2\log L$, AICC, BIC, and AIC is the better one. For determining the criteria AIC, AICC, BIC, and $-2\log L$ formulas mentioned below are applied:

$$AIC = 2k - 2\log L,\ BIC = k\log n - 2\log L \text{ and } AICC = AIC + \frac{2k(k+1)}{n-k-1}.$$

Here, n represents the size of the sample, k indicates the number of parameters inside the statistical model, and $-2\log L$ presents the maximized value of the log-likelihood function in the model.

Table 2. Comparison and performance of fitted distributions

Distributions	MLE	SE	$-2\log L$	AIC	BIC	AICC
KSPD	$\widehat{\theta} = 2.6090$	$\widehat{\theta} = 0.3019$				
	$\widehat{c} = 1.8501$	$\widehat{c} = 0.50912$	191.4293	195.3943	199.9900	195.5960
Sauleh	$\widehat{\theta} = 1.5014$	$\widehat{\theta} = 0.0697$	214.7476	216.7476	219.0243	216.8047
Exponential	$\widehat{\theta} = 0.5697$	$\widehat{\theta} = 0.0596$	224.8929	226.8929	229.1696	226.9500
Lindley	$\widehat{\theta} = 0.8697$	$\widehat{\theta} = 0.0697$	213.051	215.051	217.3277	215.1081

From Table 2, it was seen that the KSPD has the lesser $-2\log L$, BIC, AICC, and AIC values as compared to exponential, Sauleh, and Lindley distributions. Therefore, it may be found that the KSPD leads to a better fit than Sauleh, exponential as well and Lindley distributions.

10. Conclusion

The current analysis deal with a novel distribution termed KSPD was detailed. Its different statistical properties such as moments, shapes of the cdf and pdf, order statistics, moment generating function, survival function, harmonic mean, hazard rate function, Bonferroni and

Lorenz curves were examined. The distribution parameters were evaluated with MLE method. Finally, the distribution was analyzed with a real data set to demonstrate its usefulness and the findings indicate that the suggested distribution displays a better fit than exponential, Sauleh, and Lindley distributions.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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