**Communications in Mathematics and Applications**
Vol. 13, No. 4, pp. 1307–1315, 2022
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v13i4.2173

http://www.rgnpublications.com

***Special Issue***
**Recent Advances in Pure and Applied Mathematics**
***Editors*: Thangaraj Beaula, J. Joseline Manora, D. Stephen Dinagar, D. Rajan**

Research Article

# Odd Prime Labeling of Graphs Related to Circular Ladder

S. Meena and G. Gajalakshmi*

*Department of Mathematics, Government Arts College, Chidambaram, Tamil Nadu, India*
***Corresponding author:** gaja61904@gmail.com

**Received:** April 15, 2022 **Accepted:** July 25, 2022

**Abstract.** In a graph $G$ with point set $V$ a mapping $f$ is said to be an odd prime labeling if $f$ is a one-to-one function from point set $V$ to $\{1,3,5,2|V|-1\}$ satisfying the condition that for each line $uv$ in $G$ the greatest common divisor of the labels of the end points $f(u),f(v)$ is one. Investigated in this paper the odd prime labeling of circular ladder related graphs and we prove that the graphs such as $CL(n)$, $SCL(n)$, $CL(n)\odot K_1$,$CL(n)\odot \bar{K}_2$, $CL(n)\odot \bar{K}_3$ are all odd prime graphs.

**Keywords.** Odd prime graph, Circular ladder, Subdivision, Corona product

**Mathematics Subject Classification (2020).** 05C78



## 1. Introduction

In this paper, we consider only simple and finite graphs with point set $V(G)$ and line set $E(G)$. For graph theoretical notations we refer Bondy and Murthy [1].

For entire survey of graph labelling we refer [4]. Several variations of graph labeling has been developed including prime labeling. Many researchers have studied about prime graphs [2,3,8]. Meena and Kavitha in [5] proved that some star related graphs are prime graphs (cf. Meena and Vaithilingam [6].

The concept of odd prime labeling was introduced by Prajapati and Shah [7] and they have proved that the many class of graphs are odd prime graphs. Investigate in this paper the existence of odd prime labeling for some new types of graphs $CL(n)$, $SCL(n)$, $CL(n)\odot K_1$, $CL(n)\odot \bar{K}_2$, $CL(n)\odot \overline{K}_3$ are all odd prime graphs.

**Definition 1.1.** For a graph $G$, a one-to-one mapping $f : V(G) \to O_{|V|}$ is said to be odd prime labeling if for each line $uv \in E$, greatest common divisor $(f(u), f(v))$ is one. A graph is called an odd prime labeling if which admits odd prime graph. Here $O_{|V|} = \{1,3,5,\dots 2|V| - n\}$.

**Definition 1.2.** Let $S_1$ and $S_2$ be any two graphs. The corona product of $S_1 \odot S_2$ is got by one copy of $S_1$ and $|V(S_1)|$ copies of $S_2$ and by joining each vertex of the $k$-th copy of $S_2$ to the $k$-th vertex of $S_1$ where $1 \le k \le |V(S_1)|$.

**Definition 1.3.** The circular ladder graph $CL_n$ is the Cartesian product $C_n \times P_2$, where $P_2$ is the path on two nodes and $C_n$ is the cycle on $n$ nodes.

## 2. Main Results

**Theorem 2.1.** *The circular ladder $CL(n)$ is an odd prime graph for all $n$.*

*Proof.* Let $V(CL(n)) = \{u_k, v_k/1 \le k \le n\}$.
$E(CL(n)) = \{u_k, v_k/1 \le k \le n\} \cup \{u_k u_{k+1}, v_k v_{k+1}/1 \le k \le n-1\} \cup \{u_1 u_n, v_1 v_n\}$.
Here $|V(CL(n)| = 2n$ and $|E(CL(n)| = 3n$.
Define a mapping $f$ from $V(G)$ to $O_{2n}$ as follows:

$$f(u_k) = 4k - 3 \quad \text{for } 1 \le k \le n,$$

$$f(v_k) = 4k - 1 \quad \text{for } 1 \le k \le n.$$

If $n \equiv 1 (\text{mod } 3)$ then interchange the labels of $u_1$ and $v_1$ so that $f(u_1) = 3$ and $f(v_1) = 1$.
Clearly, point labels are different with this labeling for each line $e \in E$ if $\gcd(f(u), f(v)) = 1$.
If $n \not\equiv 1 (\text{mod } 3)$ then for

(i) $e = u_k v_k$, $\gcd(f(u_k), f(v_k)) = \gcd(4k-3, 4k-1) = 1$ for $1 \le k \le n$.

