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Research Article

Pendant Total Domination Polynomial of Some Families of Standard Graphs

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Abstract. In this article, our aim is to determine the pendant total domination polynomial of some families of standard graphs and obtain some properties of coefficients and nullity of the pendant total domination polynomial of a connected graph $\mathcal{G}$. Consider $\mathcal{G}$ as a simple connected graph and its vertex and edge sets are defined as $\mathcal{V}_{\mathcal{G}}$ and $\mathcal{E}_{\mathcal{G}}$, respectively. A set $\mathcal{T} \subseteq \mathcal{V}_{\mathcal{G}}$ is said to be a total dominating set of graph $\mathcal{G}$ if all the vertices of the graph must attached with some vertex of $\mathcal{T}$. A set $\mathcal{T} \subseteq \mathcal{V}_{\mathcal{G}}$ is said to be a *PTDS* if $\mathcal{T}$ is a *TDS* and $\langle \mathcal{T} \rangle$ contains at least a single pendant vertex.

Keywords. Dominating Set (*DS*), Total Dominating Set (*TDS*), Pendant Total Domination (*PTD*), Pendant Total Dominating Set (*PTDS*), Pendant Total Domination Number (*PTDN*)

Mathematics Subject Classification (2020). 05C69

1. Introduction

In this work graph $\mathcal{G}$ is considered as an undirected graph which does not contain loops and parallel edges. Vertex and edge sets are defined by $\mathcal{V}_{\mathcal{G}}$ and $\mathcal{E}_{\mathcal{G}}$, respectively. The concept of domination polynomial first defined by Arocha and Llano [3] in their research paper and then Alikhani and Peng [2] gave the definition for domination polynomial of a graph of order n which is given as: $D(\mathcal{G},x) = \sum_{i=\gamma(\mathcal{G})}^{n} d(\mathcal{G},i)x^i$, where $d(\mathcal{G},i)$ shows the total number of dominating sets of graph $\mathcal{G}$ of cardinality i.

Cockayne *et al.* [5] introduced the definition of total domination, and later in 2014 Chaluvaraju and Chaitra [4] defined the total domination polynomial which is given as: "*A total* *domination polynomial of graph $\mathcal{G}$ of order n is the polynomial $D_{td}(\mathcal{G},x) = \sum_{t=\gamma_{td}(\mathcal{G})}^{n} d_{td}(\mathcal{G},t)x^t$, where $d_{td}(\mathcal{G},t)$ is the number of total dominating sets of graph $\mathcal{G}$ of cardinality t*".

After that Nayaka [7, 8] initiated the concept of pendant domination in graph and then pendant domination polynomial of a graph. Motivated by this concept, we introduced a new parameter called pendant total domination in graph and determined the pendant total domination number of some generalized graphs (Rani and Mehra [9]). Likewise, here we define the pendant total domination polynomial of a graph which is stated as follows: A pendant total domination polynomial is a polynomial which is defined as $D_{pt}(\mathcal{G},x) = \sum_{k=\gamma_{pt}(\mathcal{G})}^{n} d_{pt}(\mathcal{G},k)x^k$, where $d_{pt}(\mathcal{G},k)$ represents the total number of pendant total dominating sets of cardinality k.

For the notion of nullity we refer Gutman and Borovićanin [6], and Chaluvaraju and Chaitra [4], "*The number of eigenvalues of $\mathcal{G}$ that are equal to zero is called the nullity of the graph $\mathcal{G}$ and the multiplicity of the zero in $D(\mathcal{G},x)$ is called the nullity of domination polynomial. It is denoted by $\eta = \eta(D(\mathcal{G},x))$*".

Let the vertex set of graph $\mathcal{G}$ is $\{v_1, v_2, \dots, v_n\}$ in which we add n new vertices $\{u_1, u_2, \dots, u_n\}$ and join each u_i to v_i. This represents the graph of $\mathcal{G} \circ K_1$ whose order is $2n$ and for corona of any two graphs refer [1].

