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Research Article

On Comparing the Time Resolution of Proposed Complex Wavelet With Morlet

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Abstract. In this study, we compare the complex Morlet wavelet's time-frequency resolution to that of a proposed complex continuous wavelet. The proposed complex continuous wavelet has a substantially lower time resolution than the complex Morlet wavelet. As a result, the proposed complex continuous wavelet is appropriate for applications requiring high time resolution. The efficiency of the proposed complex wavelet over the complex Morlet wavelet in evaluating the QRS complex and R-peak in ECG signals is demonstrated.

Keywords. Continuous wavelet transform, Time-frequency resolution, The electrocardiogram (ECG), R-peak detection

Mathematics Subject Classification (2020). 42C40, 65T60

1. Introduction

In recent years, the wavelet transform has gained popularity as a strong time-frequency analysis and signal coding technique for investigating complex non-stationary signals ([4, 5, 10]). The wavelet transformations provide a time-frequency decomposition of the signal, which allows for more effective separation of distinct signal components.

The continuous wavelet transform is a powerful tool to accurate locate signal features under the condition of choosing a good mother wavelet [4,5]. There are many wavelets in the literature, but different applications require different wavelets.

The electrocardiogram (ECG) is a test that determines how active the heart is electrically. Detecting the QRS complex, T wave and P wave in an ECG remains a challenge for those working in the field due to the signal's time-varying shape under physiological parameters and the presence of noise. Over time, a number of wavelet-based techniques for detecting these features have been proposed. Wavelet transformations' capacity to recognise and describe solitary heart beats was investigated by Senhadji *et al*. [9] and Addison [1]. Sahambi *et al*. [8] employed a wavelet that was a first-order derivative of the Gaussian function to characterise ECG waveforms. They detected and measured numerous features of the signal using wavelet analysis based on modulus maxima, including the location of the QRS complex's onset and offset, as well as the P and T waves.

We recently proposed a family of complex continuous wavelets [6, 7] and effectively applied them to signal processing and reconstruction by identifying several relevant signal reconstruction parameters. The reconstruction abilities were also compared to those of known wavelets like Morlet, Pual, and DOG.

Due to its low time resolution, the proposed complex wavelet is used in this research to analyse the QRS complex and R-peak detection in the ECG signal more efficiently than the complex Morlet wavelet in [2].

2. Continuous Wavelet Transformation (CWT)

The continuous wavelet transform of a signal $f(x) \in L^2(\mathbb{R})$ using the wavelet function $\xi_{s,\tau}(x) \in L^2(\mathbb{R})$ is defined as [1,3–5]

$$\begin{aligned} W_f(s,\tau) &= \langle f, \xi_{s,\tau} \rangle \\ &= \int_{-\infty}^{\infty} f(x)\xi^*_{s,\tau}(x)dx, \end{aligned} \tag{2.1}$$

where the wavelet function $\xi_{s,\tau}(x)$ are constructed by dilation with scale $s(s>0)$ and translating in time parameter τ. i.e.

$$\xi_{s,\tau}(x) = \frac{1}{\sqrt{s}}\xi\left(\frac{x-\tau}{s}\right) \tag{2.2}$$

Hence, the equation (2.1) as

$$W_f(s,\tau) = \int_{-\infty}^{\infty} f(x)\frac{1}{\sqrt{s}}\xi^*\left(\frac{x-\tau}{s}\right)dx. \tag{2.3}$$

The equation (2.3) is converted to the frequency domain using the Fourier-Parseval formula

$$W_f(s,\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\omega)\hat{\xi}^*_{s,\tau}(\omega)d\omega. \tag{2.4}$$

The discrete form of the continuous wavelet transformation of a signal $f(x)$ defined as [10]

$$W_f(s,n) = \sum_{a'=0}^{N-1} f_{a'}\xi^*\left[\frac{(a'-n)\delta x}{s}\right], \tag{2.5}$$

where δx is sampling period and the number of samples in time domain is N.