(ii) $e = u_k u_{k+1}$, $\gcd(f(u_k), f(u_{k+1})) = \gcd(4k-3, 4k+1) = 1$ for $1 \le k \le n-1$.

(iii) $e = v_k v_{k+1}$, $\gcd(f(v_k), f(v_{k+1})) = \gcd(4k-1, 4k+3) = 1$ for $1 \le k \le n-1$.

(iv) $e = v_1 v_n$, $\gcd(f(v_1), f(v_n)) = \gcd(3, 4n-1) = 1$.

(v) $e = u_1 u_n$, $\gcd(f(u_1), f(u_n)) = \gcd(4k-3, 4k-1) = 1$.

If $n \equiv 1 (\text{mod } 3)$ then

(vi) $e = v_1 v_n$, $\gcd(f(v_1), f(v_n)) = \gcd(1, f(v_n)) = 1$.

(vii) $e = u_1 u_n$, $\gcd(f(u_1), f(u_n)) = \gcd(3, 4n-3) = 1$.

(viii) $e = u_1 u_2$, $\gcd(f(u_1), f(u_2)) = \gcd(3, 5) = 1$.

Thus $CL(n)$ is an odd prime graph. □

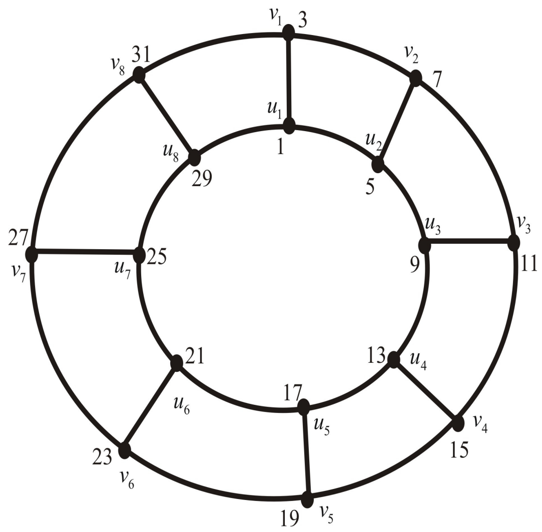


**Figure 1**

**Theorem 2.2.** *The Subdivision graph of a circular ladder $SCL(n)$ is an odd prime graph for all $n \geq 3$.*

*Proof.* Let $G$ be the subdivision graph of circular ladder $S(CL(n))$.

$$V(SCL(n)) = \{u_k, v_k, r_k, s_k, t_k / 1 \leq k \leq n\},$$

$$E(SCL(n)) = \{u_k r_k, r_k v_k, v_k s_k, u_k t_k / 1 \leq k \leq n\}.$$

Here $|V(SCL(n))| = 5n$ and $|E(SCL(n))| = 6n$.
Define a mapping $f$ from $V(G)$ to $O_{5n}$ as follows:

$$f(u_k) = 10k - 9 \quad \text{for } 1 \leq k \leq n,$$

$$f(v_k) = 10k - 5 \quad \text{for } 1 \leq k \leq n,$$

$$f(s_k) = 10k - 3 \quad \text{for } 1 \leq k \leq n,$$

$$f(r_k) = 10k - 7 \quad \text{for } 1 \leq k \leq n,$$

$$f(t_k) = 10k - 1 \quad \text{for } 1 \leq k \leq n.$$

Clearly, the point labels are different with this labeling for each line $e \in E$. The greatest common divisor $(f(u), f(v)) = 1$.

