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Research Article

Propagation of Waves in Thin Nanorod With the Effect of Thermal Field

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Abstract. In this paper, thermal response for the propagation of waves in thin nanorod is studied with the Timoshenko beam theory. The important role for vibrational analysis of the rod and characteristics of the flexural waves is discussed. Numerical calculations are derived and the scattered relations between the wavenumber and wave velocities are computed.

Keywords. Timoshenko beam theory, Wave number, Phase velocity, Wave propagation, Thermal field

Mathematics Subject Classification (2020). 74F05, 74K10

1. Introduction

Nelson [8] discovered about the material behaviors, while beam theory analysis offers analytical solutions but may be limited in complexities and nonlinearity. Elishakoff [4] establishes a beam theory that includes rotary inertia, shear deformation, and shear correction factor is commonly referred to as the Timoshenko-Ehrenfest beam theory. Wang *et al*. [10] explained about transverse shear deformation and the scale effect that are important while dealing with micro and nanobeams that are short and stubby, are both taken into account by the nonlocal Timoshenko theory. Stephen and Puchegger [9] discussed about the hinged-hinged end conditions and explained about the frequency of the beam theory which can be factorized,resulting in consecutive spectrum of natural frequencies. Dong *et al*. [3] explained about the shear correction factors that are calculated by equating the three-dimensional elasticity theories with the two transverse forces that are concurrently applied to a cross-section.

Huang [6] explained about the longitudinal vibrations in nanorod with internal long range interactions. Aydogdu [1] explained that the classical (local) rod model greatly overestimates the axial vibration frequencies by excluding the impact of small-length scale. Murmu and Adhikari [7] developed the axial vibration of double-nanorod system significantly influenced by nonlocal effects. Bahrami [2] explained about the reflection and transmission matrices of wave power reflection in nanorod. Yang *et al*. [12] discussed about the nonlocal fluid theory in fluid filled graphene tubule.

In this paper, the influence of thermal fields, on the propagation of flexural waves in graphene nanorod is discussed. The analysis is conducted within the framework of continuum mechanics. The dispersion curves obtained for the graphene nanorod with thermal effect is compared with the existing literature that considers no thermal effect. This comparison serves to highlight the accuracy and precision of findings. Overall, this paper contributes to the understanding of flexural wave propagation in graphene nanorod by considering the influence of thermal fields, providing valuable insights for researchers in the field.

2. Formulation of the Problem

Consider a graphene nanorod based on Timoshenko beam model that is subjected to shear force and moment of bending. During the bending, the plane is perpendicular to the horizontal axis. The relation between the moment of bending and curvature is as follows

$$\frac{\partial y}{\partial x} = \frac{\psi}{\gamma_o}, \tag{1}$$

where y is a displacement to the centroidal plane, x is the axial coordinate, t is the time, ψ is the effect of bending and γ_o is the shear effects.

The expressions for bending moment is given by

$$\frac{M}{EI} = \frac{\partial \psi}{\partial x}, \tag{2}$$

where M denotes moment of bending, E denotes Youngs Modulus and I is the moment of inertia. Shear strain γ and shear modulus G are applied for expressing the shear force V at the cross section, respectively as

$$V = G\int_A \gamma dA,$$

where A represents the cross sectional area. Since γ_o represents the centroidal axis for the shear strain, then $G\gamma_o A$ is the shear force. The adjustment coefficient κ is introduced to balance the equation and is given by

$$V = (G\gamma_o A)\kappa$$

$$\implies \quad V = GA\kappa\left(\frac{\partial y}{\partial x} - \psi\right) \tag{3}$$

Equation of motion in the vertical direction for the element is of the form

$$-V + \left(V + \frac{\partial V}{\partial x}dx\right) + q dx = \rho A dx \frac{\partial^2 y}{\partial t^2}$$

$$\implies \quad \frac{\partial V}{\partial x} + q = \rho A \frac{\partial^2 y}{\partial t^2} \tag{4}$$

where ρ be the element of mass density and q is the external force. Summing moments about axis that is perpendicular to x, y-plane and passing through the element center, we get

$$\frac{1}{2} V dx + \frac{1}{2}\left(V + \frac{\partial V}{\partial x} dx\right) dx + M - \left(M + \frac{\partial M}{\partial x}\right) dx = \rho I \frac{\partial^2 \psi}{\partial t^2} dx$$

$$\implies \quad V - \frac{\partial M}{\partial x} = \rho I \frac{\partial^2 \psi}{\partial t^2}. \tag{5}$$

