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Research Article

Common Fixed Point Theorem for Four Weakly Compatible Self Maps on a Complete *S*-Metric Space

A. Srinivas

Department of Mathematics, Osmania University, Hyderabad 500007, India
srinivas.arugula08@gmail.com

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Abstract. In the present paper, we establish a common fixed point theorem for four weakly compatible self maps of a S-metric space.

Keywords. S-metric space, Self map, Compatibility, Weakly compatible maps, Fixed point

Mathematics Subject Classification (2020). 47H10, 54H25

1. Introduction

The notion of metric spaces was generalized by many researchers [1–3, 7, 8, 12]. Recently, Sedghi *et al*. [11] initiated S-metric spaces as one more generalization, which generated lot of interest among researchers.

Jungck and Rhoades [6] proposed weakly compatibility as the generalization of compatibility of mappings introduced by Jungck [4, 5].

In this paper with the aid of weakly compatibility, we establish a common fixed point theorem for four self maps of a complete S-metric space. An example is provided to validate our result. This theorem generalizes the theorem proved by Sedghi *et al*. [13].

2. Preliminaries

Definition 2.1 ([11]). Let M be a non empty set. By S-metric, we mean a function $S : M^3 \to [0,\infty)$ which satisfy the following conditions:

(a) $S(\alpha',\beta',\gamma') \geq 0$,

(b) $S(\alpha',\beta',\gamma') = 0$ if and only if $\alpha' = \beta' = \gamma'$,

(c) $S(\alpha',\beta',\gamma') \leq S(\alpha',\alpha',\omega) + S(\beta',\beta',\omega) + S(\gamma',\gamma',\omega)$,

for any $\alpha',\beta',\gamma',\omega \in M$. Then (M,S) is known as S-metric space.

Lemma 2.1 ([9]). *Let (M,S) be a S-metric space. Then we have $S(\alpha',\alpha',\beta') = S(\beta',\beta',\alpha')$, for any $\alpha',\beta' \in M$.*

Definition 2.2 ([10]). Let (M,S) be a S-metric space.

(i) A sequence (α_n) in M converges to α if $S(\alpha_n,\alpha_n,\alpha) \to 0$ as $n \to \infty$, that is, for each $\epsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that for $n \geq n_0$, $S(\alpha_n,\alpha_n,\alpha) < \epsilon$. In this case, we denote it by writing $\lim_{n\to\infty} \alpha_n = \alpha$.

(ii) A sequence (α_n) is called a Cauchy sequence if for any $\epsilon > 0$, there exists an $n_0 \in \mathbb{N}$ such that $S(\alpha_n,\alpha_n,\alpha_m) < \epsilon$ for any $n,m \geq n_0$.

(iii) By a complete S-metric space, we mean a S-metric space (M,S) in which every Cauchy sequence is convergent.

Lemma 2.2 ([10]). *In a S-metric space (M,S), if there exist two sequences (α_n) and (β_n) such that $\lim_{n\to\infty} \alpha_n = \alpha$ and $\lim_{n\to\infty} \beta_n = \beta$, then $\lim_{n\to\infty} S(\alpha_n,\alpha_n,\beta_n) = S(\alpha,\alpha,\beta)$.*

Definition 2.3 ([13]). In a S-metric space (M,S), a pair of self maps (E,F) is called as compatible if $\lim_{n\to\infty} S(EF\alpha_n,EF\alpha_n,FE\alpha_n) = 0$, where (α_n) is a sequence in M such that $\lim_{n\to\infty} E\alpha_n = \lim_{n\to\infty} F\alpha_n = t$, for some $t \in M$.

Definition 2.4 ([6]). In a S-metric space (M,S), the self maps E and F of M are called as weakly compatible if $EFt = FEt$ whenever $Et = Ft$, for any $t \in M$.

