Communications in Mathematics and Applications
Vol. 5, No. 2, pp. 53–58, 2014
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
http://www.rgnpublications.com

Cycle with Parallel Chords of Pendant Edge Extension is Graceful

Research Article

A. Elumalai
Department of Mathematics, Valliammai Engineering College, SRM Nagar, Kattankulathur 603203, India
aelu2000@yahoo.com

Abstract. In this paper I have proved that cycle with parallel chords of pendant edge extension is graceful.

Keywords. Graph labeling; Graceful labeling; Cycle with parallel chords, and pendant edge extension of cycle with parallel chords.

MSC. 05C78

Received: March 22, 2013 **Accepted:** June 24, 2014

1. Introduction

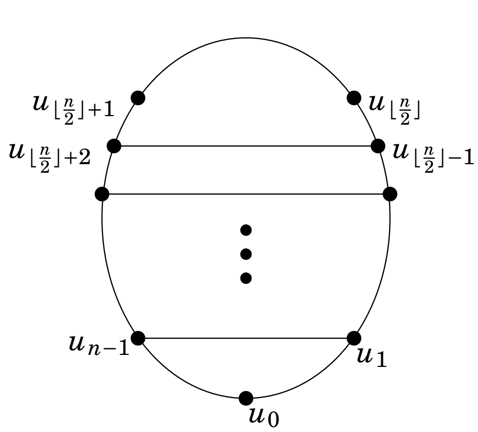
Much interest in graph labelings began in mid 1960's and a paper by Rosa (1967). The popular Ringel-Kotzig [4] conjecture that "all trees are graceful" remains unsettled. Rosa [7] introduced graceful labeling (β-valuation) as a tool to decompose complete graph K_{2n+1} into copies of a given tree on n-edges.

Characterization of graceful and other labeled graphs appears to be one of the most difficult and hard problems in graph theory. Major area of research on graph labeling is devoted to different constructoin of labeled graphs. However, the topic of graph labelings has been the subject of research for a long time in the applied fields also. It is intersting to note that the labeled graphs serve as useful models for a broad range of applications such as coding theory, X-ray crystallography, radar, astronomy, circuit design and communication network address.

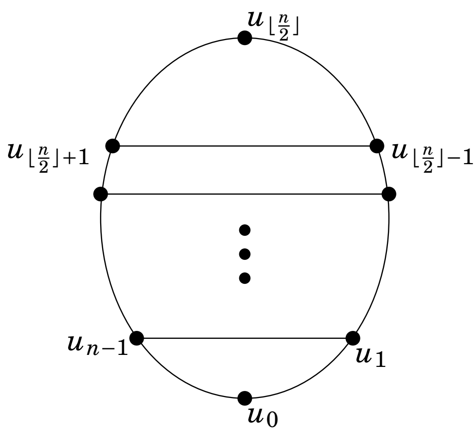
A *labeling* (or valuation) of a graph G is an assignment f of labels from a set of positive integers to the vertices of G that induce a label for each edge xy defined by the labels $f(x)$ and $f(y)$.

A function f is called a *graceful labeling* of a graph G with m edges if f is an injection from the vertex set of G to the set $\{0,1,2,\ldots,m\}$ such that, when each edge xy is assigned the label $|f(x)-f(y)|$, the resulting edge labels are distinct. For an excellent survey on graph labeling see [3].

A graph G is called *cycle with parallel chords* if G is obtained from a cycle $C_n: u_0u_1, u_1u_2, \ldots, u_{n-1}u_0$ $(n \geq 6)$ by adding the chords $u_1u_{n-1}, u_2u_{n-2}, \ldots, u_\alpha u_\beta$, where $\alpha = \left\lfloor \frac{n}{2} \right\rfloor - 1$ and $\beta = \left\lfloor \frac{n}{2} \right\rfloor + 2$, if n is odd or $\beta = \left\lfloor \frac{n}{2} \right\rfloor + 1$, if n is even.