2.1 The Complex Morlet Wavelet (CMW)

The *Complex Morlet Wavelet* (CMW) is composed of a sinusoidal wave that has been modified by a Gaussian envelope [4, 5, 11], i.e.,

$$\xi(x) = Ce^{i\eta x}e^{-\frac{x^2}{2\sigma^2}}, \tag{2.6}$$

where σ is the shape parameter, η is the center frequency and $C = \frac{1}{(\pi\sigma^2)^{1/4}}$ is the normalization constant.

The Fourier transform of complex Morlet wavelet defined in equation (2.6) is

$$\hat{\xi}(\omega) = C\sqrt{2\pi\sigma^2}e^{-\frac{\sigma^2(\omega-\eta)^2}{2}}. \tag{2.7}$$

2.2 The Proposed Complex Continuous Wavelet (PCCW)

The proposed complex continuous family of wavelets are successive derivatives of cauchy's distribution function $\gamma(x) = \frac{e^{-ix}}{1+x^2}$ ([6, 7]), i.e.

$$\xi^j(x) = \frac{(-1)^k}{\sqrt{C_j}}\frac{d^j}{dx^j}(\gamma(x)), \tag{2.8}$$

where

$$k = \begin{cases} \frac{j}{2}, & j \text{ is even,} \\ \frac{j+1}{2}, & j \text{ is odd.} \end{cases}$$

In equation (2.8), the constant C_j is normalization constant such that $\|\xi^j(x)\|^2 = 1$ and defined as

$$C_j = \int_{-\infty}^{\infty}\left|\frac{d^j}{dx^j}\gamma(x)\right|^2 dx. \tag{2.9}$$

The Fourier transform of the proposed complex wavelet $\xi^j(x)$ is

$$\hat{\xi}^j(\omega) = \frac{(-1)^k\pi}{\sqrt{C_j}}(i\omega)^j e^{-|\omega+1|}. \tag{2.10}$$

3. Time-Frequency Resolution

The wavelet transform can be used to analyze time domain function with non-stationary power at varies frequencies. In the time and frequency plane (x,ω), it can be characterized by a region whose location and width are dictated by the time and frequency spread of wavelet $\xi(x)$. For evaluating the performance of different wavelets, resolutions in the time and frequency domains are crucial. The time resolution in the time domain σ_x and the frequency resolution in the frequency domain σ_ω of continuous wavelet transform can be characterized as follows [5]:

$$\sigma_x^2 = \int_{-\infty}^{\infty}(x-u)^2|\xi(x)|^2 dx, \tag{3.1}$$

$$\sigma_\omega^2 = \frac{1}{2\pi}\int_0^{\infty}(\omega-\zeta)^2|\hat{\xi}(\omega)|^2 d\omega, \tag{3.2}$$

where

$$u = \int_{-\infty}^{\infty}x|\xi(x)|^2 dx,$$

$$\zeta = \frac{1}{2\pi}\int_0^\infty \omega|\hat{\xi}(\omega)|^2 d\omega.$$

In the time and frequency plane (x,ω), the time and frequency resolution of $\xi(x)$ is represented by a centred at (u,ζ) with a width of σ_x along time and σ_ω along frequency.

The adjustment between the width in time domain and the width in frequency domain determines the resolution of a wavelet function. A narrow wavelet function in time domain has excellent time resolution but low frequency resolution, whereas a broad (in time) function has poor time resolution but excellent frequency resolution.

Table 1. Comparison of time and frequency resolution of CMW and PCCW ($j = 4$)

Wavelet	σ_x	σ_ω	$\sigma_x \cdot \sigma_\omega$
Complex Morlet Wavelet	1.0606	0.4455	0.4725
Proposed Complex Wavelet ($j = 4$)	0.3798	0.6258	0.2377

The proposed complex wavelet ($j = 4$) has a narrower (in time) function than the complex Morlet wavelet, as shown in comparison Table 1. As a result, in time-dependent applications, the proposed complex wavelet is extremely useful.

