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Research Article

Solving Goal Programming by Alternative Simplex Method

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Abstract. It is found that the simplex algorithm is immensely used and proficient algorithm ever invented and shown extremely accurate in the formulation of optimization problems. In this paper, an alternative simplex method with some modifications has been used to solve Goal programming problem. This method is a new approach which solve goal programming problem easily and gives improved solution in comparatively less iterations.

Keywords. Goal programming problem, Optimal solution, Alternative simplex method

Mathematics Subject Classification (2020). 90C05

1. Introduction

It is impossible many times to fulfill the definite precised goals in given constraints for several queries in any organization. Then, these queries convert as one amongst maximizing degree of attainment of those goals. *Goal Programming* (GP) has an awareness which illuminate these queries of satisfying (probably differing) goals moreover seeing that feasible once a number of them have a top priority as compare to others.

Fundamentally if there is only single goal then linear programming technique is applicable, like maximizing the profit or minimizing the cost, and wherever the system might have more than single (probably differing) goals for example, there might have a collection of goals in an industry, such as stability of employment, excessive product quality, maximization of profit, minimizing overtime or price, and so on, and in today's dynamic business setting, most of the time organizations have multiple conflicting objectives to realize. Not solely do corporations explore for profit and revenue maximization or price diminution however produce other non-profit goals to cater to love social responsibilities, publicity, industrial and worker relations, etc. underneath such things, goal programming assumes utmost importance and is a strong quantitative technique capable of handling multiple call criteria. Thus, in these circumstances, different technology is desired which appears for a negotiative solution that carried on the relatively equal importance of every objective. So, one can conclude that Goal Programming is the renowned technique which helps to minimize the deviations from the goal assigned by the management. After developing the initial progress of the goal programming further in 1977, Charnes and Cooper [3] gave a survey of recent developments in goal programming and multiple objective optimizations. According to them to find out an accurate goal for every objective, the process is like to first formulate an objective function for each objective, afterward as per the basic approach of Goal programming, find a solution that minimizes the summation of deviations from their corresponding goals. Wise and Perushek [13] in 2000 presented goal programming as a solution technique. In 2005, Vieira *et al*. [7] gives an improved initial basis for the *Simplex algorithm*, which helps many researchers to know *Simplex method*. Meanwhile in 2009, Nabli [11] offered a new overview on *Simplex Algorithm*. Lokhande *et al*. [10] (2014) suggested an extremely new perspective toward *Modified Simplex Method for Optimum Solution of Linear Programming Problem*, and Khobragade *et al*. [8] recommended alternative technique to solve *Linear Programming Problem*. Again in 2014, Ghadle and Pawar [4] found a fantastic solution of *Game Theory Problem* by an *Alternative Simplex Method* that is quite advanced technique. In 2015, Narayanamoorthy and Kalyani [12] projected an *Dual Simplex Method* to solve transportation problems and a *Stratified Simplex Method* for solving fuzzy multi-objective linear programming problem is also shared by Liu and Shi [9] in 2015. Another approach for solving *Bi-Level Programming Problem* was found by Birla *et al*. [2] in 2017. Iwuji and Acha [6] suggested a *Mixed-Integer Lexicographic Goal Programming Model* in 2018. An *Easy Simplex* (*AHA Simplex*) algorithm was studied by Ansari [1] in 2019. Extensions of *Duality Results* and a *Dual Simplex Method for Linear Programming Problems* have been recommended by Goli and Nasseri [5] in 2020.

There are many inventions in *Linear Programming* (LP), but GP is still somewhere needs more attention, so it is important to focus on some new techniques while studying GP. In this article small attempt has been perform to find improved optimum solution for *Goal Programming Problem* (GPP). Hence, this paper tends to recommend new alternative approach of simplex method to solve Goal programming problem based on maximizing the profit and minimizing the cost and make the algorithm more efficient and done the analysis correctly.

2. Alternative Simplex Method for GPP

Suggested method included following stages to solve GPP:

Stage 1. Select highest value from the iteration table as a pivotal element (it may appear in any row or column) and solve:

(a) For unique highest value, the element corresponding to that row and column turn into pivotal.

