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Research Article

Convection in Ferromagnetic Nanoliquids Under Terrestrial Gravity Condition With the Effect of Non-Inertial Acceleration

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Abstract. The stability analysis of Rayleigh-Bénard convection subject to the effect of non-inertial acceleration is performed in the presence of nano-sized ferromagnetic particles-Fe_3O_4, with water, engine oil and kerosene as base fluids subject to external uniform magnetic field. The plots for thermal Rayleigh number and magnetic Rayleigh number versus wave number for different values of Lewis number (Le) and Taylor's number (Ta) are plotted and discussed in detail. Velocity profiles with the effect of non-inertial acceleration of ferrofluid has also been obtained in the study.

Keywords. Newtonian ferrofluid, Non-inertial acceleration, Fe_3O_4-magnetite nanoparticles, Rayleigh-Bénard convection

Mathematics Subject Classification (2020). 76M25, 76R10, 76R50, 76W05

1. Introduction

The main focus of the paper is on the application of ferrofluids — a step towards nanotechnology. Keeping this in mind, a ferromagnetic particle, Fe_3O_4-magnetite, sized 10nm, is suspended in the base fluids, namely, water, engine oil and kerosene. Upon recent literature survey, realized the scientific and technological importance of studying the effects of non–inertial acceleration on ferroconvection and the same inspired to analyse the influence of rotation in this paper. Convection in rotating porous media are registered in the works of Bhadauria *et al*. [2], Vadasz [17, 18] and references cited therein. Rayleigh-Bénard convection with the effect of rotation have been investigated by Bhattacharjee *et al*. [3], Kanchana *et al*. [8], Rossby [11], Zhong *et al*. [21], and references cited therein. Convection in rotating ferro-magnetic fluids have been discussed in the studies of Bhadauria *et al*. [2], Gupta and Gupta [5], Sunil and Mahajan [16]. Study of convection in magnetic fluids and nanofluids saturating a rotating porous media can be seen in the works of Mahajan and Sharma [9], Mahajan *et al*. [10], Saravanan [12], Sekar *et al*. [13], Shivakumara *et al*. [14], Vaidyanathan *et al*. [19], Yadav *et al*. [20], and references cited therein, and now in the current paper, we have considered the convection in ferromagnetic nanofluids viz., *Water-Magnetite* (WM), *Kerosene-Magnetite* (KM) and *Engine Oil-Magnetite* (EOM) under the effect of rotation. Stability curves and velocity profiles are drawn for the considered problem and finally thermophysical properties have been tabulated for WM, KM and EOM using the properties of nanofluids.

2. Mathematical Formulation

We consider a Newtonian ferro-nanoliquid layer of thickness d, parallel to the xy-plane of large horizontal extension subject to a vertical temperature gradient with gravitational acceleration, $\vec{g} = -g\hat{k}$ of an electrically non-conducting Boussinesq approximation. An external vertical magnetic field $\vec{H} = H_0\hat{k}$ is applied and the ferro-nanoliquid is rotating uniformly about z-axis with angular velocity, $\vec{\Omega} = \Omega\hat{k}$. The imposed temperatures at the layer of boundaries are, $T_{z=0} = T_0 + \Delta T$ and $T_{z=d} = T_0$. The equations governing the problem are:

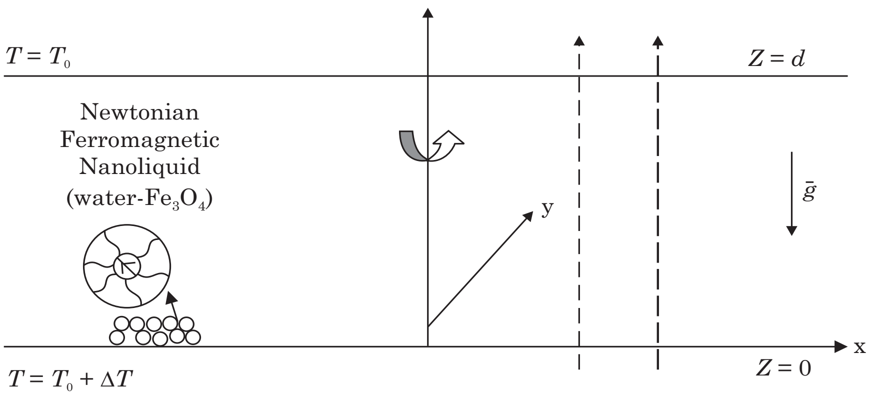

Figure 1. Physical configuration

2.1 Governing Equations

Conservation of mass:

$$u_x + v_y + w_z = 0, \tag{2.1}$$

Conservation of momentum:

$$\rho_{nf}[q_t+uq_x+vq_y+wq_z]=-(p_x,p_y,p_z)+\mu_0(M_1H_x+M_2H_y+M_3H_z) \\ -(0,0,\rho g)+2\rho_{nf}(v\Omega,-u\Omega,0)+\mu_{nf}(q_{xx}+q_{yy}+q_{zz}) \tag{2.2}$$

Conservation of energy:

$$(\rho C_p)_{nf}(T_t+uT_x+vT_y+wT_z) \\ =(\rho C_p)_{np}\left[D_B(T_x\psi_x+T_y\psi_y+T_z\psi_z)+\frac{D_T}{T_0}(T_x^2+T_y^2+T_z^2)-\frac{D_H}{H_0}(T_xH_x+T_yH_y+T_zH_z)\right] \\ +k_{nf}(T_{xx}+T_{yy}+T_{zz}) \tag{2.3}$$

Conservation of nanoparticle concentration:

$$\psi_t+u\psi_x+v\psi_y+w\psi_z \\ =\left[D_B(\psi_{xx}+\psi_{yy}+\psi_{zz})+\frac{D_T}{T_0}(T_{xx}+T_{yy}+T_{zz})-\frac{D_H}{H_0}(H_{xx}+H_{yy}+H_{zz})\right] \tag{2.4}$$

Density equation of state:

$$\rho_{nf}(\psi,T)=\psi\rho_{np}+(1-\psi)\rho_{bf}(1-\beta(T-T_0)) \tag{2.5}$$

Maxwell's equation:

$$\nabla\cdot B=0,\ \ H=\nabla\phi,\ \ B=\mu_0(M+H) \tag{2.6}$$

Magnetic equation of state:

$$\vec{M}=M_0+\chi_m(\vec{H}-H_0)-K(T-T_0) \tag{2.7}$$

where $q=(u,v,w)$ is the velocity vector, ρ is the density, t is the time, p is the pressure, μ_0 is the magnetic permeability of vacuum, μ is the viscosity, $M=(M_1,M_2,M_3)$ is the magnetisation, C_p is the specific heat, k is the thermal conductivity, D_B is the Brownian diffusion coefficient, D_H is the magnetophoretic coefficient, D_T is the thermophoretic diffusion coefficient, ψ is the ferromagnetic nanoparticle volume fraction, β is the thermal expansion coefficient, B is the magnetic induction, χ magnetic susceptibility, K_m and K_p are magnetic coefficients, T_0, H_0, M_0 and ψ_0 are the initial values.