Remark 2.1 ([6]). *Clearly, compatible maps are weakly compatible but not conversely.*

Example 2.1. Let $M = \left[\frac{5}{2},9\right]$. Define $S(\alpha,\beta,\gamma) = d(\alpha,\gamma) + d(\beta,\gamma)$, where $d(\alpha,\beta) = \max\{\alpha,\beta\}$. Define two maps E and F on M such that

$$E(\alpha) = \begin{cases} \frac{5}{2}, & \alpha \in \left\{\frac{5}{2}\right\} \cup (4,9], \\ 3, & \alpha \in \left(\frac{5}{2},4\right], \end{cases} \qquad F(\alpha) = \begin{cases} \frac{5}{2}, & \alpha = \frac{5}{2}, \\ 3+\alpha, & \alpha \in \left(\frac{5}{2},4\right], \\ \frac{\alpha+1}{2}, & \alpha \in (4,9]. \end{cases}$$

Taking $\alpha_n = 4 + \dfrac{1}{n}$, for any $n \geq 1$.

$$\lim_{n\to\infty} E\alpha_n = \lim_{n\to\infty} E\left(4+\frac{1}{n}\right) = \frac{5}{2},$$

$$\lim_{n\to\infty} F\alpha_n = \lim_{n\to\infty} F\left(4+\frac{1}{n}\right) = \frac{5}{2},$$
$$\lim_{n\to\infty} EF\alpha_n = \lim_{n\to\infty} EF\left(4+\frac{1}{n}\right) = \lim_{n\to\infty} E\left(\frac{5}{2}+\frac{1}{2n}\right) = 3,$$
$$\lim_{n\to\infty} FE\alpha_n = \lim_{n\to\infty} FE\left(4+\frac{1}{n}\right) = \lim_{n\to\infty} F\left(\frac{5}{2}\right) = \frac{5}{2}.$$

Proving that the pair (E,F) is weakly compatible but not compatible.

3. Main Result

We now state our main theorem

Theorem 3.1. *In a complete S-metric space (M,S), suppose A,B,E and F are self maps of M such that*

(i) $A(M)\subseteq F(M)$, $B(M)\subseteq E(M)$,

(ii) *the pairs (A,E) and (B,F) are weakly compatible,*

(iii) *$E(M)$ and $F(M)$ are closed subsets of M*

and

$$S(A\alpha,A\beta,B\gamma)\le d\ max\{S(E\alpha,E\beta,F\gamma),S(A\alpha,A\alpha,E\alpha),S(B\gamma,B\gamma,F\gamma),S(A\beta,A\beta,B\gamma)\} \tag{3.1}$$

for any $\alpha,\beta,\gamma\in M$ with $0<d<1$, then A,B,E and F have a unique common fixed point in M.

Proof. Let $\alpha_0\in M$. We know that $A(M)\subseteq F(M)$ then there exists $\alpha_1\in M$ such that $A\alpha_0=F\alpha_1$, and also $B\alpha_1\in E(M)$, we choose $\alpha_2\in M$ such that $B\alpha_1=E\alpha_2$. In general, $\alpha_{2n+1}\in M$ is chosen such that $A\alpha_{2n}=F\alpha_{2n+1}$, and $\alpha_{2n+2}\in M$ such that $B\alpha_{2n+1}=E\alpha_{2n+2}$, we obtain a sequence (β_n) in M such that $\beta_{2n}=A\alpha_{2n}=F\alpha_{2n+1}$, $\beta_{2n+1}=B\alpha_{2n+1}=E\alpha_{2n+2}$, $n\ge 0$.
To prove that (β_n) is a Cauchy sequence.

