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Research Article

Product Cordial Labelling for Some Bicyclic Graphs

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Abstract. A graph G with lines and points is known as a product cordial graph if there occurs a labeling g from $V(G)$ to $\{0,1\}$ such that if every line rt is given the labeled $g(r).g(t)$, then the cardinality of points with labeled zero and the cardinality of points with labeled one vary as a maximum by one and the cardinality of lines with labeled zero and the cardinality of lines with labeled one vary by as a maximum one. In this case, g is alleged the product cordial labeling of G. This paper deals with product cordial labeling for some graphs related to bicyclic graph such as $B[n,n]$, $B[n,n]*S_m$, $B[n,n]*P_2*S_m$ and $B[n,n]*P_3*S_m$, $B[n,n]\odot K_2$, $B[n,n]\odot K_3$.

Keywords. Cordial labelling, Product cordial labelling, Bicyclic graph, Corona product

Mathematics Subject Classification (2020). 05C78

1. Introduction

In this paper, we tend to consider graphs that are simple and finite p points and q lines. For an in-depth check of labelling of graphs, we relate to Gallain [3]; and we use Harary [5] and Bondy and Murthy [1] for all other notations. The notion of product cordial labelling presented by Sundaram *et al*. [7]. Meena *et al*. [6] investigated the existence of prime labelling of bicyclic graphs. We investigated the product cordial labelling of some bicyclic graphs.

Definition 1.1 ([6]). If $B[n,n]$ is the bicyclic graph obtained from two point-disjoint cycles C_m and C_n by identifying two points r of C_m and t of C_n.

Definition 1.2 ([4]). A graph is called cordial if it's attainable to label its points with zeros and ones, so when the lines are labelled with the distinction of the labels at their finish points, the quantity of points (lines) labelled with ones and zeros disagree at the most by one.

Definition 1.3 ([4]). A map g from $V(G)$ to $\{0,1\}$ is known as binary labelling of G. A binary labelling with induced line labelling g^* from $E(G)$ to $\{0,1\}$ defined by g^* from $(e = rt)$ equal to $g(r).g(t)$ is called a product cordial labelling if the absolute difference of $v_f(0)$ and $v_f(1)$ is less than or equal to 1 and the Absolute difference of $e_f(0)$ and $e_f(1)$ is less than or equal to 1. A graph which admit product cordial labelling is said to be a product cordial graphs.

Definition 1.4 ([6]). The corona product of two graphs G and H is outlined as the graph got by taking one copy of G and cardinality of $V(G)$ copies of H and attaching the ith point of G to every point in the ith copy of H.

Definition 1.5 ([6]). Complete bipartite graph $K_{1,m}$ is called star graph S_m.

2. Main Results

The product cordial labelling for some bicyclic graphs, were investigated in this paper.

Theorem 2.1. *The bicyclic graph $B[n,n] * S_m$ is a product cordial graph.*

Proof. Let $r_1, r_2, \dots, r_n$ and $t_1, t_2, \dots, t_n$ be the points of bicyclic graph $B[n,n]$ with $r_1 = t_1$ be the common point.
Let $p_i^1, p_i^2, p_i^3, p_i^4, \dots, p_i^m$ be the pendent points of S_m attached at r_i for $1 \le i \le n$ and let $s_i^1, s_i^2, s_i^3, s_i^4, \dots, s_i^m$ be the pendent points of S_m attached at t_i for $2 \le i \le n$.
Define a labelling g from $V(G)$ to $\{0,1\}$ as follows:

$$g(r_1 = t_1) = 1,$$
$$g(r_i) = 0 \quad \text{for } 2 \le i \le n,$$
$$g(t_i) = 1 \quad \text{for } 2 \le i \le n,$$
$$g(p_i^j) = 0 \quad \text{for } 2 \le i \le n,\ 1 \le j \le m,$$
$$g(s_i^j) = 1 \quad \text{for } 2 \le i \le n,\ 1 \le j \le m.$$