2.2 Properties of Ferrofluid ([4], [6])

Viscosity:

$$\frac{\mu_{nf}}{\mu_f}=\frac{1}{(1-\chi)^{2.5}} \tag{2.8}$$

Thermal conductivity:

$$\frac{k_{nf}}{k_f}=\frac{\left(\frac{k_{np}}{k_f}+2\right)-2\chi\left(1-\frac{k_{np}}{k_f}\right)}{\left(\frac{k_{np}}{k_f}+2\right)+\chi\left(1-\frac{k_{np}}{k_f}\right)} \tag{2.9}$$

Thermal diffusivity:

$$\alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}} \tag{2.10}$$

Density:

$$\frac{\rho_{nf}}{\rho_f} = (1-\chi) + \chi\frac{\rho_{np}}{\rho_f} \tag{2.11}$$

Heat capacity:

$$\frac{(\rho C_p)_{nf}}{(\rho C_p)_f} = (1-\chi) + \chi\frac{(\rho C_p)_{np}}{(\rho C_p)_f} \tag{2.12}$$

Thermal expansion coefficient:

$$\frac{(\rho\beta)_{nf}}{(\rho\beta)_f} = (1-\chi) + \chi\frac{(\rho\beta)_{np}}{(\rho\beta)_f} \tag{2.13}$$

Magnetic susceptibility, χ_{nf}, ([7]) according to:

Langevin's initial susceptibility (LIS):

$$\chi_{il} = \frac{\mu_0\pi M_d^2 d_1^3 \chi_2}{18 k_B T} \tag{2.14}$$

Modified Langevin's initial susceptibility (MLIS):

$$\chi_{iml} = \chi_{il}\left[1+\frac{\chi_{il}}{3}\right] \tag{2.15}$$

Weiss's initial susceptibility (WIS):

$$\chi_{iw} = \frac{\chi_{il}}{1-\frac{\chi_{il}}{3}} \tag{2.16}$$

Onsager's initial susceptibility (OIS):

$$\chi_{io} = \frac{3}{4}\left[\chi_{il} - 1 + \sqrt{1+\frac{2}{3}\chi_{il} + \chi_{il}^2}\right] \tag{2.17}$$

where M_d is the magnetic saturation, d_1 is the diameter of magnetite, $\chi_2 = \frac{\psi d_1^3}{(d_1+\chi_1)^3}$ is the gross volume fraction of the nanoparticles including the non-magnetic layer at the surface of the particles, χ_1 is the non-magnetic layer (= 2 nm), k_B is the Boltzmann constant ($= 1.38\times 10^{-23}$ J/K).

2.3 Basic State

Consider the solution of basic state in the below form:

$$(u,v,w) = (u_b,v_b,w_b) = (0,0,0), \quad \rho = \rho_b(z), \quad H = H_b(z), \quad M = M_b(z), \quad T = T_b(z). \tag{2.18}$$

Using equations (2.18) in basic equations, we obtain basic state solution in the below form:

$$\left.\begin{aligned} &T_b(z) = T_0 + \Delta T\left(1-\frac{z}{d}\right); \quad \psi_b(z) = \psi_0 + \lambda_1\,; \\ &H_b(z) = \left(0,0,H_0 - \lambda_2\frac{\Delta T}{d}z\right); \quad M_b(z) = \left(0,0,M_0 + \lambda_2\frac{\Delta T}{d}z\right), \end{aligned}\right\} \tag{2.19}$$

where

$$\left.\lambda_1 = \left[\frac{N_A(1+\chi_{nf}) - K_m N_A'}{1+\chi_{nf} - K_p N_A'}\right];\ \lambda_2 = \left[\frac{K_m - K_p N_A}{1+\chi_{nf} - K_p N_A'}\right];\ N_A = \frac{D_T}{D_B T_0} \text{ and } N_A' = \frac{D_H}{D_B H_0}.\right\} \tag{2.20}$$

2.4 Perturbed State

The basic state is perturbed in the below form:

$$\vec{q} = \vec{q}_b + \vec{q}', \quad p = p_b + p', \quad \vec{H}_b = H_b \hat{k} + \vec{H}', \quad \vec{M}_b = M_b \hat{k} + \vec{M}', \quad T = T_b + T'. \tag{2.21}$$

Using the above in equation (2.6) and (2.7) (and dropping $'$), we have:

$$\left.\begin{aligned} &M_1 + H_1 = \left(1 + \frac{M_0}{H_0}\right) H_1, \quad M_2 + H_2 = \left(1 + \frac{M_0}{H_0}\right) H_2, \\ &M_3 + H_3 = (1 + \chi_{nf}) H_3 - K_m \theta + K_p \psi, \end{aligned}\right\} \tag{2.22}$$

Eliminating the pressure in the momentum equation, z-component of the equation takes the form:

$$\begin{aligned} &\rho_{nf} \nabla^2 w_t - \mu_{nf} \nabla^4 w + (\psi_{xx} + \psi_{yy}) \left[g\rho_{np} - g\rho_{bf} - g\beta\rho_{bf} \frac{\Delta T}{d} z - \mu_0 \lambda_2 \frac{K_p \Delta T}{d} \right] \\ &\quad = (\theta_{xx} + \theta_{yy}) \left[g\beta\rho_{bf} - g\beta\rho_{bf}\psi_0 - g\beta\rho_{bf}\lambda_1 \frac{\Delta T}{d} z + \mu_0 \lambda_2 \frac{K_m \Delta T}{d} \right] \\ &\qquad - (1 + \chi_{nf}) \lambda_2 \frac{\Delta T}{d} (\phi_{zxx} + \phi_{zyy}) - 2\rho_{nf} \Omega \zeta_z , \end{aligned} \tag{2.23}$$

where $\zeta = v_x - u_y$ is the vorticity in z-direction.

Using equation (2.21) in momentum equation and taking curl on both sides of the equation, z-component takes the form:

$$\rho_{nf} \zeta_t = \mu_{nf} (\zeta_{xx} + \zeta_{yy} + \zeta_{zz}) + 2\rho_{nf} \Omega w_z . \tag{2.24}$$

Using equation (2.21) in energy and concentration equation:

$$\begin{aligned} \theta_t = w \frac{\Delta T}{d} + \alpha_{nf} (\theta_{xx} + \theta_{yy} + \theta_{zz}) + N_B \left[D_B \lambda_1 - 2 \frac{D_T}{T_0} + \frac{D_H}{H_0} \lambda_2 \right] \frac{\Delta T}{d} \theta_z \\ + N_B \left[-D_B \psi_z + \frac{D_H}{H_0} \phi_{zz} \right] \frac{\Delta T}{d}, \end{aligned} \tag{2.25}$$

$$\psi_t + w\lambda_1 \frac{\Delta T}{d} = D_B (\psi_{xx} + \psi_{yy} + \psi_{zz}) + \frac{D_T}{T_0} (\theta_{xx} + \theta_{yy} + \theta_{zz}) - \frac{D_H}{H_0} (H_{xx} + H_{yy} + H_{zz}). \tag{2.26}$$

Maxwell's equation (2.6), after substituting equation 2.21 and using equation (2.22), we have:

$$\left(1 + \frac{M_0}{H_0}\right) (\phi_{xx} + \phi_{yy}) + (1 + \chi_{nf}) \phi_{zz} - K_m \theta_z + K_p \psi_z = 0 . \tag{2.27}$$

The boundary conditions are considered as:

$$w = w_{zz} = T_z = \phi_z = D_B \psi_z + \frac{D_T}{T_0} T_z - \frac{D_H}{H_0} H_z = \zeta_z = 0 \ \text{ at } \ z = 0 \ \text{ and } \ d . \tag{2.28}$$

