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Research Article

On Fixed Point of Difference Polynomials with Meromorphic Function of Finite Order

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Abstract. In this paper, we investigate $f(z)$ to be a transcendental meromorphic function of finite order $\sigma(f)$ and $c \in \mathbb{C}$ be complex constants. The authors establish an fixed points about the difference polynomials $\Phi(z) = \Delta f(z) - a(f(z))^n$, where $\Delta f(z) = f(z+c) - f(z)$. These results extend the related results obtained by Wu and Wu (Fixed points of differences of meromorphic functions, *Advances in Difference Equations* **2019** (2019), Article number: 453).

Keywords. Nevanlinna theory, Fixed point, Difference polynomials, Meromorphic function

Mathematics Subject Classification (2020). 30D35

1. Introduction

In this paper, to study the value distribution of difference polynomials of meromorphic functions by using the Nevanlinna theory. Therefore, we use the basic notations of the Nevanlinna theory and assume that the reader knows these notation (see Hayman [17], Yang and Yi [23], and Zheng [26]). Let $f(z)$ be a non-constant meromorphic function defined in the complex plane $\mathbb{C}$. We will let $\sigma(f)$ and $\mu(f)$ denote the order of $f(z)$, respectively. We use $S(r,f)$ to denote any quantity of $S(r,f) = O\{T(r,f)\}$ $(r \to \infty)$, possibly outside a set E with finite logarithmic measure.

Let $f(z)$ be a function meromorphic in the complex plane $\mathbb{C}$. The order of $f(z)$ is denoted by $\sigma(f)$. For any $a \in \mathbb{C}$, We use the notations $\sigma(f)$ to denote the order of $f(z)$, $\lambda(f,a)$, and $\lambda(1/f)$, respectively, to denote the exponent of convergence of zeros of $f(z)-a$ and poles of $f(z)$. Especially, if $a=0$, we denote $\lambda(f,0)=\lambda(f)$. A point $z \in \mathbb{C}$ is called as a fixed point of $f(z)$ if $f(z)=z$. There is a considerable number of results on the fixed points for meromorphic functions in the plane, we refer the reader to Chuang and Yang [11]. It follows Chen and Shon [8], we use the notation $\tau(f)$ to denote the exponent of convergence of fixed points of f that is defined as

$$\tau(f) = 1 - \limsup_{r\to\infty} \frac{\log N\left(r, \frac{1}{f-z}\right)}{\log r}.$$

In 1993, Lahiri [18] proved the following theorem:

Theorem 1.1. *Let f be a transcendental meromorphic function in the plane. Suppose that there exists $a \in \mathbb{C}$ with $\delta(a,f)>0$ and $\delta(\infty,f)=1$. Then f has infinitely many fixed points.*

In 2004, Yang and Yi [23] have proved the following theorem:

Theorem 1.2. *Let f be a transcendental meromorphic function in $\mathbb{C}$ with a positive order. If f has two distinct Borel exceptional values, say a_1 and a_2, then the order of f is a positive integer or ∞ and $\sigma(f)=\mu(f)$, $\delta(a_1,f)=\delta(a_2,f)=1$.*

When the order of f is less than 1, Chen and Shon [8] have proved the following:

Theorem 1.3. *Let f be a transcendental meromorphic function of order of growth $\sigma(f)\le 1$. Suppose that f satisfies $\lambda(1/f)<\lambda(f)<1$ or has infinitely many zeros (with $\lambda(f)=0$) and finitely many poles. Then Δf has infinitely many fixed points and satisfies the exponent of convergence of fixed points $\tau(\Delta f)=\sigma(f)$.*

In 2014, Wu and Xu [21] have proved the following:

Theorem 1.4. *Let f be a transcendental meromorphic function of order of growth $\sigma(f)<1$ and $a \in \mathbb{C}$. Suppose that f satisfies $\lambda(1/f)<\sigma(f)$ and $\lambda(f,a)<\sigma(f)$. Then Δf has infinitely many fixed points and satisfies the exponent of convergence of fixed points $\tau(\Delta f)=\sigma(f)$.*

For the existence on the fixed points of differences, Cui and Yang [12] have proved the following theorems:

Theorem 1.5. *Let f be a function transcendental and meromorphic in the plane with the order $\sigma(f)=1$. If f has finitely many poles and infinitely many zeros with exponent of convergence of zeros $\lambda(f)\neq 1$, then Δf has infinitely many zeros and fixed points.*

Theorem 1.6. *Let f be a function transcendental and meromorphic in the plane with the order $\sigma(f)=1$, $\max\left\{\lambda(f),\lambda\frac{1}{f}\right\}=1$. If f has infinitely many zeros, then Δf has infinitely many zeros and fixed points.*

The conditions of Theorems 1.5 and 1.6 imply that 0, ∞ are Borel exceptional values. If ∞ and $d \in \mathbb{C}$ are Borel exceptional values of f, Chen [5] obtains the following theorem:

Theorem 1.7. *Let f be a finite order meromorphic function such that $\lambda\left(\frac{1}{f}\right) < \sigma(f)$, and let $c \in \mathbb{C}\{0\}$ be a constant such that $f(z+c) \not\equiv f(z)$. If $f(z)$ has a Borel exceptional value $d \in \mathbb{C}$, then $\tau(\Delta_c f) = \sigma(f)$.*

In 2016, Zhang and Chen [24] we have obtained the following result:

Theorem 1.8. *Let f be a finite order meromorphic function, and let $c \in \mathbb{C}\{0\}$ be a constant such that $f(z+c) \not\equiv f(z)$. If $f(z)$ has two Borel exceptional values, then $\tau(\Delta_c f) = \sigma(f)$.*

In 2019, Wu and Wu [22] if the order of f is not a positive integer, we have obtained the following result:

Theorem 1.9. *Let f be a transcendental meromorphic function of finite order in the plane. Suppose that $c \in \mathbb{C}\setminus\{0\}$ such that $\Delta_c f \not\equiv 0$. If there is $a \in \mathbb{C}$ with $\delta(a,f) > 0$ and $\delta(\infty,f) = 1$, then $\Delta_c f$ have infinitely many fixed points and $\tau(\Delta_c f) = \sigma(f)$.*

Theorem 1.10. *Let f be a transcendental meromorphic function of finite order in the plane. Suppose that $c \in \mathbb{C}\setminus\{0\}$ such that $\Delta_c f \not\equiv 0$. If $\delta(\infty,f) = 1$, $\delta(0,f) = 1$, then*

$$T(r,\Delta_c f) \sim T(r,f) \sim N\left(r, \frac{1}{\Delta_c f - z}\right),$$

as $r \to \infty$, $r \notin E$, where E is a possible exception set of r with finite logarithmic measure.

For c-shift difference polynomial of meromorphic functions and its certain properties, we refer to the work of Ahamed [1], Banerjee and Ahamed [4], and Mallick and Ahamed [19]. For recent developments in difference polynomials and different aspects of it, we refer to the work of Ahamed [2], Banerjee and Ahamed [3], Chen [16], and Naveenkumar *et al*. [20].

Let $f(z)$ be a meromorphic function in the complex plane $\mathbb{C}$ and $a \in \mathbb{C}_\infty = \mathbb{C} \cup \{\infty\}$. Nevanlinna's deficiency of f with respect to a is defined by

$$\delta(a,f) = 1 - \limsup_{r\to\infty} \frac{N\left(r, \frac{1}{f-a}\right)}{T(r,f)}.$$

If $a = \infty$, then one should replace $N\left(r, \frac{1}{f-a}\right)$ in the above formula by $N(r,f)$. If $\delta(a,f) > 0$, then a is called a Nevanlinna deficiency value of f.

Let $f(z)$ be a transcendental meromorphic function and $a \in \mathbb{C}\setminus\{0\}$, and $c \in \mathbb{C}\setminus\{0\}$ be complex constants satisfying. We have investigate the fixed point of mermorphic function with difference polynomials

$$\Phi(z) = \Delta f(z) - a(f(z))^n, \tag{1}$$

where $\Delta f(z) = f(z+c) - f(z)$.

In this article we generalize Theorems 1.9 and 1.10 to the case of difference polynomials defined above.