$$\begin{aligned}
S(\beta_{2n},\beta_{2n},\beta_{2n+1}) &= S(A\alpha_{2n},A\alpha_{2n},B\alpha_{2n+1})\\
&\le d\max\{S(E\alpha_{2n},E\alpha_{2n},F\alpha_{2n+1}),S(A\alpha_{2n},A\alpha_{2n},E\alpha_{2n}),\\
&\qquad S(B\alpha_{2n+1},B\alpha_{2n+1},F\alpha_{2n+1}),S(A\alpha_{2n},A\alpha_{2n},B\alpha_{2n+1})\}\\
&= d\max\{S(\beta_{2n-1},\beta_{2n-1},\beta_{2n}),S(\beta_{2n},\beta_{2n},\beta_{2n-1}),\\
&\qquad S(\beta_{2n+1},\beta_{2n+1},\beta_{2n}),S(\beta_{2n},\beta_{2n},\beta_{2n+1})\}\\
&= d\max\{S(\beta_{2n-1},\beta_{2n-1},\beta_{2n}),S(\beta_{2n},\beta_{2n},\beta_{2n+1})\}.
\end{aligned}$$

Now if, $S(\beta_{2n},\beta_{2n},\beta_{2n+1})>S(\beta_{2n-1},\beta_{2n-1},\beta_{2n})$, giving

$$S(\beta_{2n},\beta_{2n},\beta_{2n+1})<d\ S(\beta_{2n},\beta_{2n},\beta_{2n+1})$$

which is a contradiction.
Hence, $S(\beta_{2n},\beta_{2n},\beta_{2n+1})\le S(\beta_{2n-1},\beta_{2n-1},\beta_{2n})$.
Therefore, by above inequality, we get

$$S(\beta_{2n},\beta_{2n},\beta_{2n+1})\le d\ S(\beta_{2n-1},\beta_{2n-1},\beta_{2n}). \tag{3.2}$$

By a similar argument, we have

$$\begin{aligned}
S(\beta_{2n-1},\beta_{2n-1},\beta_{2n}) &= S(\beta_{2n},\beta_{2n},\beta_{2n-1}) \\
&= S(A\alpha_{2n},A\alpha_{2n},B\alpha_{2n-1}) \\
&\le d\max\{S(E\alpha_{2n},E\alpha_{2n},F\alpha_{2n-1}),S(A\alpha_{2n},A\alpha_{2n},E\alpha_{2n}), \\
&\qquad S(B\alpha_{2n-1},B\alpha_{2n-1},F\alpha_{2n-1}),S(A\alpha_{2n},A\alpha_{2n},B\alpha_{2n-1})\} \\
&= d\max\{S(\beta_{2n-1},\beta_{2n-1},\beta_{2n-2}),S(\beta_{2n},\beta_{2n},\beta_{2n-1}), \\
&\qquad S(\beta_{2n-1},\beta_{2n-1},\beta_{2n-2}),S(\beta_{2n},\beta_{2n},\beta_{2n-1})\} \\
&= d\max\{S(\beta_{2n-2},\beta_{2n-2},\beta_{2n-1}),S(\beta_{2n},\beta_{2n},\beta_{2n-1})\}.
\end{aligned}$$

Now if, $S(\beta_{2n},\beta_{2n},\beta_{2n-1}) > S(\beta_{2n-2},\beta_{2n-2},\beta_{2n-1})$, giving

$$S(\beta_{2n},\beta_{2n},\beta_{2n-1}) < d\ S(\beta_{2n},\beta_{2n},\beta_{2n-1})$$

which is a contradiction.
Hence, $S(\beta_{2n-1},\beta_{2n-1},\beta_{2n}) \le S(\beta_{2n-2},\beta_{2n-2},\beta_{2n-1})$.
Therefore, by above inequality, we get

$$S(\beta_{2n-1},\beta_{2n-1},\beta_{2n}) \le d\ S(\beta_{2n-2},\beta_{2n-2},\beta_{2n-1}). \tag{3.3}$$

Now from (3.2) and (3.3), we get

$$S(\beta_n,\beta_n,\beta_{n-1}) \le d\ S(\beta_{n-1},\beta_{n-1},\beta_{n-2}),$$

where $0 < d < 1$. Hence for $n \ge 2$, it follows that

$$S(\beta_n,\beta_n,\beta_{n-1}) \le \ldots \le d^{n-1}\ S(\beta_1,\beta_1,\beta_0). \tag{3.4}$$