Considering the normal mode expansion as:

$$[w, \theta, \psi, \phi, \zeta] = [W(z), \Theta(z), \Psi(z), \Phi(z), \xi(z)] e^{i(\kappa_x x + \kappa_y y)}, \tag{2.29}$$

where κ_x and κ_y are the wave numbers in x and y directions, respectively, equations (2.23)-(2.27), takes the form:

$$\left.\begin{aligned} &[\rho_{nf} (D^2 - \kappa^2) W_t - \mu_{nf} (D^2 - \kappa^2)^2 W] \\ &\quad = -(1 + \chi_{nf}) \lambda_2 \tfrac{\Delta T}{d} \kappa^2 D\Phi + \left[g\beta\rho_{bf} - g\beta\rho_{bf}\Psi_0 - g\beta\rho_{bf}\lambda_1 \frac{\Delta T}{d} z + \mu_0 \lambda_2 \tfrac{K_m \Delta T}{d} \right] \kappa^2 \Theta \\ &\qquad - \left[g\rho_{np} - g\rho_{bf} - g\beta\rho_{bf} \frac{\Delta T}{d} z - \mu_0 \lambda_2 \frac{K_p \Delta T}{d} \right] \kappa^2 \Psi - 2\rho_{nf} \Omega D\xi \end{aligned}\right\}, \tag{2.30}$$

$$\rho_{nf}\xi_t = \mu_{nf}(D^2-\kappa^2)\xi + 2\rho_{nf}\Omega DW, \tag{2.31}$$

$$\Theta_t = W\frac{\Delta T}{d} + \alpha_{nf}(D^2-\kappa^2)\Theta + N_B\left[D_B\lambda_1 - 2\frac{D_T}{T_0} + \frac{D_H}{H_0}\lambda_2\right]\frac{\Delta T}{d}D\Theta + N_B\left[-D_B D\Psi + \frac{D_H}{H_0}D^2\Phi\right]\frac{\Delta T}{d}, \tag{2.32}$$

$$\Psi_t + W\lambda_1\frac{\Delta T}{d} = D_B(D^2-\kappa^2)\Psi + \frac{D_T}{T_0}(D^2-\kappa^2)\Theta - \frac{D_H}{H_0}(D^2-\kappa^2)\Phi, \tag{2.33}$$

$$-\left(1+\frac{M_0}{H_0}\right)\kappa^2\Phi + (1+\chi_{nf})D^2\Phi - K_m D\Theta_z + K_p D\Psi_z = 0, \tag{2.34}$$

where $\kappa^2 = \kappa_x^2 + \kappa_y^2$.

2.5 Non-dimensionalisation

$$\left.\begin{aligned} &(x^*,y^*,z^*) = \left(\frac{x}{d},\frac{y}{d},\frac{z}{d}\right), \quad W^* = \frac{d}{\alpha_{bf}}W, \quad t^* = \frac{\alpha_{bf}}{d^2}t, \quad \xi^* = \frac{d^2}{\alpha_{bf}}\xi, \\ &\Theta^* = \frac{\Theta}{\Delta T'}, \quad \Phi^* = \frac{(1+\chi_{nf})}{K_m\Delta T d}\Phi, \quad \Psi^* = \frac{\Psi}{\Delta\Psi'}. \end{aligned}\right\} \tag{2.35}$$

After non-dimensionalising equations (2.30)–(2.34), using equation (2.35), we have:

$$\left.\begin{aligned} &[(D^2-\kappa^2)W_t - a\,\mathrm{Pr}_{nf}(D^2-\kappa^2)^2 W] \\ &\quad = -a^2\mathrm{Pr}_{nf}R_T M_1\kappa^2 D\Phi + a^2\mathrm{Pr}_{nf}R_T[1-\Psi_0-\lambda_1\Delta T z + M_1]\kappa^2\Theta \\ &\qquad - a^2\mathrm{Pr}_{nf}[R_C - R_T\Delta\Psi z + R_T M_2]\kappa^2\Psi - \sqrt{Ta}D\xi \end{aligned}\right\}, \tag{2.36}$$

$$\xi_t = a\,\mathrm{Pr}_{nf}(D^2-\kappa^2)\xi + \sqrt{Ta}DW, \tag{2.37}$$

$$\Theta_t = W + a(D^2-\kappa^2)\Theta - a\frac{N_B}{Le}\Delta\Psi D\Psi + a\frac{N_B}{Le}[\lambda_1 - 2N_A + N_A'\lambda_2]\Delta T D\Theta + a\frac{N_A' N_B}{Le}\frac{K_m\Delta T}{(1+\chi_{nf})}D^2\Phi, \tag{2.38}$$

$$\Psi_t + W\lambda_1\frac{\Delta T}{\Delta\Psi} = \frac{a}{Le}(D^2-\kappa^2)\Psi + a\frac{N_A}{Le}\frac{\Delta T}{\Delta\Psi}(D^2-\kappa^2)\Theta - a\frac{N_A'}{Le}\frac{K_m\Delta T}{(1+\chi_{nf})\Delta\Psi}D(D^2-\kappa^2)\Phi, \tag{2.39}$$

$$(D^2-\kappa^2 M3)\Phi - D\Theta + \frac{K_p}{K_m}\frac{\Delta\Psi}{\Delta T}D\Phi = 0, \tag{2.40}$$

where $\mathrm{Pr}_{nf} = \frac{\mu_{nf}}{\rho_{nf}\alpha_{nf}}$ — nanofluid Prandtl number,

$R_{nf} = \frac{\rho_{bf}\beta g\Delta T d^3}{\alpha_{nf}\mu_{nf}}$ — thermal Rayleigh number,

$R_C = \frac{(\rho_{np}-\rho_{bf})g\Delta\Psi d^3}{\alpha_{nf}\mu_{nf}}$ — concentration Rayleigh number,

$Ta = \frac{4\Omega^2 d^4}{\alpha_{bf}^2}$ — Taylors number,

$Le = \frac{\alpha_{nf}}{D_B}$ — Lewis number,

$N_B = \frac{(\rho C_p)_{np}}{(\rho C_p)_{nf}}$ — ratio of specific heat capacities,

$a = \frac{\alpha_{nf}}{\alpha_{bf}}$ — ratio of thermal diffusivity of nanofluid to base fluid,

$M_1 = \frac{\mu_0 K_m\Delta T}{gd(\rho\beta)_{nf}}\lambda_2$ and $M_2 = \frac{\mu_0 K_p\Delta\Psi}{gd(\rho\beta)_{nf}}\lambda_2$ are magnetic parameters.

Now setting,

$$[W,\Theta,\Psi,\Phi,\xi]=[W(z),\Theta(z),\Psi(z),\Phi(z),\xi(z)]e^{\tau t}, \tag{2.41}$$

where τ is the complex growth rate disturbances.
Using the above, in equations (2.36)-(2.40), we have:

$$\left.\begin{aligned}&[\tau(D^2-\kappa^2)-a\,\mathrm{Pr}_{nf}(D^2-\kappa^2)^2]W\\ &\quad=-a^2\mathrm{Pr}_{nf}R_T M_1\kappa^2 D\Phi+a^2\mathrm{Pr}_{nf}R_T[1-\Psi_0-\lambda_1\Delta Tz+M_1]\kappa^2\Theta\\ &\qquad-a^2\mathrm{Pr}_{nf}[R_C-R_T\Delta\Psi z+R_T M_2]\kappa^2\Psi-\sqrt{Ta}D\xi\end{aligned}\right\}, \tag{2.42}$$

$$\tau\xi=a\,\mathrm{Pr}_{nf}(D^2-\kappa^2)\xi+\sqrt{Ta}DW, \tag{2.43}$$

$$\tau\Theta=W+a(D^2-\kappa^2)\Theta-a\frac{N_B}{Le}\Delta\Psi D\Psi+a\frac{N_B}{Le}[\lambda_1-2N_A+N'_A\lambda_2]\Delta TD\Theta+a\frac{N'_A N_B}{Le}\frac{K_m\Delta T}{(1+\chi_{nf})}D^2\Phi, \tag{2.44}$$

$$\tau\Psi+W\lambda_1\frac{\Delta T}{\Delta\Psi}=\frac{a}{Le}(D^2-\kappa^2)\Psi+a\frac{N_A}{Le}\frac{\Delta T}{\Delta\Psi}(D^2-\kappa^2)\Theta-a\frac{N'_A}{Le}\frac{K_m\Delta T}{(1+\chi_{nf})\Delta\Psi}D(D^2-\kappa^2)\Phi, \tag{2.45}$$

$$(D^2-\kappa^2 M3)\Phi-D\Theta+\frac{K_p}{K_m}\frac{\Delta\Psi}{\Delta T}D\Phi=0. \tag{2.46}$$

The boundary conditions after non-dimensionalisation, takes the form:

$$W=W_{zz}=\Theta_z=\Phi_z=D_B\Psi_z+\frac{D_T}{T_0}\Theta_z-\frac{D_H}{H_0}H_z=\xi_z=0 \ \text{ at } \ z=0 \ \text{ and } \ 1. \tag{2.47}$$