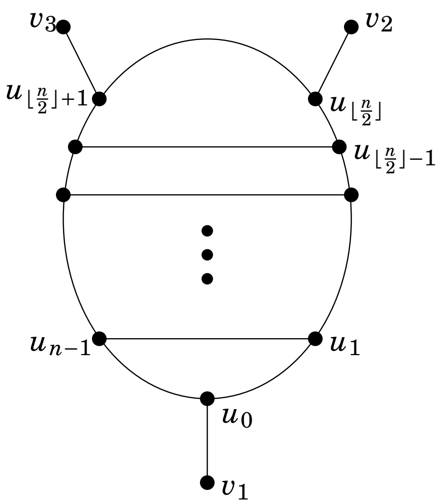


Cycle with parallel chords (with n odd)

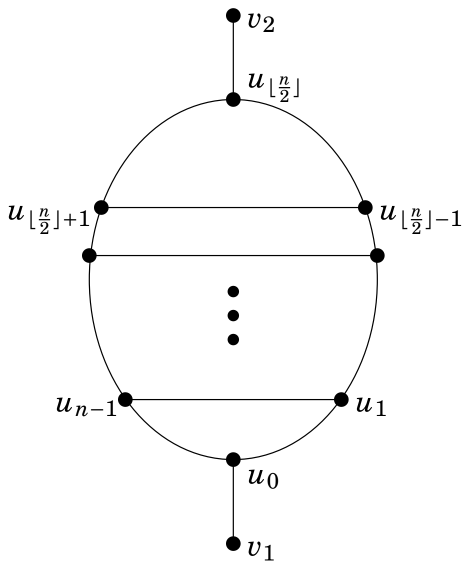


Cycle with parallel chords (with n even)

A graph G is obtained from the cycle C_n with parallel chords by adding a pendant edge at each vertex on the non-adjacent chords of the cycle is called *pendant edge extension of cycle with parallel chords*.



Pendant edge extension of cycle with parallel chords (with n odd)



Pendant edge extension of cycle with parallel chords (with n even)

Study on gracefulness of cycle related graphs initiated from Rosa's result that cycle C_n is graceful iff $n \equiv 0$ or 3(mod 4). A chord of a cycle is an edge joining two non-adjacent vertices of the cycle. Bolendiek Schumacher and Wegner conjectured in [1] that every cycle with a chord is graceful. The validity of this conjecture has been proved by Delorme, Maheo and others in [2]. A natural extension of the structure of a cycle with a chord is cycle with a P_k-chord. Koh and Yap [5] have shown that cycles with P_3-chords are graceful and conjectured that all cycles with P_k-chords are graceful. This was proved for $k \geq 4$ by Punnim and Pabhapote [6].

In this paper I have proved that cycle with parallel chords of pendant edge extension is graceful.

2. Every n-cycle with Parallel Chords of Pendant Edge Extension is Graceful

In this section I have proved that every n-cycle ($n \geq 6$) with parallel chords of pendant edge extension is graceful.

Theorem 1. *Every n-cycle ($n \geq 6$) with parallel chords of pendant edge extension is graceful.*

Proof. Let G be an n-cycle with parallel chords of pendant edge extension of graph and let $u_0, u_1, u_2, \ldots, u_{n-1}$ be the vertices of an n-cycle and v_1, v_2 and v_3 be the vertices of pendant edges. Observe that G has $n+\rho$ vertices, where $\rho = 3$, if n is odd or $\rho = 2$ if n is even and $M = \frac{3n+\rho}{2}$ edges, where $\rho = 3$, if n is odd or $\rho = 2$ if n is even.

We give labels to the vertices of G in the following four cases:

Case I: When n is odd (i.e., $n = 4r+1$, $r \geq 1$).

Define $f(u_0) = 0$

$f(u_{2i-1}) = 3i-2$, for $1 \leq i \leq \delta$, $\delta = \frac{n-1}{4}$

$f(u_{2i}) = M-3i+2$, for $1 \leq i \leq \delta$, $\delta = \frac{n-1}{4}$

$f(u_{n-2i}) = 3i$, for $1 \leq i \leq \delta$, $\delta = \frac{n-1}{4}$

$f(u_{n-(2i-1)}) = M-3(i-1)$, for $1 \leq i \leq \delta$, $\delta = \frac{n-1}{4}$

and $f(v_1) = 2$, $f(v_2) = 3i+2$, $f(v_3) = 3i+4$, for $1 \leq i \leq r$.

Case II: When n is odd (i.e., $n = 4r+3$, $r \geq 1$).