If m is even

$$g(p_1^j) = 0 \quad \text{if } 1 \le j \le \frac{m}{2},$$
$$g(p_1^j) = 0 \quad \text{if } \frac{m}{2} + 1 \le j \le m.$$

If m is odd

$$g(p_1^j) = 0 \quad \text{if } 1 \le j \le \frac{m+1}{2},$$

$g(p_1^j) = 1 \quad \text{if } \frac{m+3}{2} \le j \le m.$

Absolute difference of $v_f(0)$ and $v_f(1)$ is less than or equal to 1 and the absolute difference of $e_f(0)$ and $e_f(1)$ is less than or equal to 1.

Thus, g is a product cordial graph. □

Theorem 2.2. *The bicyclic graph $B[n,n] * P_2 * S_m$ is a product cordial graph.*

Proof. Let $r_1, r_2, \dots, r_n$ and $t_1, t_2, \dots, t_n$ be the points of bicyclic graph $B[n,n]$ with $r_1 = t_1$ be the common point.

Let r_i' be the point of path P_2 attached at r_i for $1 \le i \le n$ and let t_i' be the point of path P_2 attached at t_i for $2 \le i \le n$.

Let $p_i^1, p_i^2, p_i^3, p_i^4, \dots, p_i^m$ be the pendent points of S_m attached at r_i' for $1 \le i \le n$ and let $s_i^1, s_i^2, s_i^3, s_i^4, \dots, s_i^m$ be the points of S_m attached at t_i for $2 \le i \le n$.

Define a labelling g from $V(G)$ to $\{0,1\}$ as follows:

$$g(r_1 = t_1) = 1,$$
$$g(r_i) = 0 \quad \text{for } 2 \le i \le n,$$
$$g(t_i) = 1 \quad \text{for } 2 \le i \le n,$$
$$g(r_i^1) = 0 \quad \text{for } 1 \le i \le n,$$
$$g(t_i^1) = 1 \quad \text{for } 2 \le i \le n,$$
$$g(p_i^j) = 0 \quad \text{for } 1 \le j \le m,\ 2 \le i \le n,$$
$$g(s_i^j) = 1 \quad \text{for } 1 \le j \le m,\ 2 \le i \le n.$$

If m is even

$$g(p_1^j) = 0 \quad \text{if } 1 \le j \le \frac{m}{2},$$
$$g(p_1^j) = 1 \quad \text{if } \frac{m}{2} + 1 \le j \le m.$$

If m is odd

$$g(p_1^j) = 0 \quad \text{if } 1 \le j \le \frac{m-1}{2},$$
$$g(p_1^j) = 1 \quad \text{if } \frac{m-1}{2} \le j \le m.$$

Absolute difference of $v_f(0)$ and $v_f(1)$ is less than or equal to 1 and the absolute difference of $e_f(0)$ and $e_f(1)$ is less than or equal to 1.

Thus g is a product cordial graph. □

Theorem 2.3. *The bicyclic graph $B[n,n] * P_3 * S_m$ is a product cordial graph.*

Proof. Let $r_1, r_2, \dots, r_n$ and $t_1, t_2, \dots, t_n$ be the points of bicyclic graph $B[n,n]$ with $r_1 = t_1$ be the common point.

Let r_i' and r_i'' be the points of path P_3 attached at r_i for $1 \le i \le n$ and let t_i' and t_i'' be the points of path P_3 attached at t_i for $2 \le i \le n$.

Let $p_i^1, p_i^2, p_i^3, p_i^4, \dots, p_i^m$ be the pendent points of S_m attached at r_i'' for $1 \le i \le n$ and let $s_i^1, s_i^2, s_i^3, s_i^4, \dots, s_i^m$ be the points of S_m attached at t_i'' for $2 \le i \le n$.