Let us take an example to understand it more clearly, consider a non-stationary synthetic signal $f(x)$

$$f(x) = \begin{cases} \sin(2\pi 30x), & 0 < x \le 0.5, \\ \sin(2\pi 60x), & \text{otherwise.} \end{cases} \tag{3.3}$$

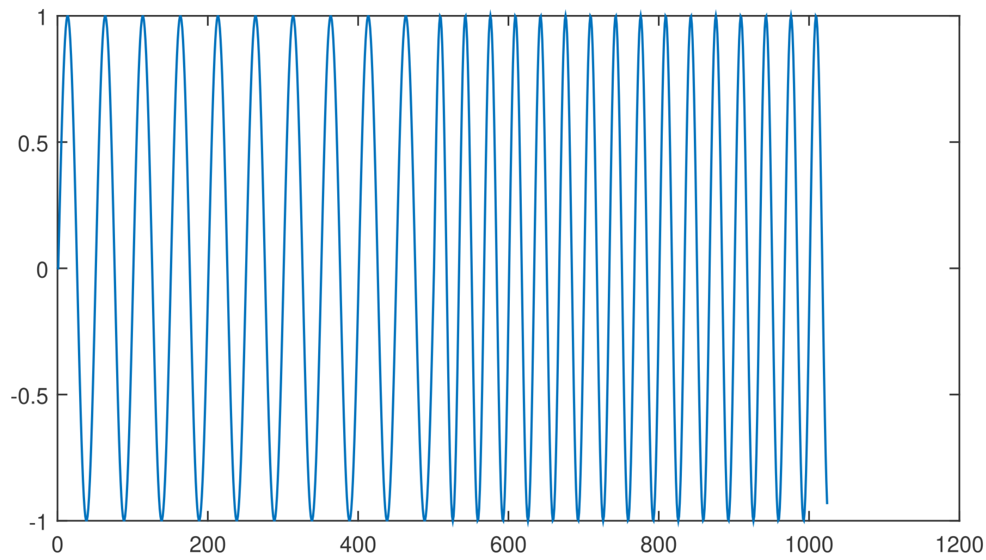

Figure 1. A synthetic signal $f(x)$

In comparison to the complex Morlet wavelet, the proposed complex wavelet ($j = 4$) appears to resolve time localization quite well, as shown in the Figures 2 and 3.

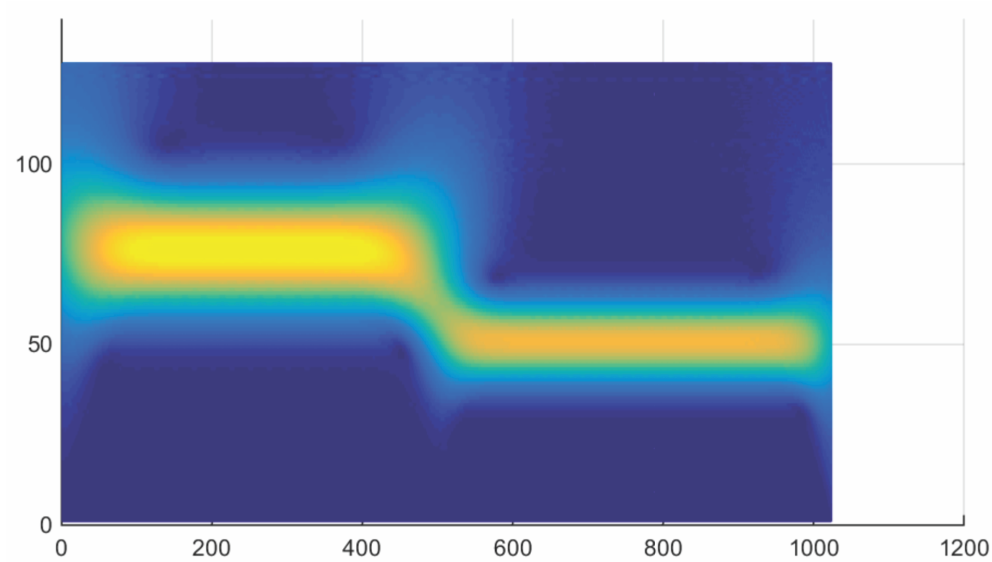

Figure 2. Absolute values of wavelet transformation coefficients of Signal $f(x)$ using CMW