(b) For two or more highest value, apply tie breaking technique.

Stage 2. Ignore corresponding row and column containing highest value. For remaining elements, go on *Step* 1 also replicate the similar procedure till the optimal solution is achieved.

Stage 3. If every single row and column is exhausted, then finest solution has been reached.

3. Formulation to Solve General GPP

In this section, the most usually applied type, general GP is discussed whose model is referred by Charnes and Cooper [3] as follows:

$$\text{Minimize: } Z = \sum_{r=1}^{m} (d_r^+, d_r^-) \tag{3.1}$$

subject to:

$$\textit{Goal Constraints:} \quad \sum_{q=1}^{n} a_{rq}x_q - d_r^+ + d_r^- = b_r, \quad \text{for } r = 1,2,3,\dots,m \tag{3.2}$$

$$\textit{System Constraints:} \quad \sum_{q=1}^{n} a_{rq}x_q \begin{bmatrix} \le \\ = \\ \ge \end{bmatrix} b_r, \quad \text{for } r = m+1,\dots,m+p \tag{3.3}$$

$$\text{where } d_r^+, d_r^-, x_q \ge 0, \quad \text{for } r = 1,\dots,m,\ q = 1,\dots,n,$$

where goal has denoted by m, system constraints by p and decision variables by n,

Z: objective function,

a_{rq}: the coefficient in the rth goal and variable q,

x_q: the qth decision variable,

d_r^-: variable with negative deviation for rth goal,

d_r^+: variable with positive deviation for rth goal.

This paper consists of some typical examples which give few experiences along with awareness to formulate as well as analyze a goal programming problem by using alternate *Simplex Approach*.

4. Supporting Examples to Solve General GPP

Problem 1.

$$\text{Minimum } z = d_1^- + d_2^- + 0x_3 + 0x_4 + 0x_5 + 0d_1^+ + 0d_2^+ \tag{4.1}$$

subject to: $2x_1 + 4x_2 + x_3 = 600$

$4x_1 + 5x_2 + x_4 = 1000$

$5x_1 + 4x_2 + x_5 = 1200$

$$20x_1 + 32x_2 + d_1^- - d_1^+ = 5400$$
$$0.3x_1 + 0.75x_2 + d_2^- - d_2^+ = 108$$

Solution. We make required calculations of the given example by the following tables.

Table 1. First table

			0	0	0	0	0	1	0	1	0
c_B	y_B	x_B	x_1	x_2	x_3	x_4	x_5	d_1^-	d_1^+	d_2^-	d_2^+
0	x_3	600	2	4	1	0	0	0	0	0	0
0	x_4	1000	4	5	0	1	0	0	0	0	0
0	x_5	1200	5	4	0	0	1	0	0	0	0
1	d_1^-	5400	20	**32**	0	0	0	1	−1	0	0
1	d_2^-	108	0.3	0.75	0	0	0	0	0	1	−1

Since $\max \sum x_{ij} = 45.75$.
Therefore, the column vector x_2 come into the next step as well as the column vector d_1^- departs.

Table 2. Enter x_2 and remove d_1^-

			0	0	0	0	0	1	0	1	0
c_B	y_B	x_B	x_1	x_2	x_3	x_4	x_5	d_1^-	d_1^+	d_2^-	d_2^+
0	x_3	−75	−1/2	0	1	0	0	−1/8	1/8	0	0
0	x_4	625/4	7/8	0	0	1	0	−5/32	5/32	0	0
0	x_5	525	**5/2**	0	0	0	1	−1/8	1/8	0	0
0	x_2	675/4	20/32	1	0	0	0	1/32	−1/32	0	0
1	d_2^-	−297/16	−27/160	0	0	0	0	−3/128	3/128	1	−1

Since $\max \sum x_{ij} = 3.33$.
Therefore, the column vector x_1 come in to the next step as well as the column vector x_5 departs.