3. Method of Solution

We assume the solution for W, ξ, Θ, Ψ and Φ in the below form, which satisfies the free-free boudary conditions, (2.47):

$$W=A_1\sin(\pi z),\quad \Theta=-\frac{B_1}{\pi}\cos(\pi z),\quad \Phi=-\frac{C_1 H_0}{\pi^2 D_H}\cos(\pi z),\quad \Psi=\frac{C_1}{\pi D_B}\sin(\pi z),\quad \xi=\frac{D_1}{\pi}\cos(\pi z), \tag{3.1}$$

where A_1, B_1, C_1 and D_1 are constants.
Substituting the above solution in equations (2.42)–(2.46), we have:

$$\begin{bmatrix} \frac{-a\,\mathrm{Pr}\,\delta^4-\delta^2\tau}{2} & \frac{a^2\mathrm{Pr}\,\kappa^2 R_T\Delta T\lambda_1}{4\pi^2} & \frac{a^2\mathrm{Pr}\,\kappa^2 R_T\pi M_1}{2N_A N'_A} & \frac{\sqrt{Ta}}{2} & \frac{a^2\mathrm{Pr}\,\kappa^2(2R_c+2R_T M_2-R_T\Delta\psi}{4\pi} \\ \frac{\pi\sqrt{Ta}}{2} & 0 & 0 & \frac{a\,\mathrm{Pr}\,\delta^2+\tau}{2\pi} & 0 \\ 0 & \frac{a\delta^2+\tau}{2\pi} & \frac{a\pi^2 N_B K_m\Delta T}{2Le(1+\chi_{nf})} & 0 & -\frac{aN_B\Delta\psi}{2Le} \\ -\frac{\Delta T\lambda_1}{2\Delta\psi} & 0 & \frac{a\pi\delta^2 K_m\Delta T}{2Le\Delta\psi(1+\chi_{nf})} & 0 & -\frac{a\delta^2+Le\tau}{2Le\pi} \\ 0 & 0 & \frac{\kappa^2 M3+\pi^2}{2N'_A} & 0 & 0 \end{bmatrix}\begin{bmatrix} A_1\\ B_1\\ C_1\\ D_1\\ E_1\end{bmatrix}$$

$$=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix} \tag{3.2}$$

where $\delta^2=\kappa^2+\pi^2$.

A non-trivial solution to the above equation, exists only when:

$$R_{Tc} = \frac{\begin{pmatrix} 2(a\delta^2+\tau)(a^3\Pr^2\delta^2(\delta^6\Delta\psi+\kappa^2 LeR_C\Delta T\lambda_1) \\ +a^2\Pr(\delta^6\Delta\psi\tau(2+Le\Pr)+\kappa^2 LeR_C\Delta T\lambda_1\tau) \\ +a\Delta\psi(\delta^4(1+2Le\Pr)\tau^2\delta^2 Ta)+Le\Delta\psi\tau(\tau^2\delta^2+\pi^2 Ta)) \end{pmatrix}}{\begin{pmatrix} (a^2\kappa^2\Pr\Delta T\lambda_1)(a\Pr\delta^2+\tau) \\ \cdot(Lea\Delta\psi\delta^2-2aLeM_2\delta^2-aN_B\Delta T\Delta\psi\lambda_1-2LeM_2\tau+Le\Delta\psi\tau) \end{pmatrix}}. \tag{3.3}$$

To analyse the stability of the system, the real part of τ is set to zero and therefore $\tau = i\tau_i$, where τ_i is the imaginary part of complex growth rate disturbance, τ, and the direct bifurcation occurs at $\tau_i = 0$. Accordingly, critical thermal Rayleigh number, is given by:

$$R_{Tc} = \frac{2(a^2\Pr^2\delta^2(\delta^6\Delta\psi+\kappa^2 LeR_C\Delta T\lambda_1)+\Delta\psi(\pi^2\delta^2 Ta))}{(a^2\kappa^2\Pr^2\Delta T\lambda_1)(Le\Delta\psi\delta^2-2LeM_2\delta^2-N_B\Delta T\Delta\psi\lambda_1)} \tag{3.4}$$

and the corresponding magnetic Rayleigh numbers are given by:

$$R_{Tc}M_1 = \frac{2(a^2\Pr^2\delta^2(\delta^6\Delta\psi+\kappa^2 LeR_C\Delta T\lambda_1)+\Delta\psi(\pi^2\delta^2 Ta))}{(a^2\kappa^2\Pr^2\Delta T\lambda_1)(Le\Delta\psi\delta^2-2LeM_2\delta^2-N_B\Delta T\Delta\psi\lambda_1)}\frac{\mu_0 K_m\Delta T}{gd(\rho\beta)_{nf}}\lambda_2 \tag{3.5}$$

and

$$R_{Tc}M_1 = \frac{2(a^2\Pr^2\delta^2(\delta^6\Delta\psi+\kappa^2 LeR_C\Delta T\lambda_1)+\Delta\psi(\pi^2\delta^2 Ta))}{(a^2\kappa^2\Pr^2\Delta T\lambda_1)(Le\Delta\psi\delta^2-2LeM_2\delta^2-N_B\Delta T\Delta\psi\lambda_1)}\frac{\mu_0 K_p\Delta\Psi}{gd(\rho\beta)_{nf}}\lambda_2. \tag{3.6}$$

The eigen value problem is solved analytically using perturbation technique with wave number κ as a perturbation parameter, and hence following the below expansion in powers of κ^2 as:

$$\left.\begin{aligned} W &= W_0+\kappa^2 W_1+\kappa^4 W_2+\dots \\ \xi &= \xi_0+\kappa^2\xi_1+\kappa^4\xi_2+\dots \\ \Theta &= \Theta_0+\kappa^2\Theta_1+\kappa^4\Theta_2+\dots \\ \Psi &= \Psi_0+\kappa^2\Psi_1+\kappa^4\Psi_2+\dots \\ \Phi &= \Phi_0+\kappa^2\Phi_1+\kappa^4\Phi_2+\dots \end{aligned}\right\}, \tag{3.7}$$

in equations (2.42)-(2.47), at zeroth order we have below set of solutions:

$$W_0 = \xi_0 = 0, \quad \Theta_0 = \Psi_0 = \Phi_0 = 1,$$

and at first order we have:

$$a\Pr_{nf}D^4W_1+a^2\Pr_{nf}R_T[1-\Psi_0-\lambda_1\Delta Tz+M_1]-a^2\Pr_{nf}[R_C-R_T\Delta\Psi z+R_TM_2]-\sqrt{Ta}D\xi_1 = 0, \tag{3.8}$$

$$a\Pr_{nf}D^2\xi_1+\sqrt{Ta}DW_1 = 0, \tag{3.9}$$

$$W_1+aD^2\Theta_1-a-a\frac{N_B}{Le}\Delta\Psi D\Psi_1+a\frac{N_B}{Le}[\lambda_1-2N_A+N_A'\lambda_2]\Delta TD\Theta_1+a\frac{N_A'N_B}{Le}\frac{K_m\Delta T}{(1+\chi_{nf})}D^2\Phi_1 = 0, \tag{3.10}$$

$$\frac{a}{Le}D^2\Psi_1-W_1\lambda_1\frac{\Delta T}{\Delta\Psi}-\frac{a}{Le}\psi_0+a\frac{N_A}{Le}\frac{\Delta T}{\Delta\Psi}D^2\Theta_1-a\frac{N_A'}{Le}\frac{K_m\Delta T}{(1+\chi_{nf})\Delta\Psi}D^3\Phi_1 = 0, \tag{3.11}$$

$$D^2\Phi_1-M3-D\Theta_1+\frac{K_p}{K_m}\frac{\Delta\Psi}{\Delta T}D\Phi_1 = 0. \tag{3.12}$$

Velocity profiles are drawn for the above set of equations for varying values of Taylors number, Ta, and magnetisation parameters, M_1 and M_2.