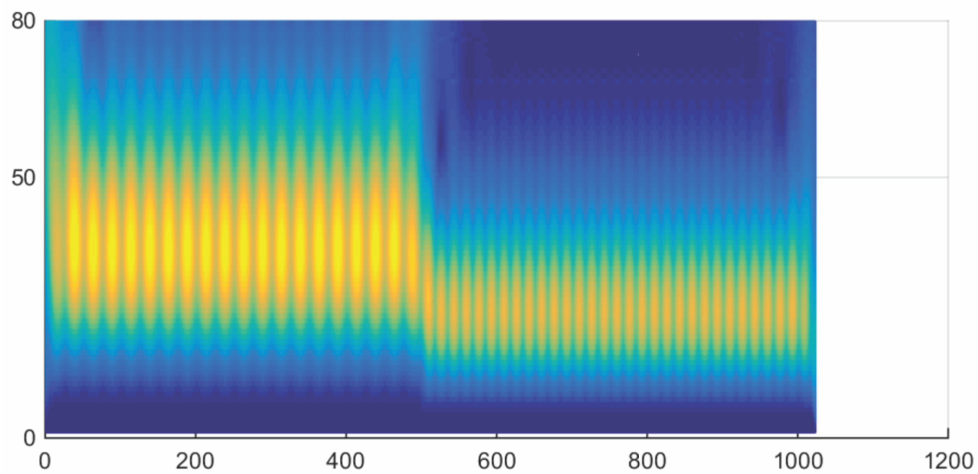

Figure 3. Absolute values of wavelet transformation coefficients of Signal $f(x)$ using PCCW ($j = 4$)

4. The electrocardiogram (ECG)

Depolarization is a term used to describe the electrical changes that occur during muscle contraction. The electrocardiogram (ECG) is a test that measures the electrical activity of the heart. Electrical changes connected with activation of the two small heart chambers, the atria, and then the two bigger heart chambers, the ventricles, produce the ECG, which is monitored at the body surface. The 'P' wave in the ECG represents the contraction of the atria, whereas the 'QRS' complex represents the contraction of the ventricles. The 'T' wave is produced when the ventricular mass returns to a resting condition (repolarization). Because of the signal's time-varying shape and the presence of noise, it is subject to physiological parameters, detecting the QRS complex, T wave and P wave in an ECG signal is still an issue that has to be solved by those working in the field. Several wavelet-based approaches for detecting these features have been proposed over time. Senhadji *et al*. [9] examined wavelet transformations' ability to distinguish and describe solitary heart beats (based on three different wavelets: Morlet, Daubechies and spline). Sahambi *et al.* [8] employed a first-order derivative of the Gaussian function as a wavelet to characterise ECG wave forms. They detected and measured numerous features of the signal using wavelet analysis based on modulus maxima, including the position of the QRS complex's onset and offset, as well as the T and P waves. The procedure was then repeated for signals with more baseline drifting and high-frequency noise. They also analysed intra-beat time intervals to figure out the position relative of the ECG's components, which are critical for assessing the heart's electrical activity.