Table 3. Enter x_1 and remove x_5

			0	0	0	0	0	1	0	1	0
c_B	y_B	x_B	x_1	x_2	x_3	x_4	x_5	d_1^-	d_1^+	d_2^-	d_2^+
0	x_3	30	0	0	1	0	1/5	−3/20	3/20	0	0
0	x_4	−55/2	0	0	0	1	**−7/20**	−9/80	9/80	0	0
0	x_1	2/10	1	0	0	0	2/5	−1/20	1/20	0	0
0	x_2	75/2	0	1	0	0	−1/4	1/16	−1/16	0	0
1	d_2^-	135/8	0	0	0	0	27/400	$\frac{-51}{1600}$	$\frac{51}{1600}$	1	−1

Since $\max \sum x_{ij} = 0.067$.
Therefore, the column vector x_5 come in to the next step as well as the column vector x_4 departs.

Table 4. Enter x_5 and remove x_4

			0	0	0	0	0	1	0	1	0
c_B	y_B	x_B	x_1	x_2	x_3	x_4	x_5	d_1^-	d_1^+	d_2^-	d_2^+
0	x_3	100/1	0	0	1	−1/5	0	−3/14	3/14	0	0
0	x_5	550/7	0	0	0	1	1	9/28	−9/28	0	0
0	x_1	1250/7	1	0	0	−2/5	0	−5/28	5/28	0	0
0	x_2	400/7	0	1	0	1/4	0	1/7	−1/7	0	0
1	d_2^-	81/7	0	0	0	$\frac{-27}{400}$	0	$\frac{-3}{56}$	$\frac{3}{56}$	1	−1

Thus, best possible solution is

$$x_1 = \frac{1250}{7},\ \ x_2 = \frac{400}{7},\ \ d_2^- = \frac{81}{7},\ \ x_3 = \frac{100}{7},\ \ x_4 = 0,\ \ x_5 = \frac{550}{7}.$$

The above result can also be represented by graphical presentation (Figure 1) which makes data easy to understand.

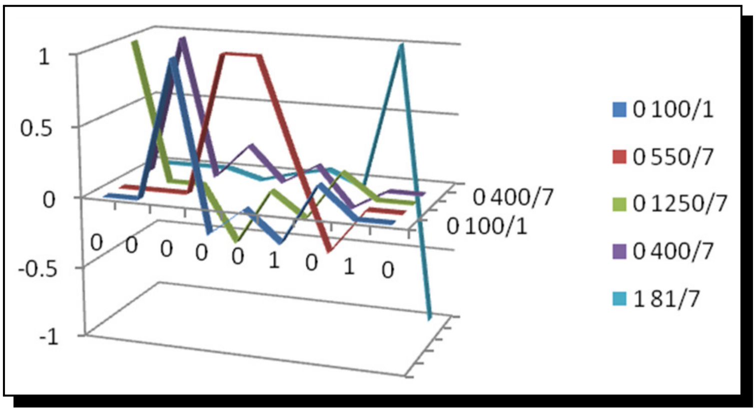

Figure 1. 3D view of optimum solution of Table 4

5. Formulation to Solve Preemptive Weighted Priority GP

In this section, the preemptive weighted priority GP is also discussed whose model is set as follows:

$$\text{Minimize: } z = \sum_{r=1}^{m} w_r P_r (d_r^- + d_r^+) \tag{5.1}$$

$$\text{subject to: } \sum_{s=1}^{n} a_{rs} x_s + d_r^- + d_r^+ = b_r, \quad r = 1,2,\dots,m \tag{5.2}$$

$$x_s, d_r^-, d_r^+ \geq 0, \qquad r = 1,2,\dots,m,\ s = 1,2,\dots,n$$

where

z: addition of the deviations of entirely essential goals with m goal constraints and n decision variables,

w_r: the relative non-negative weight allotted to deviational variables d_r^- and d_r^+ for all goal constraints,

P_r: preemptive priorities allotted to bunch of goals in rank order assembled with each other in formulation of GPP,

x_s: the sth decision variable,

a_{rs}: constant involved to each decision variable,

b_r: values at right-hand-side or goals of all constraint.