Substituting the bending moment eqn. (2) and shear force eqn. (3) into eqns. (4) and (5), the equations of the motion becomes

$$GA\kappa\left(\frac{\partial \psi}{\partial x} - \frac{\partial^2 y}{\partial x^2}\right) + \rho A \frac{\partial^2 y}{\partial t^2} = q(x,t), \tag{6}$$

$$GA\kappa\left(\frac{\partial y}{\partial x} - \psi\right) + EI \frac{\partial^2 \psi}{\partial x^2} = \rho I \frac{\partial^2 \psi}{\partial t^2}. \tag{7}$$

Due to thermal effects, the constant axial force q is given by

$$q = -EA\alpha\theta \frac{\partial^2 y}{\partial x^2}, \tag{8}$$

where the temperature is θ and the thermal expansion is α. Substituting the eqns. (8) in (6), the governing equations of the motion for Timoshenko beam theory of the graphene nanorod becomes

$$\implies \quad GA\kappa\left(\frac{\partial \psi}{\partial x} - \frac{\partial^2 y}{\partial x^2}\right) + \rho A \frac{\partial^2 y}{\partial t^2} = -EA\alpha\theta \frac{\partial^2 y}{\partial x^2} \tag{9}$$

$$\implies \quad GA\kappa\left(\frac{\partial y}{\partial x} - \psi\right) + EI \frac{\partial^2 \psi}{\partial x^2} = \rho I \frac{\partial^2 \psi}{\partial t^2} \tag{10}$$

In eqn. (9) and eqn. (10), if the thermal field terms are neglected the result matches with Graff [5].

3. Solution of the Problem

Distribution of waves in graphene nanorod under a temperature is studied by considering the harmonic wave in the infinite beam. Assuming the solutions in the form

$$y = B_1 e^{i(\gamma x - \omega t)}, \quad \psi = B_2 e^{i(\gamma x - \omega t)}, \tag{11}$$

where B_1 and B_2 are amplitudes, γ is the wavenumber and ω is the frequency. Substituting the harmonic solution eqn. (11) in the equation of motion eqns. (9)-(10), we get

$$(GA\kappa\gamma^2 - \rho A\omega^2 - EA\alpha\theta\gamma^2)B_1 + iGA\kappa\gamma B_2 = 0, \tag{12}$$

$$iGA\kappa\gamma B_1 - (GA\kappa + EI\gamma^2 - \rho I\omega^2)B_2 = 0. \tag{13}$$

Eqns. (12)-(13) can be written in the form of matrix,

$$\begin{bmatrix} (GA\kappa\gamma^2 - \rho A\omega^2 - EA\alpha\theta\gamma^2) & iGA\kappa\gamma \\ iGA\kappa\gamma & -(GA\kappa + EI\gamma^2 - \rho I\omega^2) \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = [0]. \tag{14}$$

A trivial solution is obtained by solving the matrix given in eqn. (14), so as to achieve a significant solution compare the coefficient of the determinant arrangement to nonexistent as follows

$$\begin{vmatrix} GA\kappa\gamma^2-\rho A\omega^2-EA\alpha\theta\gamma^2) & iGA\kappa\gamma \\ iGA\kappa\gamma & -(GA\kappa+EI\gamma^2-\rho I\omega^2) \end{vmatrix} = [0]. \tag{15}$$