Theorem 1.11. *Let $f(z)$ be a transcendental meromorphic function of finite order $\sigma(f)$ in the complex plan. Let $c \in \mathbb{C}$ be complex constants satisfying such that $\Phi(z) \not\equiv 0$. If there is $a \in \mathbb{C}$ with f is satisfies $\delta(\infty, f) = 1$, and a is a Nevanlinna deficiency value of f, then $\Phi(z)$ have infinitely many fixed points and $\tau(\Phi(z)) = \sigma(f)$.*

Theorem 1.12. *Let $f(z)$ be a transcendental meromorphic function of finite order $\sigma(f)$ in the complex plan. $c \in \mathbb{C}$ be complex constants satisfying such that $\Phi(z) \not\equiv 0$. If there is $a \in \mathbb{C}$ with f is satisfies $\delta(\infty, f) = 1$ and $\delta(0, f) = 1$, then*

$$T(r, \Phi(z)) \sim (n+1)T(r, f) \sim N\left(r, \frac{1}{\Phi_n(z) - z}\right),$$

as $r \to \infty$, $r \notin E$, where E is a possible exception set of r with finite logarithmic measure.

Remark 1.1. If $a = 0$, then $\Phi(z)$ becomes the forward difference $\Delta_c f$, i.e.,

$$\Phi = f(z+c) - f(z) = \Delta_c f(z).$$

Therefore, we can get Theorems 1.9 and 1.10.

Example 1.1. Let $f(z) = e^z$ and $c = \log_{10} 1$ and $a = -1$. Then, for $\Phi(z) = \Delta f(z) - (f(z))^n = e^{nz}$. Obviously, we can get $\delta(0, f) = \delta(\infty, f) = 1$ and $\Phi(z)$ have infinitely many fixed points and $\tau(\Phi(z)) = \sigma(f)$.

$$T(r, \Phi(z)) \sim (n+1)T(r, f) \sim N\left(r, \frac{1}{\Phi(z) - z}\right).$$

As $r \to \infty$, and crucially, $\Phi(z) = e^{nz} \neq 0$. Hence, the requirement $z \neq 0$ in Theorems 1.11 and 1.12 is indispensable.

2. Some Lemmas

In this section, we present several lemmas that will be necessary for the upcoming discussions.

Lemma 2.1 ([14]). *Let $f(z)$ be a transcendental meromorphic function of finite order, then*

$$m\left(r, \frac{f(z+c)}{f}\right) = S(r, f).$$

Lemma 2.2 ([9]). *Let $f(z)$ be a finite order meromorphic function, then, for each $k \in \mathbb{N}$, $\sigma(\Delta_c^k f) \le \sigma(f)$.*

Lemma 2.3 ([15]). *Let f be a transcendental meromorphic function of finite order. Then for any positive integer n, we have*

$$m\left(r, \frac{\Delta_c^n f(z)}{f(z)}\right) = S(r, f).$$

Lemma 2.4 ([10]). *Let f be a transcendental meromorphic function of finite order. Then*

$$N(r,f(z+c)) = N(r,f)+S(r,f),$$

$$T(r,f(z+c)) = T(r,f)+S(r,f),$$

where $S(r,f)=O(T(r,f))$ $(r\to\infty)$, possibly outside a set E of r with finite logarithmic measure.

Lemma 2.5 ([6]). *Suppose that $f(z)$ is a transcendental meromorphic function in the complex plane and $P(z)=a_0z^n+a_1z^{n-1}+\dots+a_n$, where $a_0(\not\equiv 0)$, $a_1,\dots,a_n$ are constants. Then*

$$T(r,P(f)) = nT(r,f)+S(r,f).$$

Lemma 2.6 ([13]). *Let $F(r)$ and $G(r)$ be monotone increasing function such that $F(r)\le G(r)$ outside of exceptional set E that is of finite logarithmic measure. Then, for any $\alpha>0$, there exists $r_0>1$ such that $F(r)\le G(\alpha r)$ for all $r>r_0$.*