Define $f(u_0) = 0$

$f(u_{2i-1}) = 3i-2$, for $1 \leq i \leq \delta$, $\delta = \frac{n+1}{4}$

$f(u_{2i}) = M-3i+2$, for $1 \leq i \leq \delta$, $\delta = \frac{n-3}{4}$

$f(u_{n-2i}) = 3i$, for $1 \leq i \leq \delta$, $\delta = \frac{n-3}{4}$

$f(u_{n-(2i-1)}) = M-3(i-1)$, for $1 \leq i \leq \delta$, $\delta = \frac{n+1}{4}$

and $f(v_1) = 2$, $f(v_2) = 3i+4$, $f(v_3) = 3i+2$, for $1 \leq i \leq r$.

Case III: When n is even (i.e., $n = 4r$, $r \geq 1$).

Define $f(u_0) = 0$

$f(u_{2i-1}) = 3i - 2$, for $1 \le i \le \delta$, $\delta = \frac{n}{4}$

$f(u_{2i}) = M - 3i + 2$, for $1 \le i \le \delta$, $\delta = \frac{n-4}{4}$

$f(u_{n-2i}) = 3i$, for $1 \le i \le \delta$, $\delta = \frac{n}{4}$

$f(u_{n-(2i-1)}) = M - 3(i-1)$, for $1 \le i \le \delta$, $\delta = \frac{n}{4}$

and $f(v_1) = 5$, $f(v_2) = 3i + 3$, for $1 \le i \le r$.

Case IV: When n is even (i.e., $n = 4r + 2$, $r \ge 1$).

Define $f(u_0) = 0$

$f(u_{2i-1}) = 3i - 2$, for $1 \le i \le \delta$, $\delta = \frac{n-2}{4}$

$f(u_{2i}) = M - 3i + 2$, for $1 \le i \le \delta$, $\delta = \frac{n-2}{4}$

$f(u_{n-2i}) = 3i$, for $1 \le i \le \delta$, $\delta = \frac{n-2}{4}$

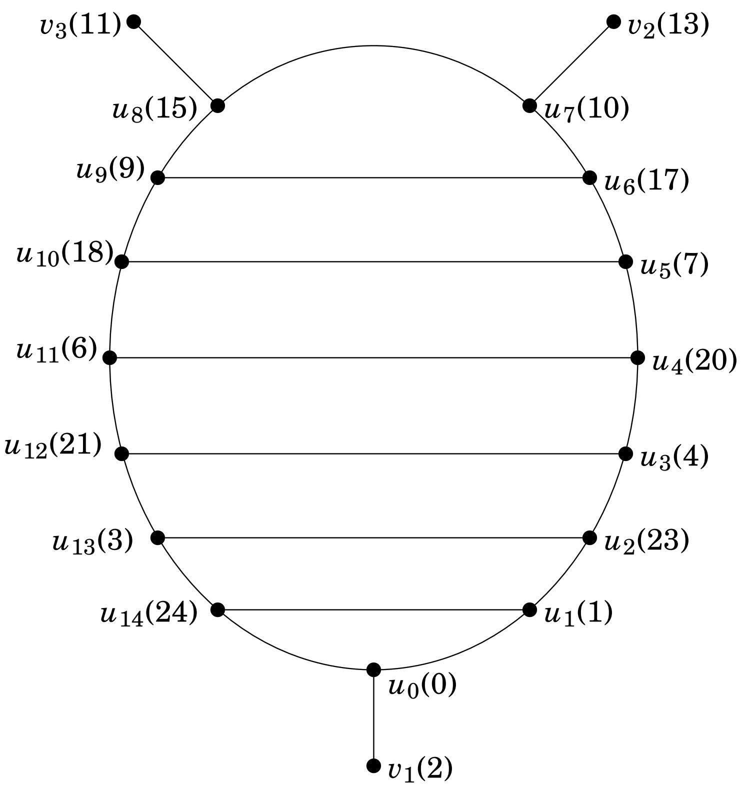
$f(u_{n-(2i-1)}) = M - 3(i-1)$, for $1 \le i \le \delta$, $\delta = \frac{n+2}{4}$

and $f(v_1) = 5$, $f(v_2) = 3i + 1$, for $1 \le i \le r$.