If we add an edge between a complete graph k_n and path P_1 then the resulted graph is said to be the lollipop graph $L_{n,1}$.

2. Main Results

Definition 2.1. Let $D_{pt}(\mathcal{G},k)$ is the set of pendant total dominating sets of graph $\mathcal{G}$ of cardinality k and suppose $d_{pt}(\mathcal{G},k) = |D_{pt}(\mathcal{G},k)|$. Then a polynomial is said to be a pendant total domination polynomial of a graph of order n which is defined as $D_{pt}(\mathcal{G},x) = \sum_{k=\gamma_{pt}(\mathcal{G})}^{n} d_{pt}(\mathcal{G},k)x^k$, where $d_{pt}(\mathcal{G},k)$ denotes the total number of pendant total dominating sets of cardinality k in $D_{pt}(\mathcal{G},k)$.

Example 2.2. Let $\mathcal{G}$ be a graph as in Figure 1.

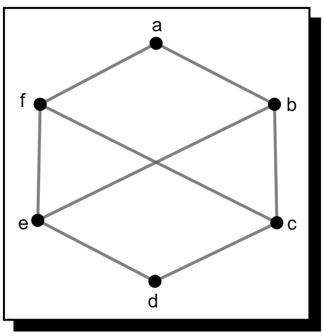

Figure 1. Graph $\mathcal{G}$

Since *PTDS* does not contain any isolated vertex, then clearly there is no *PTDS* of cardinality one.

Therefore, $d_{pt}(\mathcal{G},1)=0$,

$$D_{pt}(\mathcal{G},2)=\{\{b,c\},\{b,e\},\{c,f\},\{e,f\}\}.$$

Therefore, $d_{pt}(\mathcal{G},2)=|D_{pt}(\mathcal{G},2)|=4$

$$D_{pt}(\mathcal{G},3)=\{\{a,b,c\},\{a,b,e\},\{a,b,f\},\{a,c,f\},\{a,e,f\},\{b,c,d\},\{b,c,f\},\{b,e,f\},\{b,d,e\},\{b,c,e\},\{c,d,e\},\{c,e,f\},\{c,d,f\},\{d,e,f\}\}.$$

Therefore, $d_{pt}(\mathcal{G},3)=|D_{pt}(\mathcal{G},3)|=14$

$$D_{pt}(\mathcal{G},4)=\{\{a,b,c,d\},\{a,b,d,e\},\{a,c,d,f\},\{a,d,e,f\}\}.$$

Therefore, $d_{pt}(\mathcal{G},4)=|D_{pt}(\mathcal{G},4)|=4$

$$D_{pt}(\mathcal{G},5)=\{\{a,b,c,d,e\},\{a,c,d,e,f\},\{a,b,d,e,f\},\{a,b,c,d,f\}\}.$$

Therefore, $d_{pt}(\mathcal{G},5)=|D_{pt}(\mathcal{G},5)|=14$.

According to definition of *PTDS*, there is no *PTDS* of cardinality 6. Therefore, $d_{pt}(\mathcal{G},6)=0$,

$$\begin{aligned}D_{pt}(\mathcal{G},x)&=\sum_{k=\gamma_{pt}(\mathcal{G})}^{n} d_{pt}(\mathcal{G},k)x^k\\&=\sum_{k=2}^{5} d_{pt}(\mathcal{G},k)x^k\\&=d_{pt}(\mathcal{G},2)x^2+d_{pt}(\mathcal{G},3)x^3+d_{pt}(\mathcal{G},4)x^4+d_{pt}(\mathcal{G},5)x^5\\&=4x^2+14x^3+4x^2+4x^5.\end{aligned}$$

Now, we find the *TD* polynomial of same graph $\mathcal{G}$. Since *TDS* does not contain any isolated vertex then there is no *TDS* of single vertex.