Lemma 2.7. *Let $f(z)$ be a transcendental meromorphic function of finite order $\sigma(f)$ in the complex plan. Let $c\in\mathbb{C}$ be complex constants satisfying such that $\Phi(z)\not\equiv 0$, and $\delta(0,f)>0$. Then $\Phi(z)$ a transcendental and meromorphic function of finite order.*

Proof. Since $\delta(0,f)>0$, from Lemma 2.2, we know that $\sigma(\Phi(z))\le\sigma(f)<+\infty$. If $\Phi(z)$ is a transcendental meromorphic function. Suppose that $\Phi(z)$ is not a transcendental meromorphic function. Then, there is a rational $Q(z)$ such that $Q(z)\Phi(z)\equiv 1$, i.e.,

$$\begin{aligned}\frac{1}{f^{n+1}} &\equiv Q(z)\frac{\Phi(z)}{f^{n+1}}\\ &\equiv Q(z)\left(\left(\frac{f(z+c)}{f}\right)-\left(\frac{f(z)}{f}\right)-a\left(\frac{f(z)}{f}\right)^n\right).\end{aligned}$$

Apply Lemma 2.1 and note that $f(z)$ is transcendental, we can get

$$m\left(r,\frac{1}{f^{n+1}}\right)\le m(r,Q(z))+m\left(r,\frac{\Phi(z)}{f^{n+1}}\right)=S(r,f).$$

Therefore

$$\begin{aligned}m\left(r,\frac{1}{f^{n+1}}\right)+N\left(r,\frac{1}{f^{n+1}}\right)&\le N\left(r,\frac{1}{f^{n+1}}\right)+S(r,f)\\ &\le(n+1)N\left(r,\frac{1}{f}\right)+S(r,f).\end{aligned}$$

Apply Lemma 2.5 and the first fundamental theorem of Nevanlinna theory, we can get

$$(n+1)T(r,f)\le(n+1)N\left(r,\frac{1}{f}\right)+S(r,f).$$

This contradicts with $\delta(0,f)>0$. Thus $\Phi(z)$ is a transcendental and meromorphic function of finite order. □

Lemma 2.8. *Let $f(z)$ be a transcendental meromorphic function of finite order $\sigma(f)$ in the complex plan. Let $c\in\mathbb{C}$ be complex constants satisfying that such that $\Phi(z)\not\equiv 0$ and $\delta(0,f)>0$,* *then*

$$(n+1)T(r,f) \le (n+1)N\left(r,\frac{1}{f}\right) + 2(n+1)N(r,f) + N\left(r,\frac{1}{\Phi(z)-z}\right) + S(r,f).$$

Proof. By Lemma 2.7, we know that $\Phi(z)$ is a transcendental meromorphic function, then there is $\eta \in \mathbb{C}\{0\}$ such that $z\Delta_\eta\Phi(z)-\Phi(z) \not\equiv 0$.

Noticing

$$\frac{1}{f^{n+1}} = \frac{\Phi(z)}{zf^{n+1}} - \frac{z\Delta_\eta\Phi(z)-\Phi(z)}{zf^{n+1}}\frac{\Phi(z)-z}{z\Delta_\eta\Phi(z)-\Phi(z)}, \tag{2}$$

then

$$\begin{aligned} m\left(r,\frac{1}{f^{n+1}}\right) &\le m\left(r,\frac{\Phi(z)}{zf^{n+1}}\right) + m\left(r,\frac{z\Delta_\eta\Phi(z)-\Phi(z)}{zf^{n+1}}\right) + m\left(r,\frac{\Phi(z)-z}{z\Delta_\eta\Phi(z)-\Phi(z)}\right) + O(1) \\ &\le 2m\left(r,\frac{\Phi(z)}{f^{n+1}}\right) + m\left(r,\frac{\Delta_\eta\Phi(z)}{f^{n+1}}\right) + m\left(r,\frac{\Phi(z)-z}{z\Delta_\eta\Phi(z)-\Phi(z)}\right) + O(\log r). \end{aligned} \tag{3}$$