6. Supporting Examples to Solve Preemptive Weighted Priority GP

Problem 2.

$$\text{Mininize: } z = P_1 d_1^- + 5P_3 d_2^- + 3P_3 d_3^- + P_2 d_4^+ + P_4 d_1^+ \tag{6.1}$$

$$\text{subject to: } x_1 + x_2 + d_1^- - d_1^+ = 80$$

$$x_1 + x_2 + d_4^- - d_4^+ = 90$$

$$x_1 + d_2^- = 70$$

$$x_2 + d_3^- = 45$$

$$x_1, x_2, d_1^+, d_1^-, d_2^-, d_3^-, d_4^-, d_4^+ \geq 0$$

Solution. We make required calculations of the given example by the following tables.

Table 5. First table

			0	0	P_1	$5P_3$	$3P_3$	0	P_4	P_2
c_B	y_B	x_B	x_1	x_2	d_1^-	d_1^+	d_2^-	d_3^-	d_4^-	d_4^+
P_1	d_1^-	80	1	1	1	−1	0	0	0	0
0	d_4^-	90	1	1	1	0	0	0	1	−1
$5P_3$	d_2^-	70	**1**	0	0	0	1	0	0	0
$3P_3$	d_3^-	45	0	1	0	0	0	1	0	0

Since $\max \sum x_{ij} = 3$.
Therefore, the column vector x_1 come in to the next step as well as the column vector d_2^- departs.

Table 6. Enter x_1 and remove d_2^-

			0	0	P_1	$5P_3$	$3P_3$	0	P_4	P_2
c_B	y_B	x_B	x_1	x_2	d_1^-	d_1^+	d_2^-	d_3^-	d_4^-	d_4^+
P_1	d_1^-	10	0	**1**	1	−1	−1	0	0	0
0	d_4^-	20	0	1	0	0	−1	0	1	−1
0	x_1	70	1	0	0	0	1	0	0	0
$3P_3$	d_3^-	45	0	1	0	0	0	1	0	0

Since $\max \sum x_{ij} = 3$.
Therefore, the column vector x_2 come in to the next step as well as the column vector d_1^- departs.

Table 7. Enter x_2 and remove d_1^-

			0	0	P_1	$5P_3$	$3P_3$	0	P_4	P_2
c_B	y_B	x_B	x_1	x_2	d_1^-	d_1^+	d_2^-	d_3^-	d_4^-	d_4^+
0	x_2	10	0	1	1	−1	−1	0	0	0
0	d_4^-	10	0	0	−1	1	0	0	**1**	−1
0	x_1	70	1	0	0	0	1	0	0	0
$3P_3$	d_3^-	35	0	0	−1	1	1	1	0	0

Thus, best possible solution is

$$x_1 = 70,\ \ x_2 = 20,\ \ d_4^- = 10,\ \ d_3^- = 35.$$

The above result can also be represented by graphical presentation (Figure 2) which makes data easy to understand.

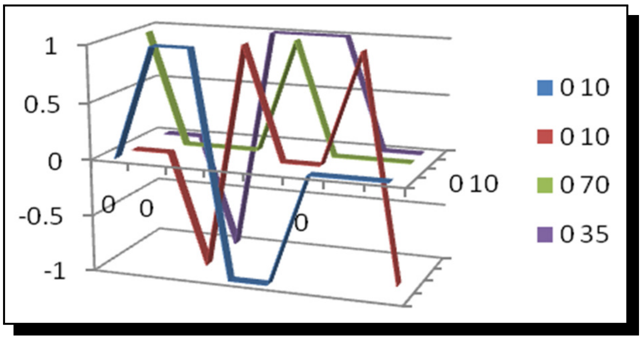

Figure 2. 3D view of optimum solution of Table 3

7. Conclusions

We had done an attempt to explain GPP by means of alternative simplex method. It will give a fresh approach which is simple to explain GPP. Above powerful method will use to catch improved solution which takes minimum quantity of iterations as well skip calculations of net evaluation, so save the precious time.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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