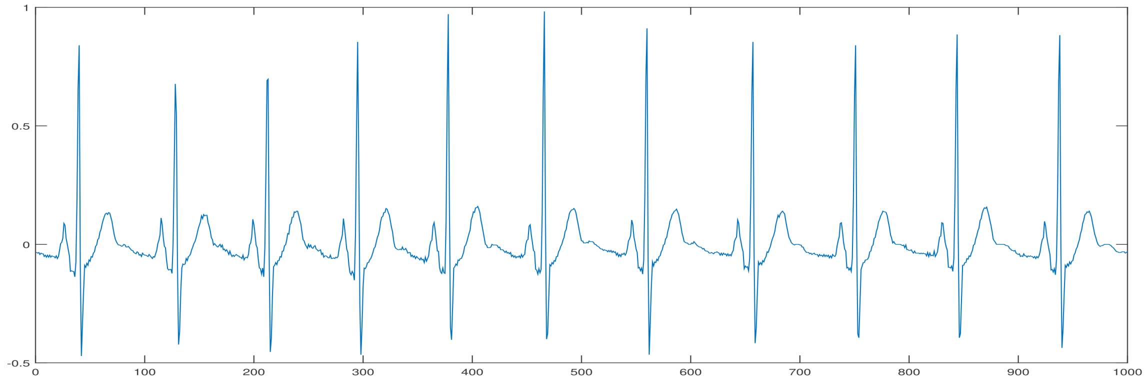

Figure 4. An ECG signal

An ECG signal is obtained from record $a01m$ in the apnea database[1]. Both complex wavelets are used to compute the wavelet transforms. Figure 5 shows the modulus plots of the coefficients. The QRS complex of the ECG signal emerges more distinctly in time location with the proposed complex wavelet ($j = 4$) than the complex Morlet wavelet.

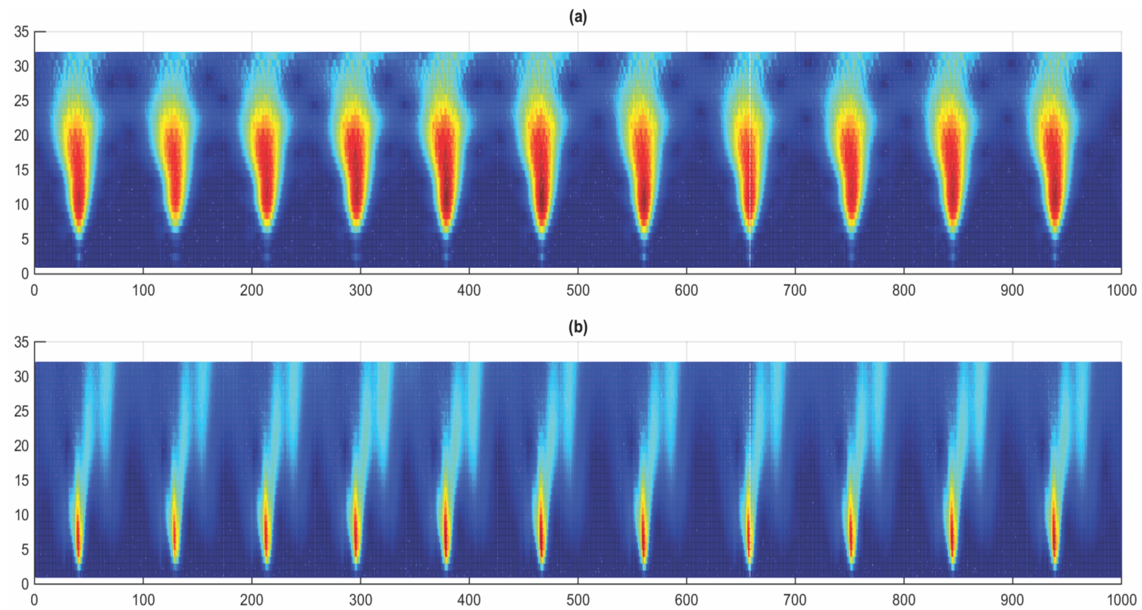

Figure 5. Absolute values of wavelet transform coefficients of ECG signal using with (a)CMT and (b) PCCW ($j = 4$)

[1]Physiobank [Online], http://www.physionet.org/physiobank/database/apnea-ecg.

5. R-peak Detection

Due to the time varying morphology of the signal responsive to physiological parameters and the presence of noise, developing an algorithm for detecting the QRS complex, T wave and P wave in an ECG signal is a difficult problem [1].

For the study of ECG signals, R wave detectors are particularly useful. They are used to compute the R-R time series on which a number of heart rate variability (HRV) measures can be derived, as well as to determine the fiducial points utilised in ensemble averaging analytic methods.

We took the following steps to automatically select a scale s_α for detecting R peaks:

Step I: The analyzed ECG signal is taken from record $a01m$ in the apnea database[1] and the wavelet transform coefficients $W_{a01m}(s,n)$ were calculated.

Step II: The maximum of absolute values of $W_{a01m}(s,n)$ at each time point was collected and stored in a vector V_m, as shown in Figure 7 in the simulation of this stage.