(i) $e = u_k r_k, \gcd(f(u_k), f(r_k)) = \gcd(10k - 9, 10k - 7) = 1$ for $1 \leq k \leq n$.

(ii) $e = v_k r_k, \gcd(f(v_k), f(r_k)) = \gcd(10k - 5, 10k - 7) = 1$ for $1 \leq k \leq n$.

(iii) $e = v_k s_k, \gcd(f(v_k), f(s_k)) = \gcd(10k - 5, 10k - 3) = 1$ for $1 \leq k \leq n$.

(iv) $e = u_k t_k, \gcd(f(u_k), f(t_k)) = \gcd(10k - 9, 10k - 1) = 1$ for $1 \leq k \leq n$.

(v) $e = s_k v_{k+1}, gcd(f(s_k), f(v_{k+1})) = \gcd(10k - 3, 10k + 5) = 1$ for $1 \leq k \leq n - 1$.

(vi) $e = t_k u_{k+1}, gcd(f(y_k), f(u_{k+1})) = \gcd(10k - 1, 10k + 1) = 1$ for $1 \leq k \leq n - 1$.

(vii) $e = u_1 t_n, gcd(f(u_1), f(t_n) = \gcd(1, f(t_n) = 1$.

(viii) $e = v_1 s_n, gcd(f(v_1), f(s_n) = \gcd(5, f(s_n) = 1.$

Thus $SCL(n)$ is an odd prime graph. □

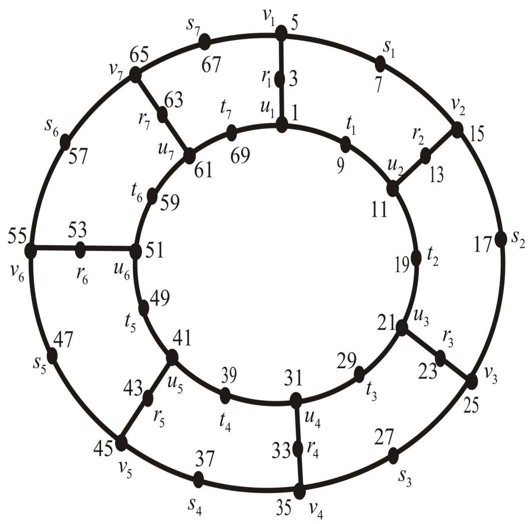


**Figure 2**

**Theorem 2.3.** *The carona product of circular ladder $CL(n) \odot K_1$ is an odd prime graph for all $n$.*

*Proof.* Let $V(CL(n)) = \{u_k, v_k, x_k, y_k / 1 \le k \le n\}$.

$E(CL(n)) = \{u_k v_k, u_k x_k, v_k y_k / 1 \le k \le n\} \cup \{u_k u_{k+1}, v_k v_{k+1} / 1 \le k \le n-1\}$.

Define a mapping $f$ from $V(G)$ to $O_{4n}$ as follows:

$$f(u_k) = 8k - 7 \quad \text{for } 1 \le k \le n,$$

$$f(v_k) = 8k - 3 \quad \text{for } 1 \le k \le n,$$

$$f(x_k) = 8k - 5 \quad \text{for } 1 \le k \le n,$$

$$f(y_k) = 8k - 1 \quad \text{for } 1 \le k \le n.$$

Clearly, the point labels are different with this labeling for each line $e \in E$.

The greatest common divisor $(f(u), f(v)) = 1$.

(i) $e = u_k v_k, \gcd(f(u_k), f(v_k)) = \gcd(8k-7, 8k-3) = 1$ for $1 \le k \le n$.

(ii) $e = u_k x_k, \gcd(f(u_k), f(x_k)) = \gcd(8k-7, 8k-5) = 1$ for $1 \le k \le n$.

(iii) $e = v_k y_k, \gcd(f(v_k), f(y_k)) = \gcd(8k-3, 8k-1) = 1$ for $1 \le k \le n$.