By solving the determinant given in eqn. (15), a fourth order frequency equation is obtained in the form

$$(G\kappa A\gamma^2-\rho A\omega^2-EA\alpha\theta\gamma^2)(GA\kappa+EI\gamma^2-\rho I\omega^2)-G^2A^2\kappa^2\gamma^2=0$$

$$\implies \frac{EI}{\rho A}\left(1-\frac{E\alpha\theta}{G\kappa}\right)\gamma^4-\frac{I}{A}\left(1+\frac{E}{G\kappa}+\frac{E\alpha\theta}{G\kappa}\right)\gamma^2\omega^2-\omega^2+\frac{\rho I}{G\kappa A}\omega^4-\frac{E\alpha\theta\gamma^2}{\rho^2}=0. \tag{16}$$

Eqn. (16) represents the relationship between the frequency and wavenumber of the graphene nanorod based on Timoshenko beam theory. Letting the wave propagation's phase velocity $c=\frac{\omega}{\gamma}$, the dispersive characteristics of the nanorod are analyzed.

4. Numerical Results

The frequency equation is derived using beam theory and parameters of the beam are taken in the order from Yang *et al.* [11] as $E=1\times10^{12}$ pa, $\rho=2.27\times10^3\,\mathrm{kg\,m^{-3}}$, $G=0.4\times10^{12}$ pa, $A=3\times10^{-19}\,\mathrm{m}^2$, $I=1.78\times10^{-38}\,\mathrm{m}^4$, $T=10\,\mathrm{K}$ and $\alpha=-1.6\times10^{-6}\,\mathrm{k}^{-1}$.

Dispersion relation are drawn between the wave number and phase velocity of the graphene nanorod with the effect thermal field and is shown in Figure 1. From Figure 1, it is observed that the Timoshenko beam theory as function of wavenumber and phase velocity with the effect of thermal field. Also it determines that when the wavenumber rises the phase velocity also increases in certain period and then it flows down in same period without any disturbance in the entire process.

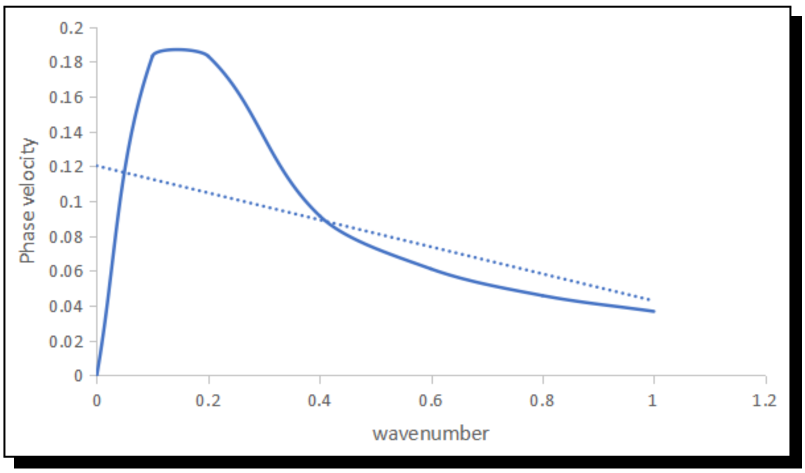

Figure 1. Dispersion relation between wavenumber and phase velocity with temperature

Dispersion relation are drawn between the wave number and phase velocity of the graphene nanorod exposed to distinct thermal field and is shown in Figure 2. From Figure 2, it is seen that as the wave number rises the phase velocity increases at different temperature for higher modes of vibration it represents the wave characteristics in nature.

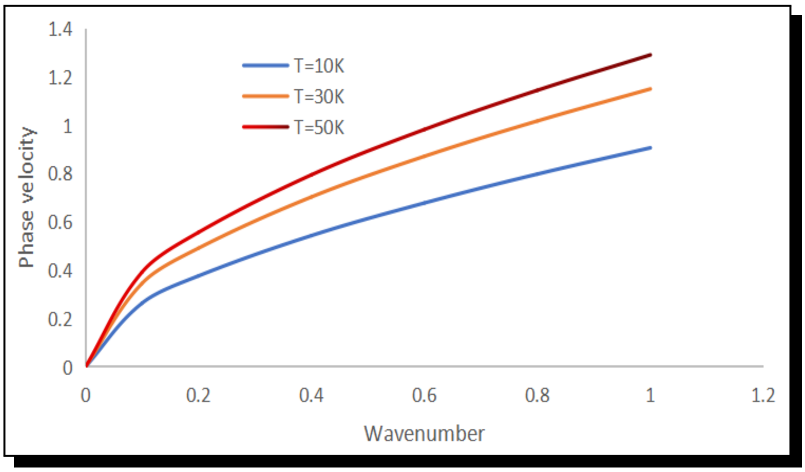

Figure 2. Dispersion relation between wavenumber and phase velocity with temperature for higher modes