For $n > m$, we get

$$\begin{aligned}
S(\beta_n,\beta_n,\beta_m) &\le 2S(\beta_m,\beta_m,\beta_{m+1}) + 2S(\beta_{m+1},\beta_{m+1},\beta_{m+2}) + \ldots + S(\beta_{n-1},\beta_{n-1},\beta_n) \\
&< 2S(\beta_m,\beta_m,\beta_{m+1}) + 2S(\beta_{m+1},\beta_{m+1},\beta_{m+2}) + \ldots + 2S(\beta_{n-1},\beta_{n-1},\beta_n).
\end{aligned}$$

Hence from (3.4), it follows that

$$\begin{aligned}
S(\beta_n,\beta_n,\beta_m) &\le 2(d^m + d^{m+1} + \ldots + d^{n-1})S(\beta_1,\beta_1,\beta_0) \\
&= 2d^m[1 + d + d^2 + \ldots]S(\beta_1,\beta_1,\beta_0) \\
&= \frac{2d^m}{1-d}S(\beta_1,\beta_1,\beta_0) \to 0 \text{ as } n \to \infty.
\end{aligned}$$

It follows that (β_n) is a Cauchy sequence in complete S-metric space. Therefore, there is an η in M such that

$$\lim_{n\to\infty} A\alpha_{2n} = \lim_{n\to\infty} F\alpha_{2n+1} = \lim_{n\to\infty} B\alpha_{2n+1} = \lim_{n\to\infty} E\alpha_{2n+2} = \eta.$$

We now establish that η is a common fixed point of A,B,E and F.
As $F(M)$ is a closed subset of M, we have

$$Fv = \eta = \lim_{n\to\infty} F\alpha_{2n+1}, \quad \text{for some } v \in M.$$

Keeping $\alpha = \beta = \alpha_{2n}$ and $\gamma = v$ in (3.1), we get

$$S(A\alpha_{2n}, A\alpha_{2n}, Bv) \le d \max\{S(E\alpha_{2n}, E\alpha_{2n}, Fv), S(A\alpha_{2n}, A\alpha_{2n}, E\alpha_{2n}), S(Bv, Bv, Fv), S(A\alpha_{2n}, A\alpha_{2n}, Bv)\}. \tag{3.5}$$

On passing to the limits

$$\begin{aligned} S(\eta,\eta,Bv) &\le d \max\{S(\eta,\eta,Fv), S(\eta,\eta,\eta), S(Bv,Bv,Fv), S(\eta,\eta,Bv)\} \\ &\le d\ S(\eta,\eta,Bv), \end{aligned}$$

which implies $Bv = \eta$.
Hence $Fv = Bv = \eta$.
Therefore, $BFv = FBv$, which gives

$$B\eta = F\eta. \tag{3.6}$$

Putting $\alpha = \beta = \alpha_{2n}$ and $\gamma = \eta$ in (3.1), we get

$$S(A\alpha_{2n}, A\alpha_{2n}, B\eta) \le d \max\{S(E\alpha_{2n}, E\alpha_{2n}, F\eta), S(A\alpha_{2n}, A\alpha_{2n}, E\alpha_{2n}), S(B\eta, B\eta, F\eta), S(A\alpha_{2n}, A\alpha_{2n}, B\eta)\}. \tag{3.7}$$