(iv) $e = u_k u_{k+1}, \gcd(f(u_k), f(u_{k+1})) = \gcd(8k-7, 8k+1) = 1$ for $1 \le k \le n-1$.

(v) $e = v_k v_{k+1}, \gcd(f(v_k), f(v_{k+1})) = \gcd(8k-3, 8k+5) = 1$ for $1 \le k \le n-1$.

Thus $CL(n) \odot K_1$ is an odd prime graph. □

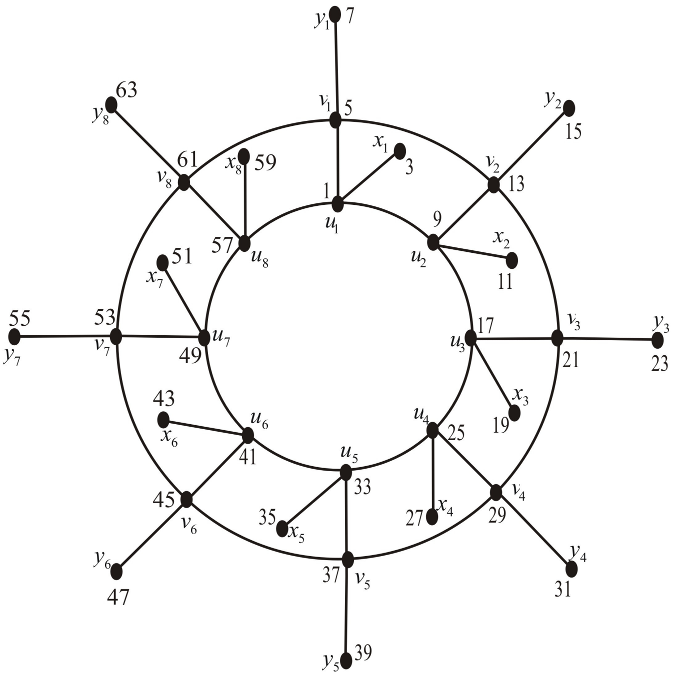


**Figure 3**

**Theorem 2.4.** *The carona product of circular ladder $CL(n)\bigodot\bar{K}_2$ is an odd prime graph for all $n$.*

*Proof.* Let $V(CL(n)) = \{u_k, v_k, x_k, y_k, p_k, q_k / 1 \le k \le n\}$.
$E(CL(n)) = \{u_kv_k, u_kx_k, v_ky_k, v_kp_k, v_kq_k / 1 \le k \le n\} \cup \{u_ku_{k+1}, v_kv_{k+1} / 1 \le k \le n-1\}$.
Define a mapping $f$ from $V(G)$ to $O_{6n}$ as follows:

$$f(u_k) = 12k - 11 \quad \text{for } 1 \le k \le n,$$

$$f(v_k) = 12k - 5 \quad \text{for } 1 \le k \le n,$$

$$f(x_k) = 12k - 9 \quad \text{for } 1 \le k \le n,$$

$$f(y_k) = 12k - 7 \quad \text{for } 1 \le k \le n,$$

$$f(p_k) = 12k - 3 \quad \text{for } 1 \le k \le n,$$

$$f(q_k) = 12k - 1 \quad \text{for } 1 \le k \le n.$$

Clearly, the point labels are different with this labeling for each line $e \in E$.
The greatest common divisor $(f(u), f(v)) = 1$.

(i) $e = u_kv_k$, $\gcd(f(u_k), f(v_k)) = \gcd(12k-11, 12k-5) = 1$ for $1 \le k \le n$.

(ii) $e = u_kx_k$, $\gcd(f(u_k), f(x_k)) = \gcd(12k-11, 12k-9) = 1$ for $1 \le k \le n$.

(iii) $e = v_ky_k$, $\gcd(f(v_k), f(y_k)) = \gcd(12k-5, 12k-7) = 1$ for $1 \le k \le n$.

(iv) $e = v_kp_k$, $\gcd(f(v_k), f(p_k)) = \gcd(12k-5, 12k-3) = 1$ for $1 \le k \le n$.