Applying the first fundamental theorem, we get

$$m\left(r,\frac{1}{f^{n+1}}\right) = (n+1)T(r,f) - (n+1)N\left(r,\frac{1}{f}\right) + O(1), \tag{4}$$

$$\begin{aligned} m\left(r,\frac{\Phi(z)-z}{z\Delta_\eta\Phi(z)-\Phi(z)}\right) &= m\left(r,\frac{z\Delta_\eta\Phi(z)-\Phi(z)}{\Phi(z)-z}\right) + N\left(r,\frac{z\Delta_\eta\Phi(z)-\Phi(z)}{\Phi(z)-z}\right) \\ [-2pt] &\quad - N\left(r,\frac{\Phi(z)-z}{z\Delta_\eta\Phi(z)-\Phi(z)}\right) + O(1) \\ &\le m\left(r,\frac{z\Delta_\eta\Phi(z)-\Phi(z)}{\Phi(z)-z}\right) + N\left(r,\frac{z\Delta_\eta\Phi(z)-\Phi(z)}{\Phi(z)-z}\right) + O(1). \end{aligned} \tag{5}$$

Combining (2)-(5), we have

$$\begin{aligned} (n+1)T(r,f) &\le (n+1)N\left(r,\frac{1}{f}\right) + 2m\left(r,\frac{\Phi(z)}{f^{n+1}}\right) + m\left(r,\frac{\Delta_\eta\Phi(z)}{f^{n+1}}\right) + m\left(r,\frac{z\Delta_\eta\Phi(z)-\Phi(z)}{\Phi(z)-z}\right) \\ &\quad + N\left(r,\frac{z\Delta_\eta\Phi(z)-\Phi(z)}{\Phi(z)-z}\right) + O(\log r) \\ &\le (n+1)N\left(r,\frac{1}{f}\right) + N\left(r,\frac{1}{\Phi(z)-z}\right) + N(r,z\Delta_\eta\Phi(z)-\Phi(z)) \\ &\quad + 2m\left(r,\frac{\Phi(z)}{f^{n+1}}\right) + m\left(r,\frac{\Delta_\eta\Phi(z)}{f^{n+1}}\right) + m\left(r,\frac{z\Delta_\eta\Phi(z)-\Phi(z)}{\Phi(z)-z}\right) + O(\log r). \end{aligned} \tag{6}$$

Since

$$\begin{aligned} \Delta_\eta\Phi(z) &= \Delta_\eta(f(z+c)-f(z)-a(f(z))^n \\ &= f(z+c+\eta)-f(z+\eta)-a((f(z+\eta))^n-(f(z))^n)-f(z+c)+f(z), \\ \Delta_\eta(\Phi(z)-z) &= f(z+c+\eta)-f(z+\eta)-a((f(z+\eta))^n-(f(z))^n)-f(z+c)+f(z)-(z+\eta), \end{aligned}$$

then, we can get

$$\begin{aligned} z\Delta_\eta\Phi(z)-\Phi(z) &= z(f(z+c+\eta)-f(z+\eta))-az((f(z+\eta))^n-2(f(z))^n) \\ &\quad + a(f(z))^n-(z+1)(f(z+c)-f(z)). \\ z\Delta_\eta(\Phi(z)-z)-(\Phi(z)-z) &= z(f(z+c+\eta)-f(z+\eta))-az((f(z+\eta))^n-2(f(z))^n) \\ &\quad + a(f(z))^n-(z+1)(f(z+c)-f(z))-z(z+\eta-1). \end{aligned}$$

Therefore,

$$\frac{z\Delta_\eta\Phi(z)-\Phi(z)}{\Phi(z)-z}=\frac{z\Delta_\eta(\Phi(z)-z)-(\Phi(z)-z)}{\Phi(z)-z}=\frac{z\Delta_\eta(\Phi(z)-z)}{\Phi(z)-z}-1, \tag{7}$$

$$\begin{aligned}N(r,z\Delta_\eta\Phi(z)-\Phi(z))\le{}& N(r,f(z+c+\eta)-f(z+\eta))+N(r,(f(z+\eta))^n-2(f(z))^n)\\ &+N(r,(f(z))^n)+N(r,f(z+c)-f(z)).\end{aligned} \tag{8}$$