Step III: We found all the scales that corresponded to the peaks of V_m by computing them. The selected scale, s_α is obtained by taking the average of these scales.

Step IV: The value corresponding to this scale s_α gives R-peak.

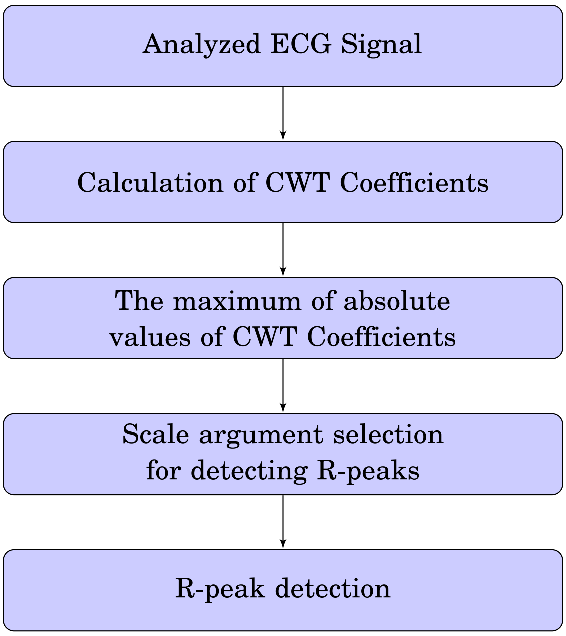

Figure 6. The analyzed ECG signal

Figures 10 and 11 shows that the R-peaks recognised using the proposed complex wavelet ($j = 4$) are sharper in time than those detected using the complex Morlet wavelet, which needs a threshold point and sliding window [2].