(v) $e = v_kq_k$, $\gcd(f(v_k), f(q_k)) = \gcd(12k-5, 12k-1) = 1$ for $1 \le k \le n$.

(vi) $e = u_ku_{k+1}$, $\gcd(f(u_k), f(u_{k+1})) = \gcd(12k-11, 12k+1) = 1$ for $1 \le k \le n-1$.

(vii) $e = v_k v_{k+1}$, $\gcd(f(v_k), f(v_{k+1})) = \gcd(12k-5, 12k+7) = 1$ for $1 \le k \le n-1$.

Thus $CL(n) \odot \bar{K}_2$ is an odd prime graph. □

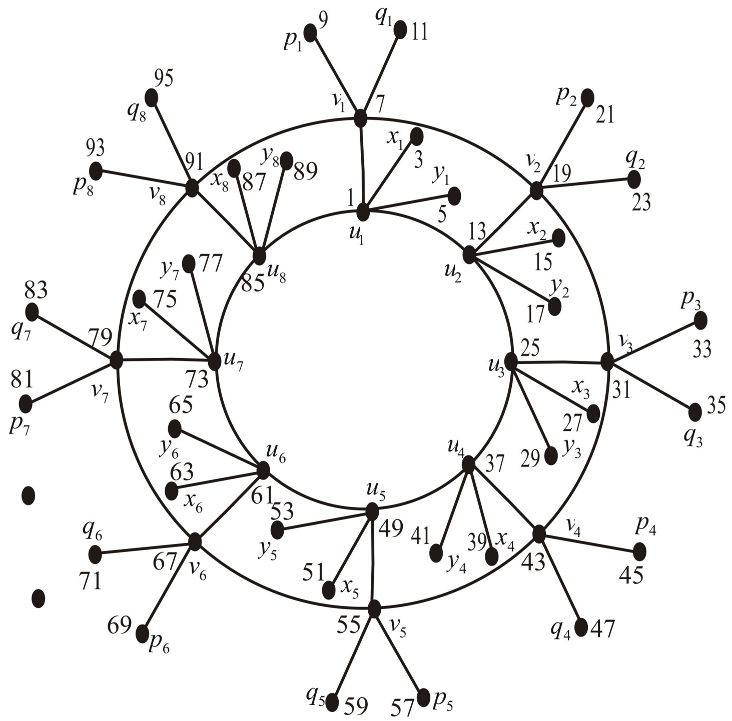


**Figure 4**

**Theorem 2.5.** *The carona product of circular ladder $CL(n) \odot \bar{K}_3$ is an odd prime graph.*

*Proof.* Let $V(CL(n)) = \{u_k, v_k, x_k, y_k, z_k, p_k, q_k, r_k / 1 \le k \le n\}$.

$E(CL(n)) = \{u_k v_k, u_k x_k, v_k y_k, u_k z_k, v_k p_k, v_k q_k, v_k r_k / 1 \le k \le n\} \cup \{u_k u_{k+1}, v_k v_{k+1} / 1 \le k \le n-1\}$.

Define a mapping $f$ from $V(G)$ to $O_{8n}$ as follows:

$f(u_k) = 16k - 15$ for $1 \le k \le n$, $k \not\equiv 0 \pmod 3$

$f(u_k) = 16k - 13$ for $1 \le k \le n$, $k \equiv 0 \pmod 3$

$f(v_k) = 16k - 5$ for $1 \le k \le n$,

$f(x_k) = 16k - 13$ for $1 \le k \le n$, $k \not\equiv 0 \pmod 3$

$f(x_k) = 16k - 15$ for $1 \le k \le n$, $k \equiv 0 \pmod 3$

$f(y_k) = 16k - 11$ for $1 \le k \le n$,

$f(z_k) = 16k - 9$ for $1 \le k \le n$,

$f(p_k) = 16k - 7$ for $1 \le k \le n$,

$f(q_k) = 16k - 3$ for $1 \le k \le n$,

$f(r_k) = 16k - 1$ for $1 \le k \le n$.