It is clear that f is injective and the edge values are distinct and range from 1 to M. Thus f is graceful labeling. Hence the graph G is graceful. □

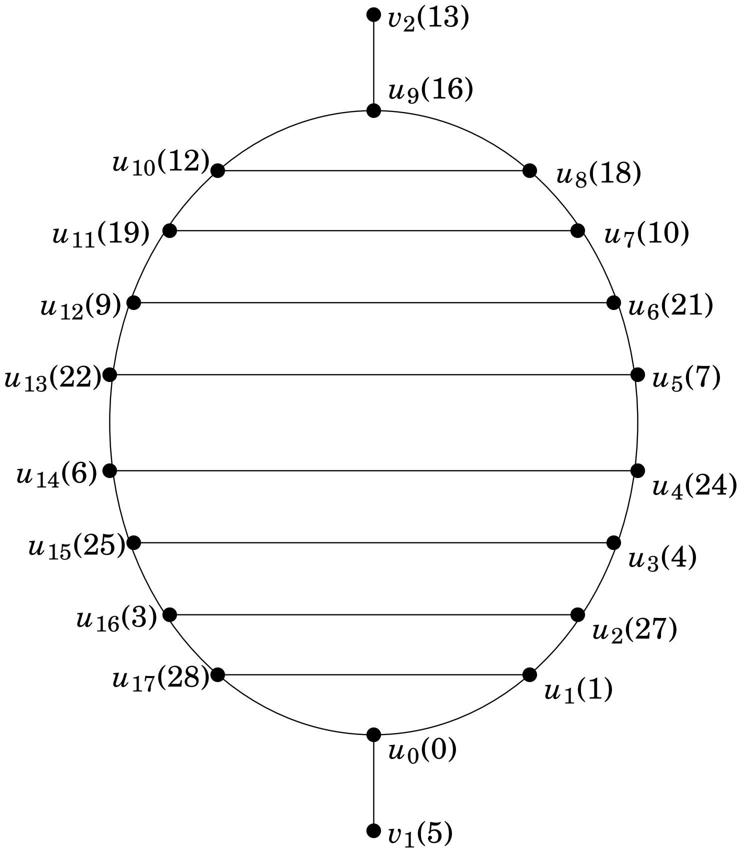
3. Illustrative Examples

Example 1.



Graceful labeled C_{15} with parallel chords of pendant edge extension

Example 2.



Graceful labeled C_{18} with parallel chords of pendant edge extension

4. Conclusion

In Theorem 1, I have shown that the cycle C_n with parallel chords of pendant edge extension of non-adjacent chords is graceful. It appears that the cycle C_n with parallel chords of pendant edge extension may not be graceful for all $n \geq 4$. However, I strongly feel that the cycle C_n with parallel chords of pendant edge extension of non-adjacent chords is graceful. Is it true for every cycle with parallel chords and every cycle with parallel P_k-chords of pendant edge extension is graceful?

Acknowledgement

The author would like to thank the referees for their valuable comments and suggestions.

References

[1] R. Bodendiek, H. Schumacher, H. Wegner, Uber eine spezielle Klasse grozioser Eulerscher Graphen, *Mitt. Math. Gesellsch, Hamburg* **10** (1975), 241–248.

[2] C. Delorme, M. Maheo, H. Thrillier, K. M. Koh, H. K. Teo, Cycles with a chord are graceful, *J. Graph Theory* **4** (1980), 409–415.

[3] J. A. Gallian, A dynamic survey of graph labeling, *Electronic J. Combin.* (Nov. 2011), #DS6.

[4] C. Huang, A. Kotzig, A. Rosa, Further results on tree labellings, *Util. Math.* **210** (1982), 31–48.

[5] K. M. Koh, K. Y. Yap, Graceful numberings of cycles with a P_3-chord, *Bull. Inst. Math. Acad. Sinica* **12** (1985), 41–48.

[6] N. Punnium, N. Pabhapote, On graceful graphs: cycles with a P_k-chord, $k \geq 4$, *Ars Combin.* **23A** (1987), 225–228.

[7] A. Rosa, On certain valuations of the vertices of a graph, *Theory of Graphs* (*Internat. Symposium*, Rome, July 1966), Gordon and Breach, N.Y.— Dunod Paris (1967), 349–355.