Therefore, $d_{td}(\mathcal{G},1)=0$

$$D_{td}(\mathcal{G},2)=\{\{b,c\},\{b,e\},\{c,f\},\{e,f\}\}.$$

Therefore, $d_{td}(\mathcal{G},2)=|D_{td}(\mathcal{G},2)|=4$

$$D_{td}(\mathcal{G},3)=\{\{a,b,c\},\{a,b,e\},\{a,b,f\},\{a,c,f\},\{a,e,f\},\{b,c,d\},\{b,c,f\},\{b,e,f\},\{b,d,e\},\{b,c,e\},\{c,d,e\},\{c,e,f\},\{c,d,f\},\{d,e,f\}\}.$$

Therefore, $d_{td}(\mathcal{G},3)=|D_{td}(\mathcal{G},3)|=14$

$$D_{td}(\mathcal{G},4)=\{\{a,b,c,d\},\{a,b,c,f\},\{a,b,e,f\},\{a,b,d,e\},\{a,c,d,f\},\{a,d,e,f\},\{b,c,d,e\},\{b,c,e,f\},\{c,d,e,f\}\}.$$

Therefore, $d_{pt}(\mathcal{G},4)=|D_{pt}(\mathcal{G},4)|=9$

$$D_{td}(\mathcal{G},5)=\{\{a,b,c,d,e\},\{a,c,d,e,f\},\{a,b,d,e,f\},\{a,b,c,d,f\},\{a,b,c,e,f\},\{b,c,d,e,f\}\}.$$

Therefore, $d_{pt}(\mathcal{G},5)=|D_{pt}(\mathcal{G},5)|=6$

$$D_{td}(\mathcal{G},6)=\{\{a,b,c,d,e,f\}\}.$$

Therefore, $d_{pt}(\mathcal{G},6)=|D_{pt}(\mathcal{G},6)|=1$

$$D_{td}(\mathcal{G},x)=\sum_{k=\gamma_{td}(\mathcal{G})}^{n} d_{td}(\mathcal{G},k)x^k$$

$$= \sum_{k=2}^{5} d_{td}(\mathcal{G},k)x^k$$
$$= d_{td}(\mathcal{G},2)x^2 + d_{td}(\mathcal{G},3)x^3 + d_{td}(\mathcal{G},4)x^4 + d_{td}(\mathcal{G},5)x^5 + d_{td}(\mathcal{G},6)x^6$$
$$= 4x^2 + 14x^3 + 9x^2 + 6x^5 + x^6.$$

Remark 2.3. For the given graph $\mathcal{G}$ in Figure 1, *TD* polynomial and *PTD* polynomial are not same. In fact, *PTD* polynomial is a five degree polynomial and *TD* polynomial is a six degree polynomial.

Theorem 2.4. *Let us consider a connected graph $\mathcal{G}$ (order $n \geq 2$) and $\{v_1, v_2, \dots, v_n\}$ be its vertex set, then*

(i) $d_{pt}(\mathcal{G},n) = \begin{cases} 1, & \text{if } \mathcal{G} \text{ has a pendant vertex,} \\ 0, & \text{if } \mathcal{G} \text{ has no pendant vertex.} \end{cases}$

(ii) *For any connected graph $\mathcal{G}$, $d_{pt}(\mathcal{G},1) = 0$.*

(iii) $d_{pt}(\mathcal{G},i) = 0$ *iff* $i < \gamma_{pt}(\mathcal{G})$ *or* $i > n$.

(iv) $D_{pt}(\mathcal{G},x)$ *has no constant term and no first degree term.*

(v) *If $\mathcal{G}$ contains a pendant vertex, then* $\deg(D_{pt}(\mathcal{G},x)) = n$.

(vi) *If $\mathcal{G}$ does not contain any pendant vertex, then* $\deg(D_{pt}(\mathcal{G},x)) < n$.

(vii) 0 *is the zero of $D_{pt}(\mathcal{G},x)$ of order $\gamma_{pt}(\mathcal{G})$.*

(viii) *If $\mathcal{G}$ contains a pendant vertex, then* $\deg(D_{td}(\mathcal{G},x)) = \deg(D_{pt}(\mathcal{G},x))$.