On passing to the limits

$$\begin{aligned} S(\eta,\eta,B\eta) &\le d \max\{S(\eta,\eta,B\eta), S(\eta,\eta,\eta), S(B\eta,B\eta,F\eta), S(\eta,\eta,B\eta)\} \\ &\le d\ S(\eta,\eta,B\eta), \end{aligned}$$

which implies $B\eta = \eta$.
From (3.6),

$$B\eta = F\eta = \eta. \tag{3.8}$$

We have $Eu = \eta = \lim_{n\to\infty} E\alpha_{2n+2}$, for some $u \in M$ as $E(M)$ is a closed.
Putting $\alpha = \beta = u$ and $\gamma = \alpha_{2n+1}$ in (3.1), we get

$$S(Au, Au, B\alpha_{2n+1}) \le d \max\{S(Eu, Eu, F\alpha_{2n+1}), S(Au, Au, Eu), S(B\alpha_{2n+1}, B\alpha_{2n+1}, F\alpha_{2n+1}), S(Au, Au, B\alpha_{2n+1})\}. \tag{3.9}$$

On passing to the limits

$$\begin{aligned} S(Au,Au,\eta) &\le d \max\{S(\eta,\eta,\eta), S(Au,Au,\eta), S(\eta,\eta,\eta), S(Au,Au,\eta)\} \\ &\le d\ S(Au,Au,\eta), \end{aligned}$$

which gives $Au = \eta$.
Hence $Eu = Au = \eta$.
Therefore, $AEu = EAu$, which gives

$$A\eta = E\eta. \tag{3.10}$$

Putting $\alpha = \beta = \eta$ and $\gamma = v$ in (3.1), we get

$$S(A\eta, A\eta, Bv) \le d \max\{S(E\eta, E\eta, Fv), S(A\eta, A\eta, E\eta), S(Bv, Bv, Fv), S(A\eta, A\eta, Bv)\}$$

$$S(A\eta, A\eta, \eta) \le d \max\{S(A\eta, A\eta, \eta), S(A\eta, A\eta, A\eta), S(\eta,\eta,\eta), S(A\eta, A\eta, \eta)\}$$

$$S(A\eta, A\eta, \eta) \le d\ S(A\eta, A\eta, \eta),$$

which implies $A\eta = \eta$.
From (3.10),

$$A\eta = E\eta = \eta. \tag{3.11}$$

From (3.8) and (3.11), we get

$$A\eta = E\eta = E\eta = F\eta = \eta. \tag{3.12}$$

Therefore η is a fixed point of A, B, E and F.
We now prove η is unique, for if $\zeta(\zeta \neq \eta)$ in M is such that

$$A\zeta = B\zeta = E\zeta = F\zeta = \zeta.$$

Keeping $\alpha = \beta = \eta$ and $\gamma = \zeta$ in (3.1), we get

$$S(A\eta, A\eta, B\zeta) \le d \max\{S(E\eta, E\eta, F\zeta), S(A\eta, A\eta, E\eta), S(B\zeta, B\zeta, F\zeta), S(A\eta, A\eta, B\zeta)\}$$

$$S(\eta, \eta, \zeta) \le d \max\{S(\eta, \eta, \zeta), S(\eta, \eta, \eta), S(\zeta, \zeta, \zeta), S(\eta, \eta, \zeta)\}$$

$$S(\eta, \eta, \zeta) \le d\ S(\eta, \eta, \zeta),$$

showing $\zeta = \eta$, proving the uniqueness of common fixed point of A, B, E and F. □

As an illustration, we have the following example.

Example 3.1. Let $M = \left[\frac{5}{2}, 9\right]$ and $S(\alpha, \beta, \gamma) = d(\alpha, \gamma) + d(\beta, \gamma)$, where $d(\alpha, \beta) = \max\{\alpha, \beta\}$. Define mappings A, B, E and F on M such that

$$A(\alpha) = \begin{cases} \frac{5}{2} & \alpha \in \left\{\frac{5}{2}\right\} \cup (4, 9], \\ 3, & \alpha \in \left(\frac{5}{2}, 4\right], \end{cases} \qquad B(\alpha) = \begin{cases} \frac{5}{2} & \alpha \in \left\{\frac{5}{2}\right\} \cup (4, 9], \\ 4, & \alpha \in \left(\frac{5}{2}, 4\right], \end{cases}$$