Thus, from Lemma 2.4 and (8), we deduce

$$N(r,z\Delta_\eta\Phi(z)-\Phi(z))\le 2(n+1)N(r,f(z))+O(\log r). \tag{9}$$

By Lemmas 2.2 and 2.7 we know that $\Phi(z)-z$ is a transcendental meromorphic function of finite order. It follows from Lemma 2.3 and (7), we get

$$\begin{aligned}&m\left(r,\frac{\Phi(z)}{f^\psi}\right)=S(r,f),\\ &m\left(r,\frac{\Delta_\eta\Phi(z)}{f^\psi}\right)=S(r,f),\\ &m\left(r,\frac{z\Delta_\eta\Phi(z)-\Phi(z)}{\Phi(z)-z}\right)=S(r,f).\end{aligned} \tag{10}$$

From (6) and (9)-(10), we have

$$(n+1)T(r,f)\le nN\left(r,\frac{1}{f}\right)+2(n+1)N(r,f)+N\left(r,\frac{1}{\Phi(z)-z}\right)+S(r,f). \tag{11}$$

□

3. Proof of Theorems

Proof of Theorem 1.11. Denoting $g=f-a$ by (11) we derive,

$$\begin{aligned}(n+1)T(r,f)&\le(n+1)T(r,g)+O(1)\\ &\le(n+1)N\left(r,\frac{1}{g}\right)+2(n+1)N(r,g)+N\left(r,\frac{1}{\Phi(z)-z}\right)+S(r,g)\\ &=(n+1)N\left(r,\frac{1}{f-a}\right)+2(n+1)N(r,f)+N\left(r,\frac{1}{\Phi(z)-z}\right)+S(r,f).\end{aligned} \tag{12}$$

Since $\delta(0,f)>0$ and $\delta(\infty,f)=1$, there is a positive number $\theta<1$ such that

$$N\left(r,\frac{1}{f-a}\right)<\theta T(r,f), \tag{13}$$

$$N(r,f)=O(1)T(r,f). \tag{14}$$

If $\Phi(z)$ has only a finite number of fixed points, then from (12)-(14), we would have

$$(n+1)(1-O(1)-\theta)T(r,f)\le N\left(r,\frac{1}{\Phi(z)-z}\right),\quad r\notin E,\ r\to\infty, \tag{15}$$

where E is a possible exceptional set with finite logarithmic measure. Noticing f is transcendental, applying Lemma 2.6 and (15), we can get that $\Phi(z)$ assumes has infinitely many fixed points and $\tau(\Phi(z))=\sigma(f)$. □

Proof of Theorem 1.12. Since $\delta(0,f)=1$ and $\delta(\infty,f)=1$,

$$N\left(r,\frac{1}{f}\right)=S(r,f), \tag{16}$$

$$N(r,f)=S(r,f). \tag{17}$$

Since

$$T(r,\Phi(z))\le T(r,f(z+c)-f(z))+T(r,(f(z))^n). \tag{18}$$

Using Lemma 2.4, we can derive from (18) that

$$T(r,\Phi(z))\le(1+n)T(r,f)+S(r,f). \tag{19}$$

From (16)-(19), we have

$$\begin{aligned}(n+1)T(r,f)&\le N\left(r,\frac{1}{\Phi(z)-z}\right)+S(r,f)\\ &\le T(r,\Phi(z))+S(r,f)\\ &\le(n+1)T(r,f)+S(r,f).\end{aligned} \tag{20}$$

Since f is transcendental, (20) means that $\Phi(z)$ assumes has infinitely many fixed points and

$$T(r,\Phi(z))\sim(n+1)T(r,f)\sim N\left(r,\frac{1}{\Phi(z)-z}\right),$$

as $r\notin E$, $r\to\infty$, where E is a possible exception set of r with finite logarithmic measure. □

4. Conclusion

Theorems 1.11 and 1.12 seem to deal with transcendental meromorphic functions in the complex plane. Theorem 1.11 connects the behavior of $\Phi(z)$, a function constructed from $f(z)$, to the existence of in finitely many fixed points and the order of $f(z)$. Theorem 1.12 appears to establish an asymptotic relation involving the Nevanlinna characteristics of $f(z)$ and $\Phi(z)$ under certain conditions.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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