(ix) *If $\mathcal{G}$ does not contain any pendant vertex, then* $\deg(D_{td}(\mathcal{G},x)) > \deg(D_{pt}(\mathcal{G},x))$.

(x) $D_{pt}(\mathcal{G},x)$ *is a strictly increasing function.*

Proof. (i) There is only one way to select n vertices because $\mathcal{G}$ has n vertices and degree of at least one vertex is one. Thus, there is only one *PTDS* of cardinality n. But if $\mathcal{G}$ has no pendant vertex then by definition there is no *PTDS* of cardinality n.

(ii) As *PTDS* does not contains an isolated vertex. Therefore, $d_{pt}(\mathcal{G},1) = 0$.

(iii) $d_{pt}(\mathcal{G},i) = 0$ since $\gamma_{pt}(\mathcal{G})$ is a minimum *PTDN* so there is no *PTDS* exist whose cardinality is less than $\gamma_{pt}(\mathcal{G})$ and also the order of graph is n so maximum cardinality of *PTDS* is n. Hence $d_{pt}(\mathcal{G},i) = 0$.

(iv) A single vertex can not totally dominates itself. So, at least two vertices needs to totally dominate the vertices of $\mathcal{G}$. Hence, the *TD* polynomial has no constant term as well as no first degree term. Therefore, *PTD* polynomial also has no constant term as well as no first degree term.

(v) Since graph contains a pendant vertex, then by (i) it is clear that $d_{pt}(\mathcal{G},n) = 1$ and by definition of *PTD* polynomial a term x^n must present in the polynomial. Thus degree of *PTD* polynomial is n.

(vi) If $\mathcal{G}$ has no pendant vertex, then there is no *PTDS* of cardinality n. Thus $d_{pt}(\mathcal{G},n) = 0$. Hence $deg(D_{pt}(\mathcal{G},x)) < n$.

(vii) It is obvious by definition of *PTD* polynomial.

(viii) Since $\mathcal{G}$ has no pendant vertex, then by (iii) there is no *PTDS* of cardinality n but in case of *TDS* the vertex set of graph is always a *TDS*. This shows that $\deg(D_{td}(\mathcal{G},x)) > \deg(D_{pt}(\mathcal{G},x))$.

(ix) Since $\mathcal{G}$ has a pendant vertex, then from (i) the vertex set is always a *TDS* as well as *PTDS* of $\mathcal{G}$. Hence $\deg(D_{td}(\mathcal{G},x)) = \deg(D_{pt}(\mathcal{G},x))$.

(x) According to definition of *PTD* polynomial $D_{pt}(G,x)$. □

Lemma 2.5. *The* PTDN *of corona of two graphs* $\mathcal{G}$ (*any connected graph of order* n) *and* K_1 *is as*

$$\gamma_{pt}(\mathcal{G}\circ K_1) = \begin{cases} n, & \text{if } \mathcal{G} \text{ has a pendant vertex,} \\ n+1, & \text{if } \mathcal{G} \text{ has no pendant vertex.} \end{cases}$$

Proof. Let $\{v_1, v_2, \dots, v_n\}$ be the vertex set of a graph $\mathcal{G}$ and degree of at least one vertex is one. Now let $\mathcal{T} = \{v_1, v_2, \dots, v_n\}$, then $\mathcal{T}$ will be the *PTDS* of $\mathcal{G}\circ K_1$ as vertices of $\mathcal{T}$ totally dominates itself and $\{u_1, u_2, \dots, u_n\}$. Also, the subgraph induced by $\mathcal{T}$ has a pendant vertex. Hence $\gamma_{pt}(\mathcal{G}\circ K_1) = n$.