Clearly, the point labels are different with this labeling for each line $e \in E$.

The greatest common divisor $(f(u), f(v)) = 1$.

(i) $e = u_k v_k$, $\gcd(f(u_k), f(v_k)) = \gcd(16k-15, 16k-5) = 1$ for $1 \le k \le n$, $k \not\equiv 0 (\text{mod } 3)$.

(ii) $e = u_k v_k$, $\gcd(f(u_k), f(v_k)) = \gcd(16k-15, 16k-5) = 1$ for $1 \le k \le n$, $i \equiv 0 (\text{mod } 3)$.

(iii) $e = u_k x_k$, $\gcd(f(u_k), f(x_k)) = \gcd(16k-15, 16k-13) = 1$ for $1 \le k \le n$, $i \not\equiv 0 (\text{mod } 3)$.

(iv) $e = u_k x_k$, $\gcd(f(u_k), f(x_k)) = \gcd(16k-13, 16k-15) = 1$ for $1 \le k \le n$, $i \equiv 0 (\text{mod } 3)$.

(v) $e = u_k y_k$, $\gcd(f(u_k), f(y_k)) = \gcd(16k-15, 16k-11) = 1$ for $1 \le k \le n$, $i \not\equiv 0 (\text{mod } 3)$.

(vi) $e = u_k y_k$, $\gcd(f(u_k), f(y_k)) = \gcd(16k-13, 16k-11) = 1$ for $1 \le k \le n$, $i \not\equiv 0 (\text{mod } 3)$.

(vii) $e = u_k z_k$, $\gcd(f(u_k), f(z_k)) = \gcd(16k-15, 16k-9) = 1$ for $1 \le k \le n$, $i \not\equiv 0 (\text{mod } 3)$.

(viii) $e = u_k z_k$, $\gcd(f(u_k), f(z_k)) = \gcd(16k-13, 16k-9) = 1$ for $1 \le k \le n$, $i \equiv 0 (\text{mod } 3)$.

(ix) $e = v_k p_k$, $\gcd(f(v_k), f(p_k)) = \gcd(16k-5, 16k-7) = 1$ for $1 \le k \le n$.

(x) $e = v_k q_k$, $\gcd(f(v_k), f(q_k)) = \gcd(16k-5, 16k-13) = 1$ for $1 \le k \le n$.

(xi) $e = v_k r_k$, $\gcd(f(v_k), f(r_k)) = \gcd(16k-5, 16k-1) = 1$ for $1 \le k \le n$.

(xii) $e = u_k u_{k+1}$, $\gcd(f(u_k), f(u_{k+1})) = \gcd(16k-15, 16k+3) = 1$ for $1 \le k \le n-1$.

(xiii) $e = u_k u_{k+1}$, $\gcd(f(u_k), f(u_{k+1})) = \gcd(16k-13, 16k+3) = 1$ for $1 \le k \le n-1$.

(xiv) $e = v_k v_{k+1}$, $\gcd(f(v_k), f(v_{k+1})) = \gcd(16k-5, 16k+1) = 1$ for $1 \le k \le n-1$.