$$E(\alpha) = \begin{cases} \frac{5}{2}, & \alpha = \frac{5}{2}, \\ 3 + \alpha, & \alpha \in \left(\frac{5}{2}, 4\right], \\ \frac{\alpha+1}{2}, & \alpha \in (4, 9], \end{cases} \qquad F(\alpha) = \begin{cases} \frac{5}{2}, & \alpha = \frac{5}{2}, \\ 7, & \alpha \in \left(\frac{5}{2}, 4\right], \\ \frac{\alpha+1}{2}, & \alpha \in (4, 9]. \end{cases}$$

Clearly, $A(M) = \left\{\frac{5}{2}, 3\right\}$, $B(M) = \left\{\frac{5}{2}, 4\right\}$, $E(M) = \left[\frac{5}{2}, 5\right] \cup \left(\frac{11}{2}, 7\right]$ and $F(M) = \left[\frac{5}{2}, 5\right] \cup \{7\}$.
We observe that $A(M) \subseteq F(M)$ and $B(M) \subseteq E(M)$ and (A, E), (B, F) are weakly compatible.
Conditions (i) and (ii) of Theorem 3.1 are satisfied.
Taking $\alpha_n = 4 + \dfrac{1}{n}$, for any $n \ge 1$.

$$\lim_{n\to\infty} A\alpha_n = \lim_{n\to\infty} A\left(4 + \frac{1}{n}\right) = \frac{5}{2},$$

$$\lim_{n\to\infty} B\alpha_n = \lim_{n\to\infty} B\left(4 + \frac{1}{n}\right) = \frac{5}{2},$$

$$\lim_{n\to\infty} E\alpha_n = \lim_{n\to\infty} E\left(4 + \frac{1}{n}\right) = \frac{5}{2},$$

$$\lim_{n\to\infty} F\alpha_n = \lim_{n\to\infty} F\left(4 + \frac{1}{n}\right) = \frac{5}{2},$$

$$\lim_{n\to\infty} AE\alpha_n = \lim_{n\to\infty} AE\left(4+\frac{1}{n}\right) = \lim_{n\to\infty} A\left(\frac{5}{2}+\frac{1}{2n}\right) = 3,$$
$$\lim_{n\to\infty} EA\alpha_n = \lim_{n\to\infty} EA\left(4+\frac{1}{n}\right) = \lim_{n\to\infty} E\left(\frac{5}{2}\right) = \frac{5}{2},$$
$$\lim_{n\to\infty} BF\alpha_n = \lim_{n\to\infty} BF\left(4+\frac{1}{n}\right) = \lim_{n\to\infty} B\left(\frac{5}{2}+\frac{1}{2n}\right) = 7,$$
$$\lim_{n\to\infty} FB\alpha_n = \lim_{n\to\infty} FB\left(4+\frac{1}{n}\right) = \lim_{n\to\infty} F\left(\frac{5}{2}\right) = \frac{5}{2}.$$

Therefore, (A,E) and (B,F) are weakly compatible but not compatible.
Now we check the condition stated in inequality (3.1) of Theorem 3.1 in different cases.

Case (i): If $\alpha,\beta,\gamma \in \left(\frac{5}{2},4\right]$.
Then, $A\alpha = 3$, $A\beta = 3$, $B\gamma = 4$, $E\alpha = 3+\alpha$, $E\beta = 3+\beta$, $F\gamma = 7$, $S(A\alpha,A\beta,B\gamma) = 8$, $S(E\alpha,E\beta,F\gamma) = 14$, $S(A\alpha,A\alpha,E\alpha) = 14$, $S(B\gamma,B\gamma,F\gamma) = 14$, $S(A\beta,A\beta,B\gamma) = 8$.
From (3.1), $8 \le d\max\{14,14,14,8\}$, which shows $\frac{4}{7} \le d < 1$.