If the graph $\mathcal{G}$ has no pendant vertex, then $\mathcal{T}$ must be a *TDS* but not a *PTDS* as we know the subgraph $\langle\mathcal{T}\rangle$ has no pendant vertex. Thus to construct a *PTDS* we required to add a vertex with vertices of $\mathcal{T}$ which is attached with any vertex of $\mathcal{T}$ then there will be $n+1$ vertices in *PTDS*. Hence $\gamma_{pt}(\mathcal{G}\circ K_1) = n+1$. □

Theorem 2.6. *The* PTD *polynomial of corona of two graphs* $\mathcal{G}$ *(any connected graph of order* n*) and* K_1 *is as*

$$D_{pt}(\mathcal{G}\circ K_1, x) = \begin{cases} x^n(1+x)^n, & \text{if } \mathcal{G} \text{ has a pendant vertex,} \\ x^n[(1+x)^n - 1], & \text{if } \mathcal{G} \text{ has no pendant vertex.} \end{cases}$$

Proof. Let $\mathcal{G}\circ K_1$ is a graph of order $2n$ where $\mathcal{G}$ is a graph with vertex set $\{v_1, v_2, \dots, v_n\}$ and at least one of them is a pendant vertex. Let $\mathcal{T} = \{v_1, v_2, \dots, v_n\}$ then by using Lemma 2.5 $\mathcal{T}$ will be the *PTDS* of $\mathcal{G}$ and $\gamma_{pt}(\mathcal{G}\circ K_1) = n$. If we consider an another set $\mathcal{T}_1 = \{u_1, u_2, \dots, u_n\}$ then this will not be a *PTDS* as it contains all isolated vertices. Thus there is only one *PTDS* of cardinality n and the *PTD* polynomial contains a term x^n. Now to construct *PTDS* of cardinality $n+1, n+2, \dots, 2n$ add vertices of $\mathcal{T}_1$ in $\mathcal{T}$ one by one. Hence there are $\binom{n}{k-n}$ ways to select $(k-n)$ vertices from n vertices of $\mathcal{T}_1$. Then the *PTD* polynomial of $\mathcal{G}\circ K_1$ will be

$$\begin{aligned} D_{pt}(\mathcal{G}\circ K_1, x) &= \sum_{k=n}^{2n} d_{pt}(\mathcal{G}\circ K_1, k)x^k \\ &= x^n + \binom{n}{1}x^{n+1} + \binom{n}{2}x^{n+2} + \dots + \binom{n}{n}x^{2n} \\ &= x^n\left[1 + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n\right] \\ &= x^n[(1+x)^n]. \end{aligned}$$

Now consider graph $\mathcal{G}$ has no pendant vertex. Then by Lemma 2.5, $\gamma_{pt}(\mathcal{G}\circ K_1) = n+1$ such that for $k \le n, d_{pt}(\mathcal{G},k) = 0$. Similarly to construct *PTD* sets of cardinality $n+1, n+2, \dots, 2n$ add vertices of $\mathcal{T}_1$ in $\mathcal{T}$ one by one. Hence there are $\binom{n}{k-n}$ ways to select $(k-n)$ vertices from n vertices of $\mathcal{T}_1$. Then, the *PTD* polynomial of $\mathcal{G}\circ K_1$ will be

$$\begin{aligned} D_{pt}(\mathcal{G}\circ K_1,x) &= \sum_{k=n+1}^{2n} d_{pt}(\mathcal{G}\circ K_1,k)x^k \\ &= \binom{n}{1}x^{n+1} + \binom{n}{2}x^{n+2} + \dots + \binom{n}{n}x^{2n} \\ &= x^n\left[\binom{n}{1}x^1 + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n\right] \\ &= x^n[(1+x)^n - 1]. \end{aligned}$$

□

Theorem 2.7. *The PTD polynomial of complete graph k_n is $\binom{n}{2}x^2$.*

Proof. Clearly, $\gamma_{pt}(\mathcal{G}) = 2$ and every edge of complete graph is a *PTDS* of cardinality 2. Thus $d_{pt}(k_n,2) = \binom{n}{2}$. Since every *TDS* of k_n having cardinality $k \ge 3$ does not contain any pendant vertex, then $d_{pt}(k_n,k) = 0$ for $k \ge 3$. Hence, $D_{pt}(k_n,x) = \binom{n}{2}x^2$. □