4. Results

Figure 2 shows the linear relationship between the thermal Rayleigh number and concentration Rayleigh number for different values of Le and Ta. Also, we can see that, decrease in Le and increase in Ta has the powers to stabilize the system.

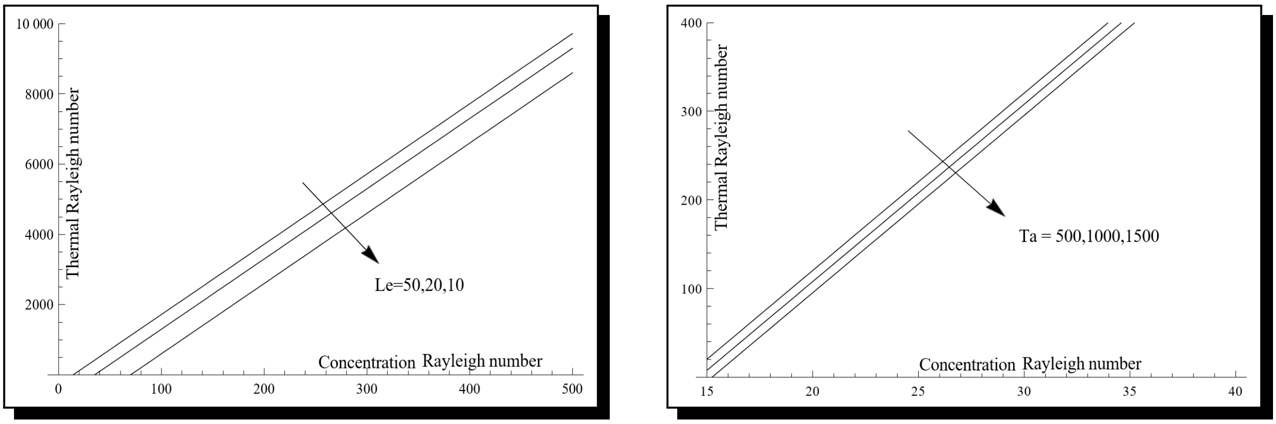

Figure 2. R_T versus R_C for different values of Le and Ta

Neutral stability curves are shown in Figures 3–9. In Figures 3, 5 and 7, plots for thermal and magnetic Rayleigh numbers versus wave number for different values of Le are shown and can be seen that, as Le increases, both thermal and magnetic Rayleigh numbers decreases denoting the early onset of convection. In Figures 4, 6 and 8, plots for thermal and magnetic Rayleigh numbers versus wave number for different values of Ta can be seen and one can observe that as Ta number increases, thermal and magnetic Rayleigh number increases which implies the effect of non-inertial acceleration is to stabilize the system. Therefore, rotation can be used as a technique to regulate the stability of the system. Figure 9 shows the plot of thermal Rayleigh number versus wave number for different values of concentration Rayleigh number, thereby implying that, increase in the concentration of ferromagnetic nanoparticles triggers the early onset of convection. The minimum points in these graphs denotes the critical thermal and critical magnetic Rayleigh numbers corresponding to critical wave number.

Velocity profiles for different values of magnetic parameters M_1, M_2 and Taylor number, Ta has been presented in Figures 10 and 11 respectively for all three ferrofluids viz., WM, KM and EOM. Varying the above mentioned parametrs, one can notice that, velocity is high in EOM ferrofluid and gradually decreases for KM and WM ferrofluids. There is an upward shift in the velocity profile as we increase the magnetic parameter M_2 and decrease M_1. Also, the effect of non-inertial acceleration can be seen in Figure 11. As Ta increases, velocity profile shifts down gradually. Thermophysical properties of basefluids and nanoparticle are pulled from earlier studies and are shown in Table 1 and thermophysical properties for ferrofluids are calculted as shown in Table 2 and 3 for nanoparticle volume fraction $\psi = 0.1$ and $\psi = 0.2$, respectively.