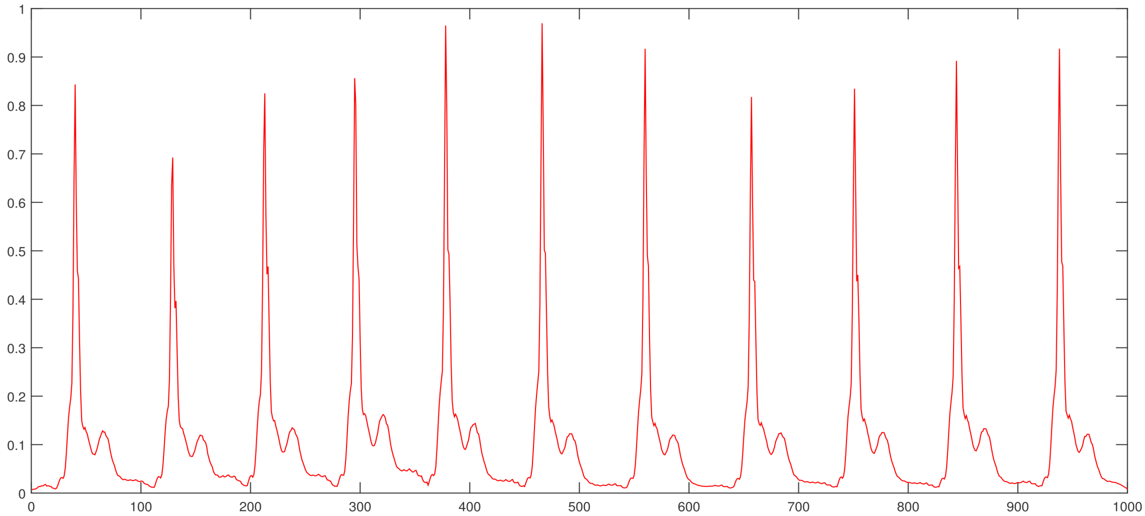

Figure 7. The maximum of absolute values of PCCW transform coefficients

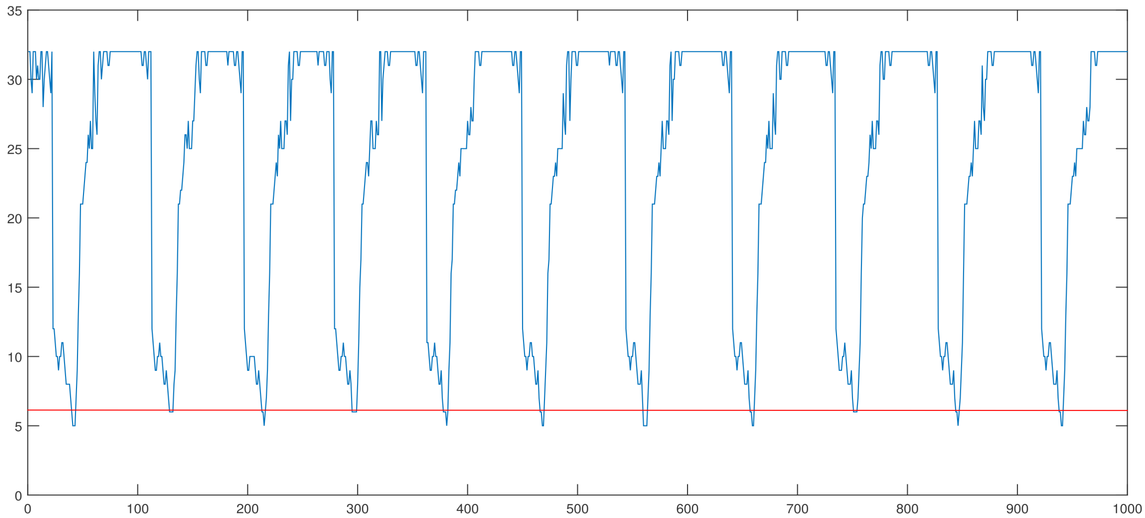

Figure 8. Selected scale(red line) for R peak detection

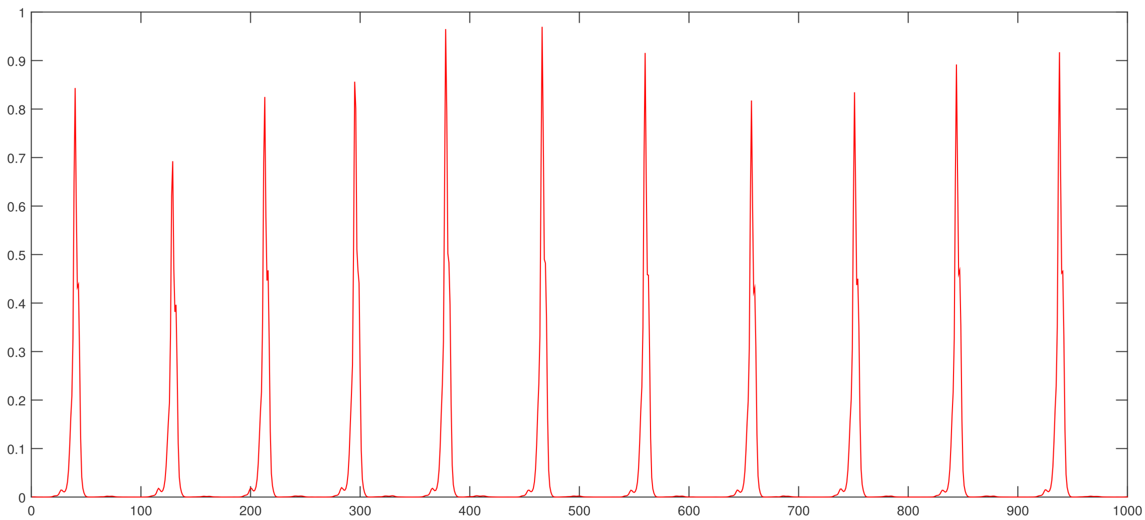

Figure 9. The absolute values of PCCW transform coefficients at selected scale s_α