Thus $CL(n) \odot \bar{K}_3$ is an odd prime graph. □

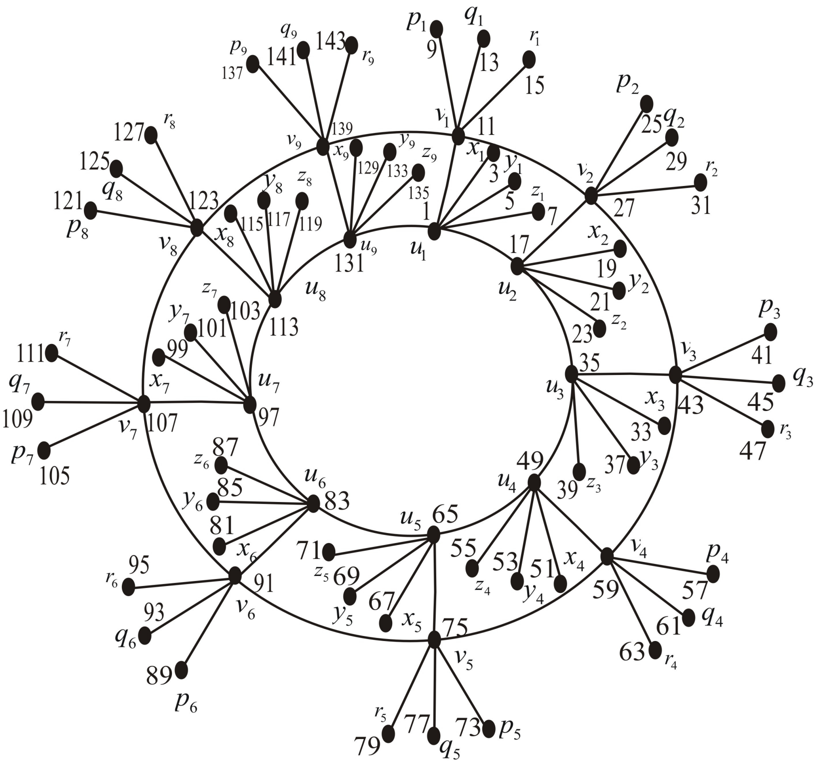


**Figure 5**

## 3. Conclusion

The odd prime labelings of various classes of graphs such as $CL(n)$, $SCL(n)$, $CL(n)\odot K_1$, $CL(n)\odot \bar{K}_2$, $CL(n)\odot \bar{K}_3$ were investigated. To derive similar results for other graph families and other graph labelings in an open area research.

## Acknowledgment

The authors are thankful to the referees for their kind suggestions and comments.

### Competing Interests

The authors declare that they have no competing interests.

### Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

## References

[1] J. A. Bondy and U. S. R. Murty, *Graph Theory and Applications*, North-Holland, New York (1976), URL: https://www.zib.de/groetschel/teaching/WS1314/BondyMurtyGTWA.pdf.

[2] T. Deretsky, S. M. Lee and J. Mitchem, On vertex prime labelings of graphs, in: *Graph Combinatorics and Applications*, *J. Alavi, G. Chartrand, O. Oellerman and A. Schwenk* (eds.), *Proceedings of the 6th International Conference Theory and Applications of Graphs*, Wiley, New York, Vol. **1** (1991), 359 – 369.

[3] H.-L. Fu and K.-C. Huang, On prime labeling, *Discrete Mathematics* **127** (1994), 181 – 186, URL: https://www.math.nycu.edu.tw/research/DOWNLOAD_FILES/hlfu/34.%20On%20prime%20labeling.pdf.

[4] J. A. Gallian, A dynamic survey of graph labeling, *The Electronic Journal of Combinations* **16** (2009), DS6, URL: https://www.combinatorics.org/ojs/index.php/eljc/article/view/DS6/pdf.

[5] S. Meena and P. Kavitha, Prime labeling of duplication of some star related graphs, *International Journal of Mathematics Trends and Technology* **23**(1) (2015), 26 – 32, DOI: 10.14445/22315373/IJMTT-V23P504.

[6] S. Meena and K. Vaithilingam, Prime labeling for some crown related graphs, *International Journal of Scientific & Technology Research* **2**(3) (2013), 92 – 95, URL: https://www.ijstr.org/final-print/mar2013/Prime-Labeling-For-Some-Crown-Related-Graphs.pdf.

[7] U. M. Prajapati and K. P. Shah, On odd prime labeling, *International Journal of Research and Analytical Reviews* **5**(4) (2018), 284 – 294, URL: http://ijrar.com/upload_issue/ijrar_issue_20542373.pdf.

[8] A. Tout, A. N. Dabboucy and K. Howalla, Prime labeling of graphs, *National Academy Science Letters* **11** (1982), 365 – 368.

**[9]** S. K. Vaidya and K. K. Kanani, Prime labeling for some cycle related graphs, *Journal of Mathematics Research* **2**(2) (2010), 98 – 104, DOI: 10.5539/jmr.v2n2p98.