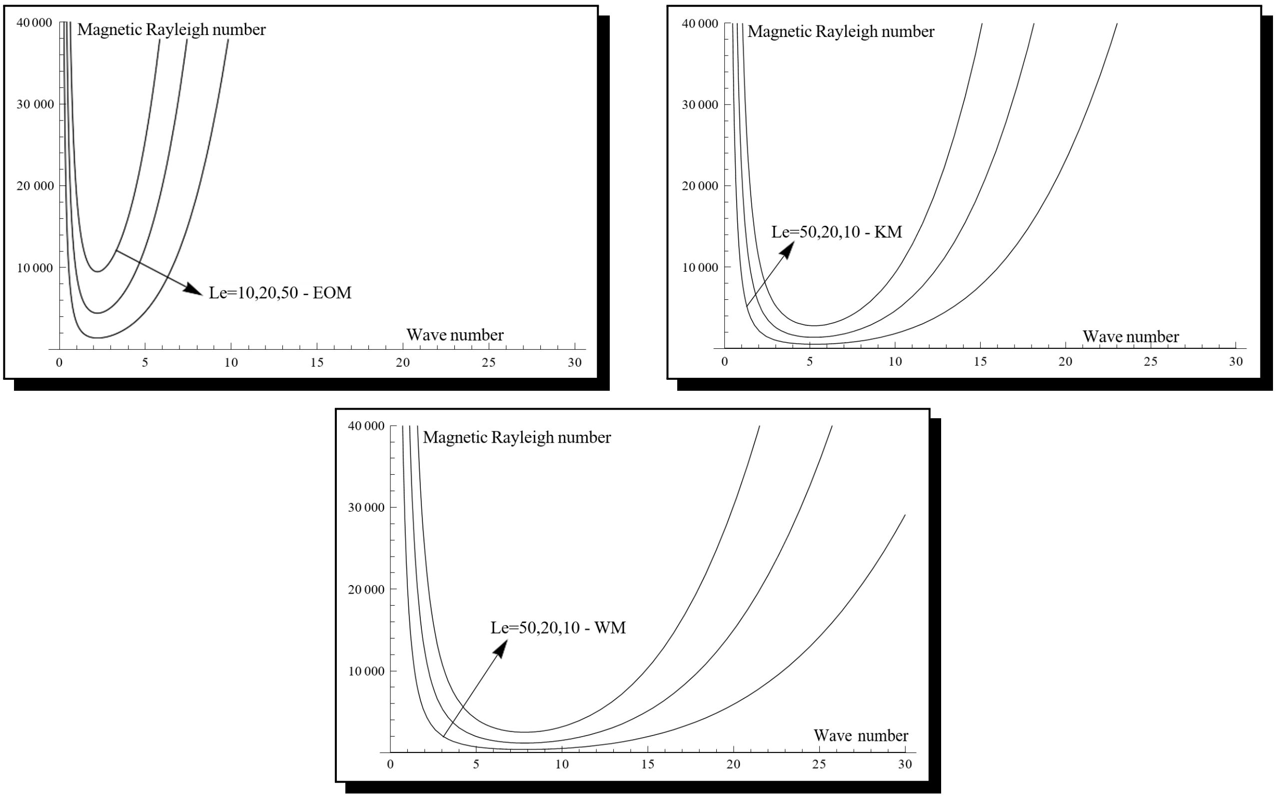

Figure 3. R_T versus κ for variant values of Le

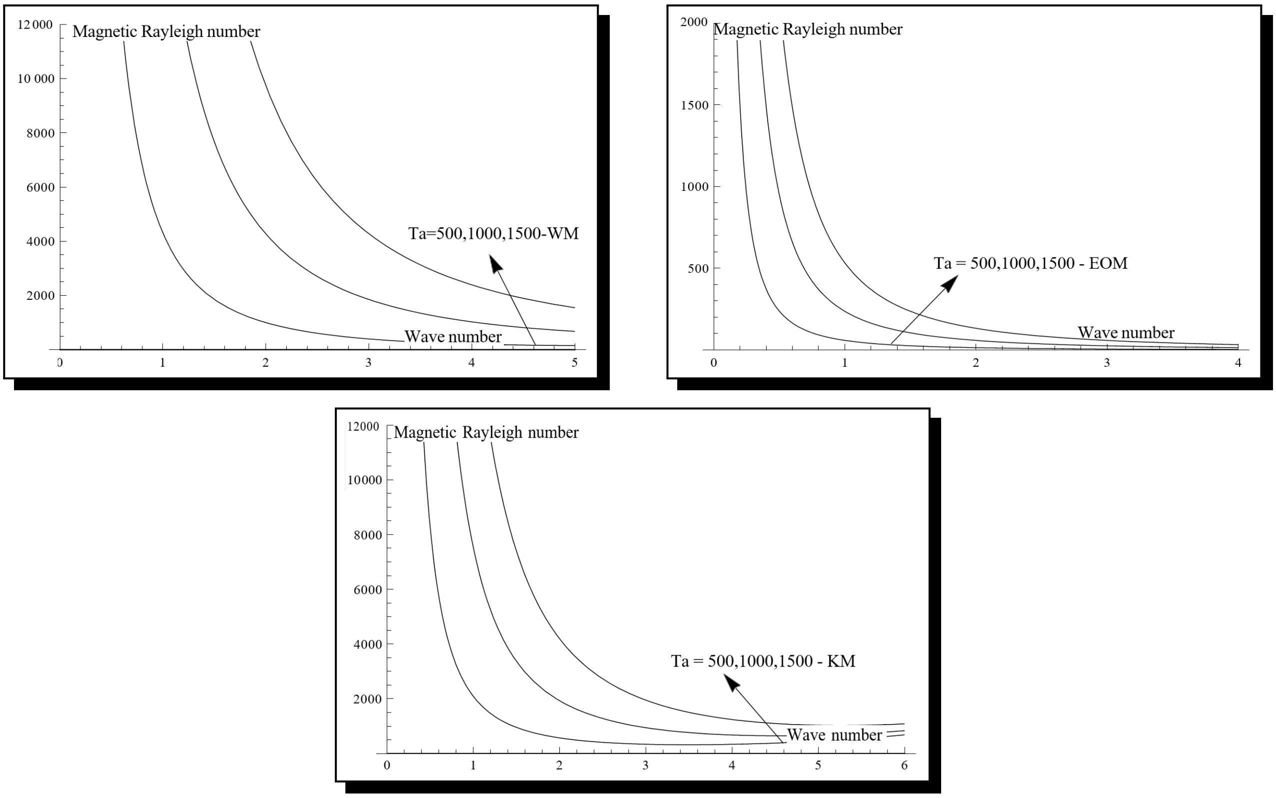

Figure 4. R_T versus κ for variant values of Le

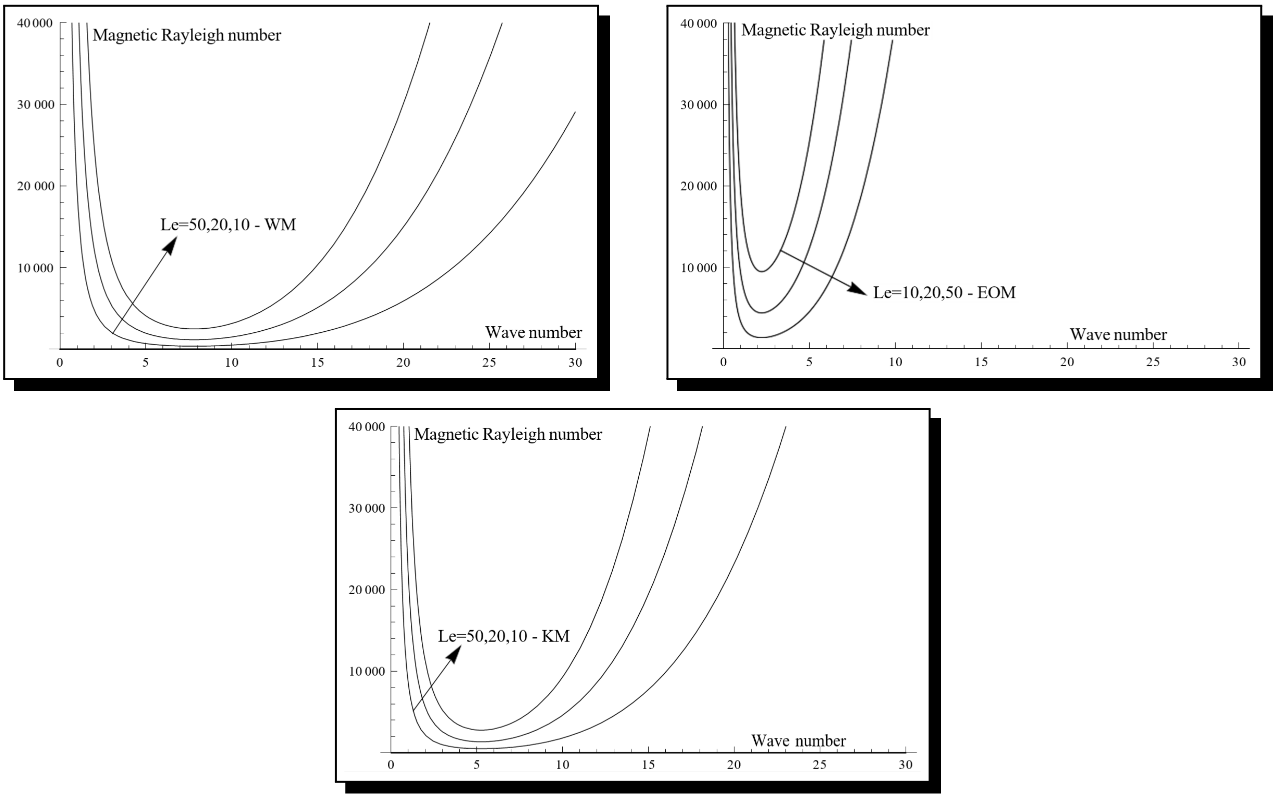

Figure 5. R_T versus κ for variant values of Le

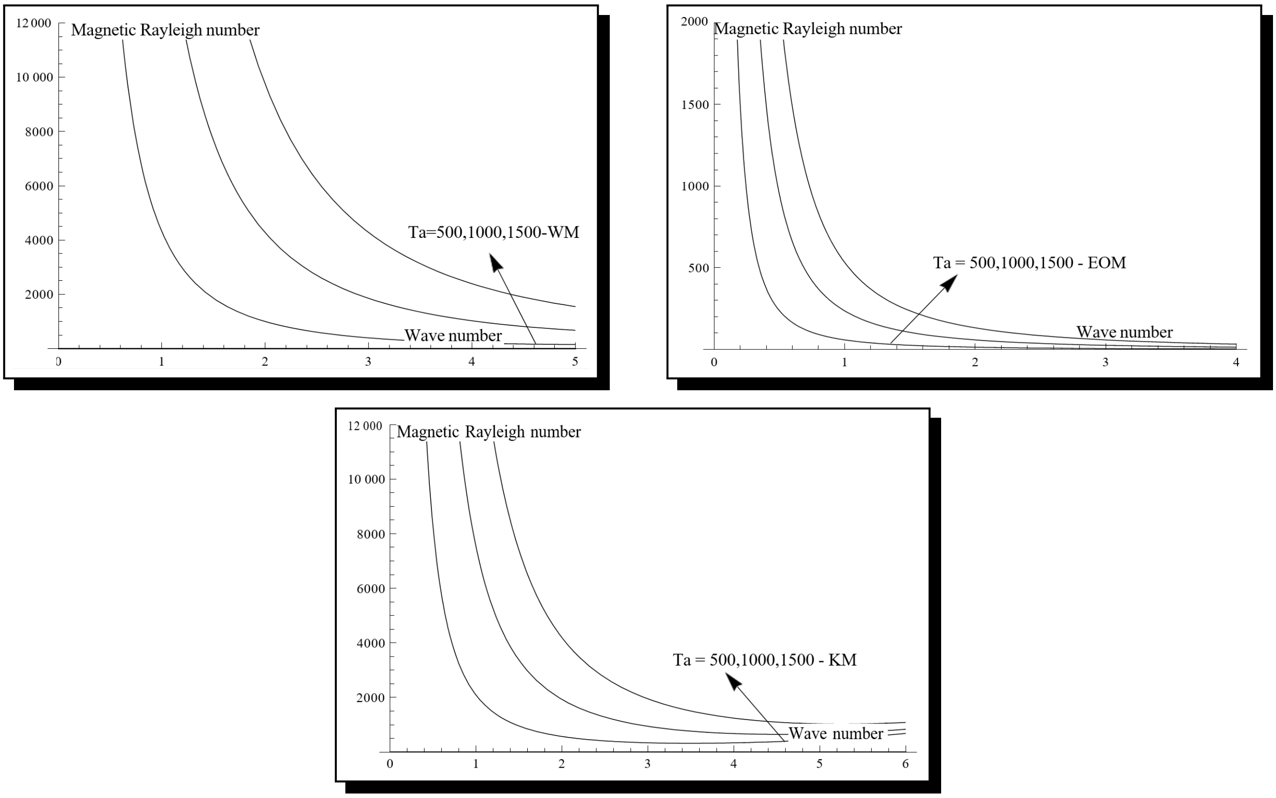

Figure 6. R_T versus κ for variant values of Le

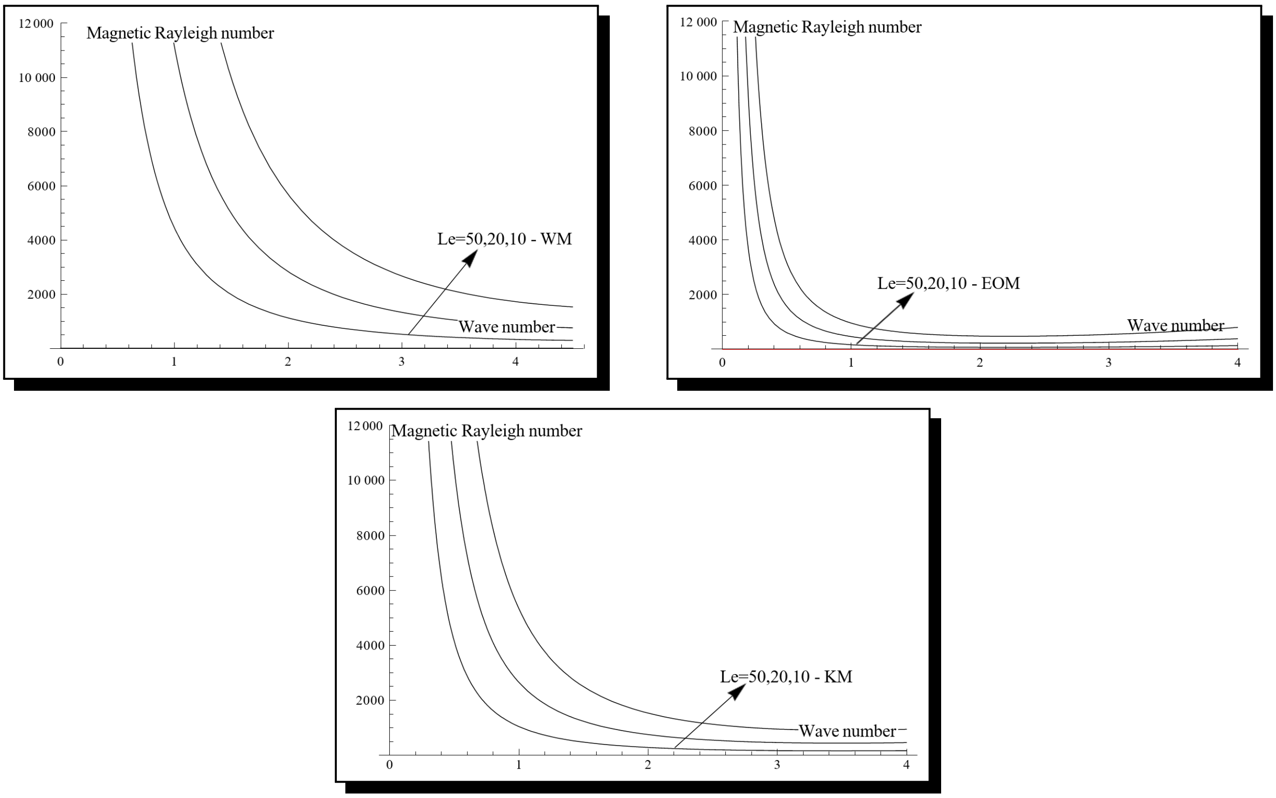

Figure 7. R_T versus κ for variant values of Le

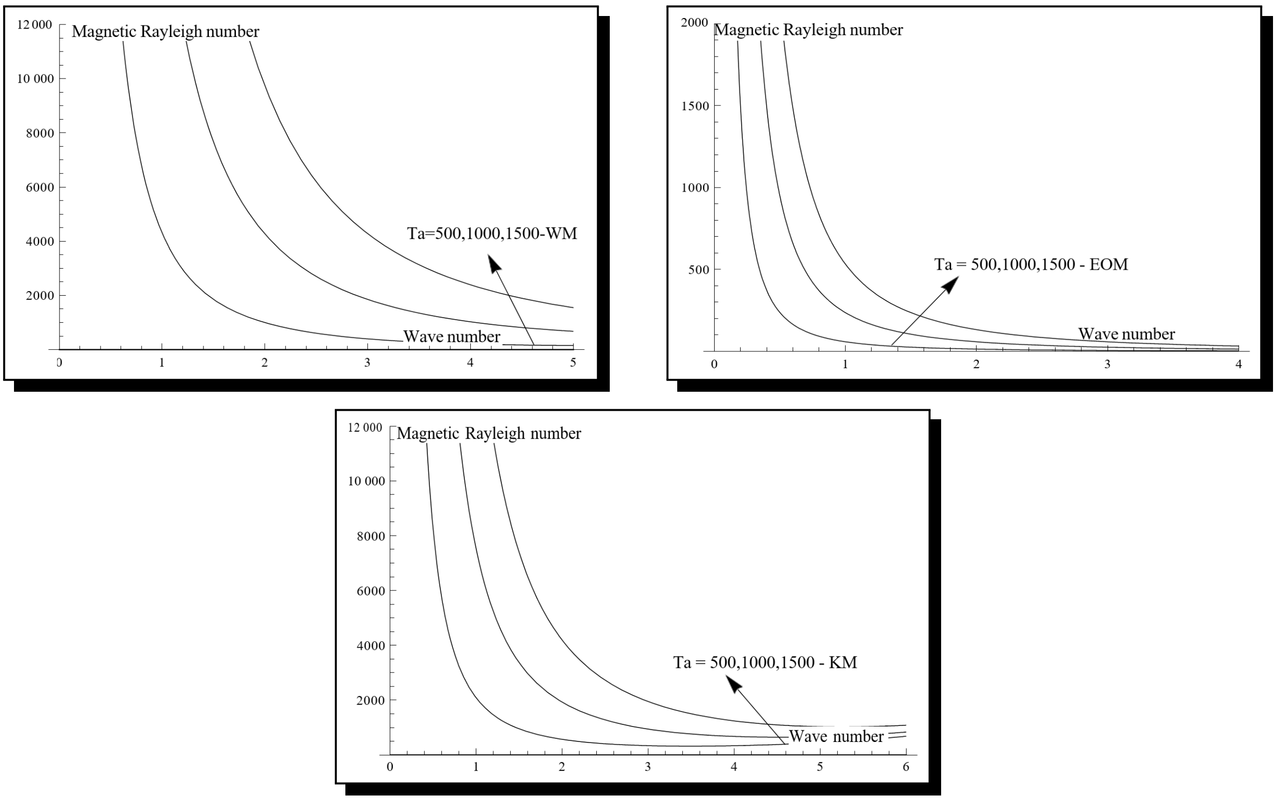

Figure 8. R_T versus κ for variant values of Le

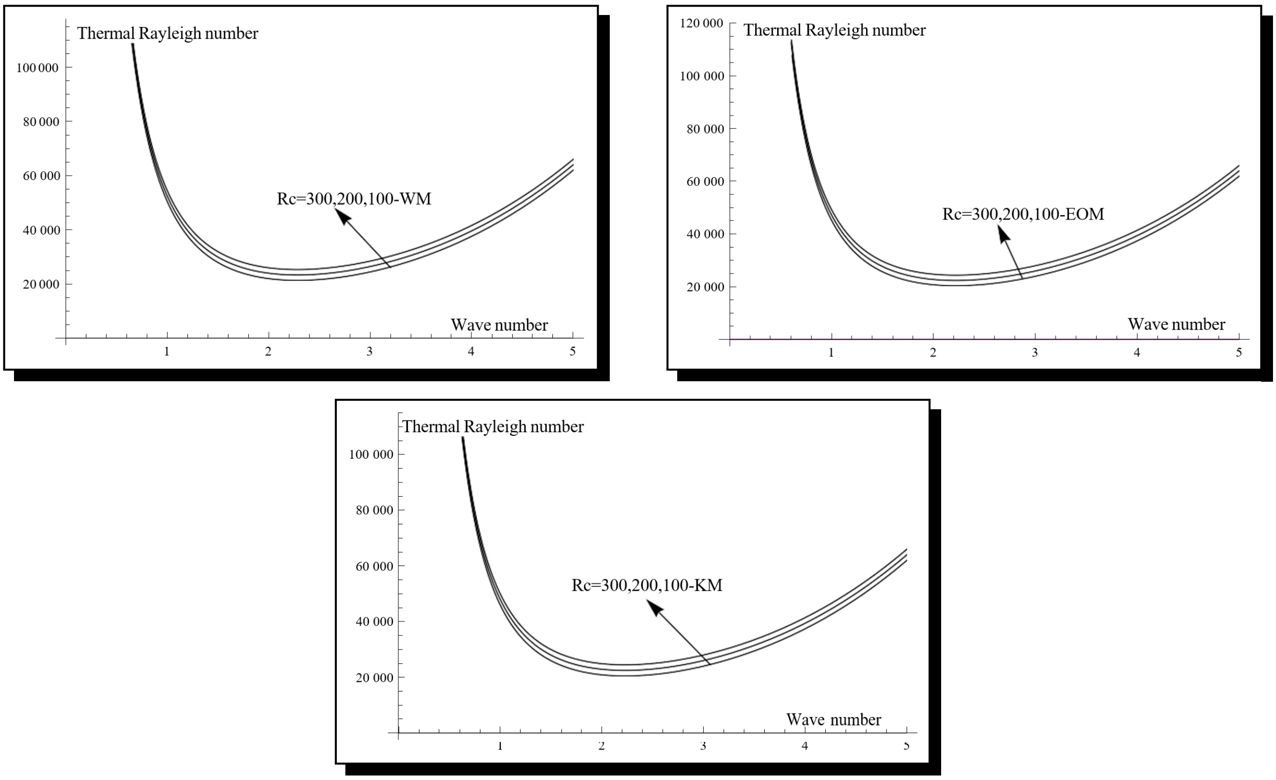

Figure 9. R_T versus κ for variant values of Le

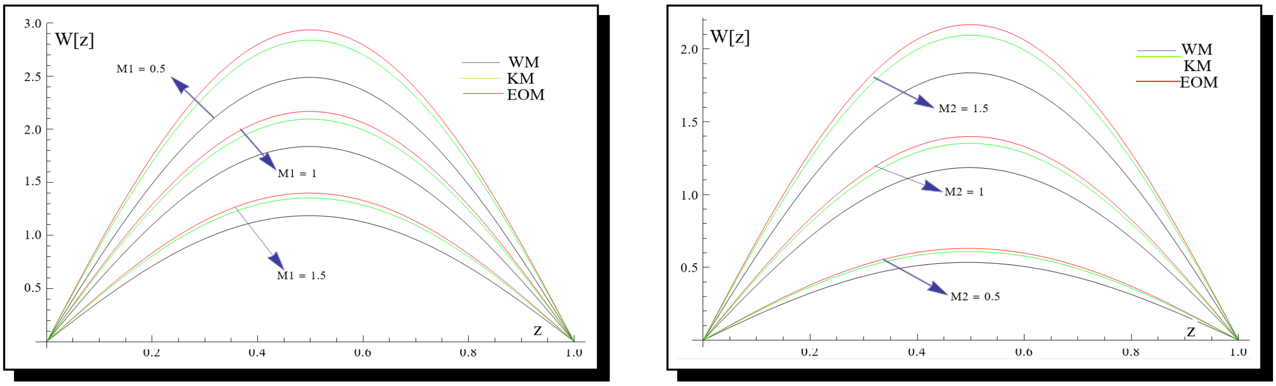

Figure 10. R_T versus κ for variant values of Le