Theorem 2.8. *The PTD polynomial of a Lollipop graph $L_{n,1}$ is $x^2[n+(1+x)^{n-1}-1]$.*

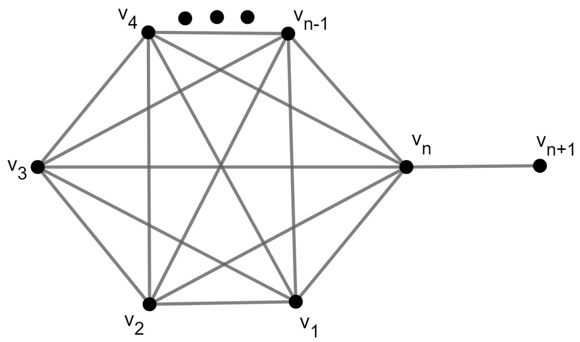

Figure 2. Lollipop graph $L_{n,1}$

Proof. According to definition of *PTDS*, it is easy to see that there is no *PTDS* of cardinality one and n *PTDS* of cardinality two namely $\{v_1,v_n\},\{v_2,v_n\},\dots,\{v_{n-1},v_n\},\{v_n,v_{n+1}\}$. Similarly, for *PTDS* of cardinality 3 we need to select two vertices v_n, v_{n+1} and select one vertex from $\{v_1,v_2,\dots,v_{n-1}\}$. Thus, there are $\binom{n-1}{1}$ *PTDS* of cardinality three. Proceeding in the same manner, we determine other *PTDS* of cardinality $4,5,\dots,n+1$.
Hence,

$$\begin{aligned} D_{pt}(L_{n,1},x) &= nx^2 + \binom{n-1}{1}x^3 + \binom{n-1}{2}x^4 + \dots + \binom{n-1}{n-1}x^{n+1} \\ &= nx^2 + x^2\left[\binom{n-1}{1}x^1 + \binom{n-1}{2}x^2 + \dots + \binom{n-1}{n-1}x^{n-1}\right] \end{aligned}$$

$$= nx^2 + x^2\left[\sum_{k=1}^{n-1}\binom{n-1}{k}x^k\right]$$
$$= x^2[n + (1+x)^{n-1} - 1]. \qquad \square$$

Theorem 2.9. *The pendant total domination polynomial of a complete bipartite graph $K_{n,n}$ of order $2n$ is $n^2x^2 + 2\binom{n}{1}\sum\limits_{k=2}^{n}\binom{n}{k}x^{1+k}$.*

Proof. Let V_1 and V_2 be the partite set of complete bipartite graph $K_{n,n}$. To construct $PTDS$, select at least one vertex from V_1 and at least one from V_2. Thus $\gamma_{pt}(K_{n,n}) = 2$. Now it is easy to see that there are n^2 $PTDS$ of cardinality two. To construct $PTDS$ of cardinality 3 we need to select one vertex from V_1 (or V_2) and other two from V_2 (or V_1). Therefore, there are $2\binom{n}{1}\binom{n}{2}$ $PTDS$ of cardinality three. Similarly, to construct $PTDS$ of cardinality 4 we need to select one vertex from V_1 (or V_2) and remaining vertices from V_2 (or V_1). Proceeding in the same manner, we determine other $PTDS$ of cardinality $5,\dots,n+1$ and $d_{pt}(K_{n,n},k) = 0$ for $\forall\ k > n+1$ as every TDS of cardinality greater than $n+1$ does not have any pendant vertex. Thus, we have

$$D_{pt}(K_{n,n},x) = n^2x^2 + 2\binom{n}{1}\binom{n}{2}x^3 + 2\binom{n}{1}\binom{n}{3}x^4 + \dots + 2\binom{n}{1}\binom{n}{n}x^{n+1}$$
$$= n^2x^2 + 2\binom{n}{1}\sum_{k=2}^{n}\binom{n}{k}x^{1+k}. \qquad \square$$