Case (ii): If $\alpha,\beta,\gamma \in (4,9]$.
Then, $A\alpha = \frac{5}{2}$, $A\beta = \frac{5}{2}$, $B\gamma = \frac{5}{2}$, $E\alpha = \frac{\alpha+1}{2}$, $E\beta = \frac{\beta+1}{2}$, $F\gamma = \frac{\gamma+1}{2}$, $S(A\alpha,A\beta,B\gamma) = 5$, $S(E\alpha,E\beta,F\gamma) = 10$, $S(A\alpha,A\alpha,E\alpha) = 10$, $S(B\gamma,B\gamma,F\gamma) = 10$, $S(A\beta,A\beta,B\gamma) = 5$.
From (3.1), $5 \le d\max\{10,10,10,5\}$, which shows $\frac{1}{2} \le d < 1$.
Similarly, the other cases can be checked with suitable modifications wherever they are necessary. Clearly, $\frac{5}{2}$ is a unique common fixed point of A,B,E and F in M.

Competing Interests

The author declares that he has no competing interests.

Authors' Contributions

The author wrote, read and approved the final manuscript.

References

[1] B.C. Dhage, Generalized metric spaces and mappings with fixed point, *Bulletin of the Calcutta Mathematical Society* **84** (1992), 329 – 336.

[2] S. Gähler, 2-metrische Räume und ihre topologische Struktur, *Mathematische Nachrichten* **26**(1-4) (1963), 115 – 148, DOI: 10.1002/mana.19630260109.

[3] S. Gahler, Zur geometric 2-metriche raume, *Revue Roumaine de Mathematiques Pures et Appliquees* **11** (1966), 665 – 667.

[4] G. Jungck, Common fixed points for non-continuous non self maps on non metric spaces, *Far East Journal of Mathematical Sciences* **4** (1996), 199 – 215.

[5] G. Jungck, Compatible mappings and common fixed points, *International Journal of Mathematics and Mathematical Sciences* **9(4)** (1986), 771 – 779, DOI: 10.1155/S0161171286000935.

[6] G. Jungck and B. E. Rhoades, Fixed point for set valued functions without continuity, *Indian Journal of Pure and Applied Mathematics* **29(3)** (1998), 227 – 238.

[7] Z. Mustafa and B. Sims, Some remarks concerning D-metric spaces, *Proceedings of the International Conference on Fixed Point Theory and its Applications* (2003), 189 – 198, URL: https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=b403e87dcede715031c6355182d997f89beb7bf1.

[8] Z. Mustafa and B. Sims, A new approach to generalized metric spaces, *Journal of Nonlinear and Convex Analysis* **7**(2006), 289 – 297.

[9] S. Sedghi, I. Altun, N. Shobe and M.A. Salahshour, Some properties of S-metric spaces and fixed point results, *Kyungpook Mathematical Journal* **54(1)** (2014), 113 – 122, DOI: 10.5666/KMJ.2014.54.1.113.

[10] S. Sedghi and N.V. Dung, Fixed point theorems on S-metric spaces, *Matematicki Vesnik* **66** (2014), 113 – 124.

[11] S. Sedghi, N. Shobe and A. Aliouche, A generalization of fixed point theorems in S-metric spaces, *Matematicki Vesnik* **64** (2012), 258 – 226.

[12] S. Sedghi, N. Shobe and H. Zhou, A common fixed point theorem in D^*-metric spaces, *Fixed Point Theory and Applications* **2007** (2007), 27906, 13 pages, URL: https://fixedpointtheoryandalgorithms.springeropen.com/articles/10.1155/2007/27906.

[13] S. Sedghi, N. Shobkolaei, M. Shahraki and T. Došenović, Common fixed point of four maps in S-metric spaces, *Mathematical Sciences* **12** (2018), 137 – 143, DOI: 10.1007/s40096-018-0252-6.