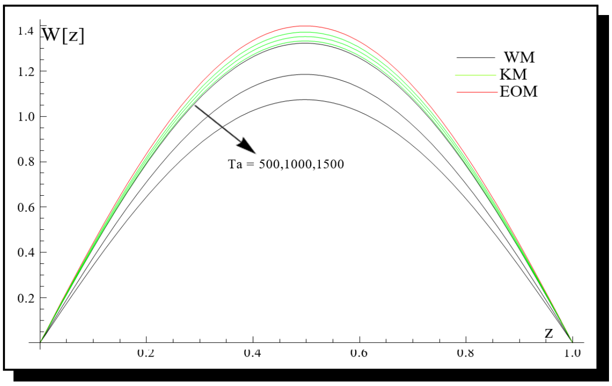

Figure 11. R_T versus κ for variant values of Le

Table 1. Thermophysical properties of basefluid and nanoparticle [2]

Quantity	Water	Engine oil	Kerosene	Fe_3O_4
Density, ρ (kg/m^3)	997.1	888.23	780	5180
Thermal expansion coefficient, $\beta(10^{-5}\ K^{-1})$	21	70	99	20.6
Specific heat, C_p (J/kg.K)	4179	1880.3	2090	670
Thermal conductivity, k (W/m.K)	0.613	0.145	0.149	80.4
Dynamic viscosity, μ (kg/m.s)	0.001003	0.845	0.0016	—

Table 2. Thermophysical properties of nanoliquid for $\psi = 0.1$

Quantity	Water-Fe_3O_4	Engine Oil-Fe_3O_4	Kerosene-Fe_3O_4
ρ	1415.39	1317.41	1220
k_{nf}	0.761142	0.189648	0.194781
μ_{nf}	0.00130525	1.09964	0.00208216
$(Cp)_{nf}$	2894.79	1404.41	1487.08
$(\rho Cp)_{nf} * 10^6$	4.09725	1.85018	1.81424
$(\rho\beta)_{nf}$	29516	66629.3	80168.8
β_{nf}	20.8536	50.5761	65.7121
$\alpha_{nf} * 10^{-7}$	1.85769	1.02502	1.07362
P_{nf}	4.96416	8143.26	15.8966

Table 3. Thermophysical properties of nanoliquid for $\psi = 0.2$

Quantity	Water-Fe_3O_4	Engine Oil-Fe_3O_4	Kerosene-Fe_3O_4
ρ_{nf}	1833.68	1746.58	1660
k_{nf}	0.935242	0.244509	0.251009
μ_{nf}	0.00175217	1.47615	0.00279508
$(Cp)_{nf}$	2196.47	1162.4	1203.78
$(\rho Cp)_{nf} * 10^6$	4.02762	2.03023	1.99828
$(\rho\beta)_{nf}$	8092.93	71082.5	83117.6
β_{nf}	20.774	40.698	50.0708
$\alpha_{nf} * 10^{-7}$	2.32207	1.20434	1.25612
P_{nf}	4.11507	7017.68	13.4046

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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