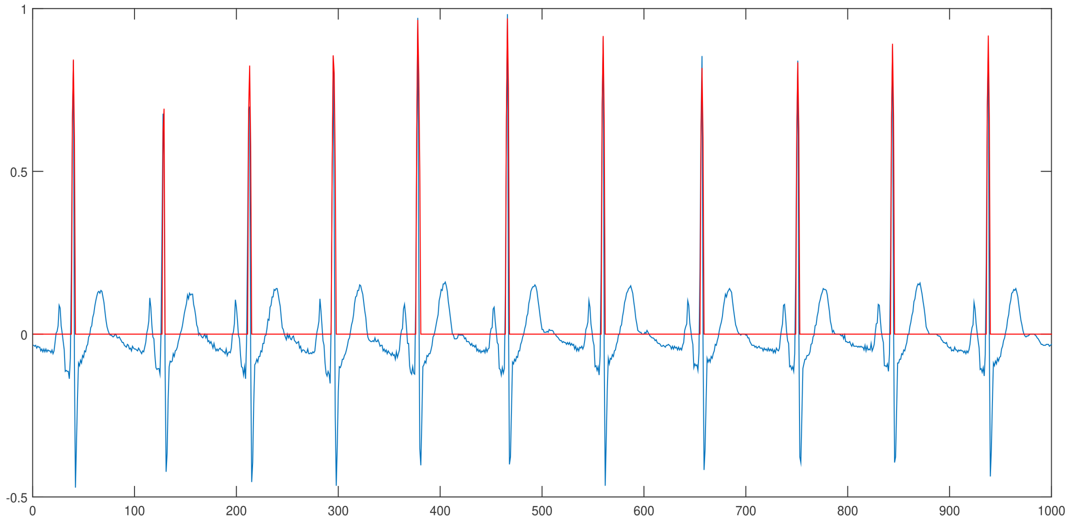

Figure 10. R peak detection using Proposed complex wavelet ($j = 4$)

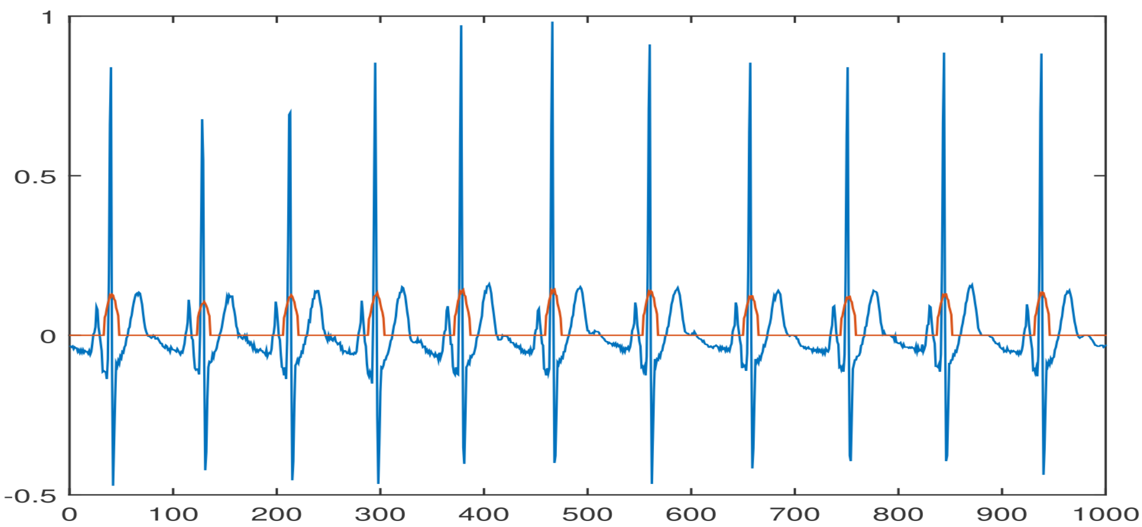

Figure 11. R peak detection using Complex morlet wavelet

6. Conclusion

The proposed complex wavelet (PCCW) is more successful at detecting QRS complex in ECG signals since it has a lower time resolution than the complex Morlet wavelet (CMW). Furthermore, detecting R-peaks with a CMW necessarily requires numerous computations, such as finding the threshold point and sliding window, whereas the PCCW does not require all of these parameters and still provides sharper time resolution for R-peaks than the CMW.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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