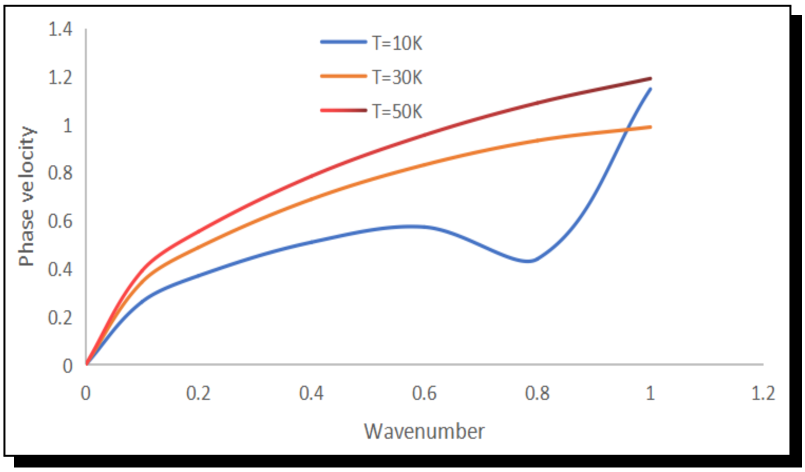

Figure 3. Dispersion relation between wavenumber and phase velocity with temperature for lower modes

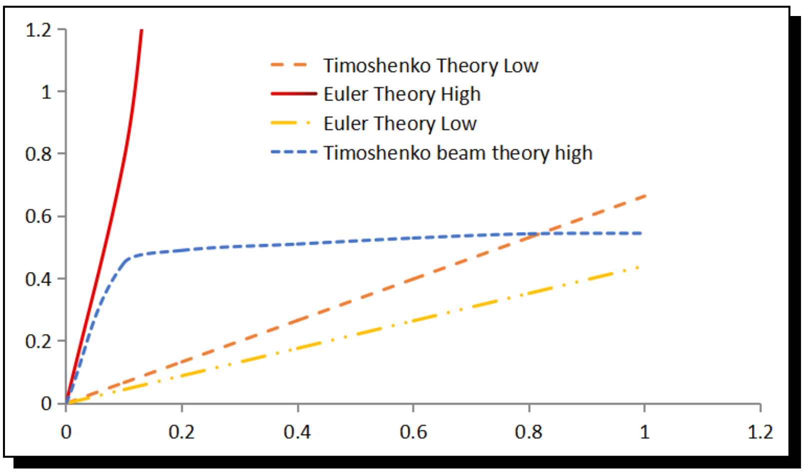

Figure 4. Dispersion curves between wavenumber and phase velocity

Dispersion relation are drawn between the wave number and phase velocity of the graphene nanorod exposed to distinct thermal field and is shown in Figure 3. From Figure 3, it is seen that as the wave number rises the phase velocity also increases at different temperature for higher modes of vibration. For different temperature it is determined as the wave number increases the phase velocities increases.

Dispersion curves of graphene tubule in the absence of thermal effect is drawn and is proven in Figure 4. From Figure 4, it is determined that the graphene nanorod increases the wavenumber as the phase velocities increases also. Further, the dispersion curves of graphene nanorod in the absence of external force matches the curves in Figure 4 of Graff [5]. This demonstrates the existence of the current result.

5. Concluding Remarks

The scattered relations between the phase velocity and wavenumber is derived by finding the solution in matrix form. Frequency equations for flexural modes of vibration are derived and the numerical results are plotted in form of dispersion curves to study the characteristics of propagation of waves. Also it drives reasoned that the phase velocity increases as wave number climbs for any mode of vibrations.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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