Theorem 2.10. *The pendant total domination polynomial of any connected graph of order $n \geq 2$ has nullity greater than or equal to two.*

Proof. Suppose $\mathcal{T} \subseteq \mathcal{V}_{\mathcal{G}}$ be a $PTDS$ of $\mathcal{G}$ then $\mathcal{T}$ must be a TDS and the subgraph induced by $\mathcal{T}$ contains atleast one pendant vertex. Let $|\mathcal{T}| = \gamma_{pt}(\mathcal{G})$. As per definition of TDS, $\mathcal{T}$ should have atleast two vertices which are adjacent to each other so that $\gamma_{td}(\mathcal{G}) \geq 2$ and also $\gamma_{pt}(\mathcal{G}) \geq 2$. In $D_{pt}(\mathcal{G},x)$, k ranges from $PTDN$ to order of $\mathcal{G}$ so that $deg(x) \geq 2$ in $D_{pt}(\mathcal{G},x)$. If we take $D_{pt}(\mathcal{G},x) = 0$, then $x = 0$ will be the zero of order 2 or more. Hence nullity of $D_{pt}(\mathcal{G},x)$ is greater than or equal to two. $\square$

3. Conclusion

We determined the pendant total dominating sets and pendant total domination polynomial of some standard graphs like complete graph, lollipop graph, complete bipartite graph and corona of a connected graph with k_1. Also, we studied some properties of pendant total domination polynomial and determined the lower bound for nullity of pandant total domination polynomial. Further, pendant total domination polynomial can also be determined for different graphs and different graph operations like total, cartesian, middle, join of graphs etc.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] S. Alikhani, On the domination polynomial of some graph operations, *International Scholarly Research Notices* **2013** (2013), Article ID 146595, 3 pages, DOI: 10.1155/2013/146595.

[2] S. Alikhani and Y. H. Peng, Introduction to Domination Polynomial of a Graph, *Ars Combinatoria* **114**(2014), 257 – 266, URL: http://combinatoire.ca/ArsCombinatoria/ACgetit.php?vol=114&paper=26.

[3] J. L. Arocha and B. Llano, Mean value for the matching and dominating polynomial, *Discussiones Mathematicae Graph Theory* **20**(1) (2000), 57 – 69, DOI: 10.7151/dmgt.1106.

[4] B. Chaluvaraju and V. Chaitra, Total domination polynomial of a graph, *Journal of Informatics and Mathematical Sciences* **6**(2) (2014), 87 – 92, DOI: 10.26713/jims.v6i2.256.

[5] E. J. Cockayne, R. M. Dawes and S. T. Hedetniemi, Total domination in graphs, *Networks* **10**(3) (1980), 211 – 219, DOI: 10.1002/net.3230100304.

[6] I. Gutman and B. Borovićanin, Nullity of graphs: An updated survey, *Zbornik Radova* **22** (2011), 137 – 154, URL: http://elib.mi.sanu.ac.rs/files/journals/zr/22/zbr14137.pdf.

[7] S. R. Nayaka, Puttaswamy and S. Purushothama, Pendant domination in graphs, *Journal of Combinatorial Mathematics and Combinatorial Computing* **112**(1) (2020), 219 – 230.

[8] S. R. Nayaka, Puttaswamy and S. Purushothama, Pendant domination polynomial of a graph, *International Journal of Pure and Applied Mathematics* **117**(11) (2017), 193 – 199, URL: https://acadpubl.eu/jsi/2017-117-11-14/articles/11/23.pdf.

[9] J. Rani and S. Mehra, Pendant total domination number of some generalized graphs, *Advances and Applications in Discrete Mathematics* **33** (2022), 19 – 44, DOI: 10.17654/0974165822036.