Define a labelling g from $V(G)$ to $\{0,1\}$ as follows:

$$g(r_1 = t_1) = 1,$$
$$g(r_i) = 0 \quad \text{for } 2 \le i \le n,$$
$$g(t_i) = 1 \quad \text{for } 2 \le i \le n,$$
$$g(r_1') = 0,\ g(r_1'') = 1,$$
$$g(r_i') = 0,\ g(r_i'') = 0 \quad \text{for } 2 \le i \le n,$$
$$g(t_i') = 0,\ g(t_i'') = 1 \quad \text{for } 2 \le i \le n,$$
$$g(p_i^j) = 0 \quad \text{for } 2 \le i \le n,\ 1 \le j \le m,$$
$$g(s_i^j) = 1 \quad \text{for } 2 \le i \le n,\ 1 \le j \le m.$$

If m is even

$$g(p_1^j) = 0 \quad \text{if } 1 \le i \le \frac{m}{2},$$
$$g(p_1^j) = 1 \quad \text{if } \frac{m}{2} + 1 \le i \le m.$$

If m is odd

$$g(p_1^j) = 0 \quad \text{if } 1 \le i \le \frac{m+1}{2},$$
$$g(p_1^j) = 1 \quad \text{if } \frac{m+3}{2} + 1 \le j \le m.$$

Absolute difference of $v_f(0)$ and $v_f(1)$ is less than or equal to 1 and the absolute difference of $e_f(0)$ and $e_f(1)$ is less than or equal to 1.

Thus g is a product cordial graph. □

Theorem 2.4. *The corona product of bicyclic graph $B[n,n] \odot K_2$ is a product cordial graph.*

Proof. Let $r_1, r_2, \dots, r_n$ and $t_1, t_2, \dots, t_n$ be the points of bicyclic graph $B[n,n]$ with $r_1 = t_1$ be the common point.

Let p_i^1, p_i^2 be the points of K_2 attached at r_i for $1 \le i \le n$ and let s_i^1, s_i^2 be the points of K_2 attached at t_i for $2 \le i \le n$.

Define a labelling g from $V(G)$ to $\{0,1\}$ as follows:

$$g(r_1 = t_1) = 1,$$
$$g(r_i) = 0 \quad \text{for } 2 \le i \le n,$$
$$g(t_i) = 1 \quad \text{for } 2 \le i \le n,$$
$$g(p_1^1) = 0,\ g(p_1^2) = 0,$$
$$g(p_i^1) = g(p_i^2) = 0 \quad \text{for } 2 \le i \le n,$$
$$g(s_i^1) = g(s_i^2) = 1 \quad \text{for } 2 \le i \le n.$$

Absolute difference of $v_f(0)$ and $v_f(1)$ is less than or equal to 1 and the absolute difference of $e_f(0)$ and $e_f(1)$ is less than or equal to 1. Thus g is a product cordial graph. □

Theorem 2.5. *The corona product of bicyclic graph $B[n,n]\odot K_3$ admits product cordial labeling.*

Proof. Let $r_1,r_2,\dots,r_n$ and $t_1,t_2,\dots,t_n$ be the points of bicyclic graph $B[n,n]$ with $r_1=t_1$ be the common point.

Let p_i^1,p_i^2,p_i^3 be the points of K_3 attached at r_i for $1\le i\le n$ and let s_i^1,s_i^2,s_i^3 be the points of K_3 attached at t_i for $2\le i\le n$.

Define a labelling g from $V(G)$ to $\{0,1\}$ as follows:

$$g(r_1=t_1)=1,$$
$$g(r_i)=0 \quad \text{for } 2\le i\le n,$$
$$g(t_i)=1 \quad \text{for } 2\le i\le n,$$
$$g(p_i^1)=0,$$
$$g(p_i^2)=1,$$
$$g(p_i^3)=1,$$
$$g(p_1^1)=f(p_1^2)=g(p_i^3)=0 \quad \text{for } 2\le i\le n,$$
$$g(s_i^1)=f(s_i^2)=g(s_i^2)=1 \quad \text{for } 2\le i\le n.$$

Absolute difference of $v_f(0)$ and $v_f(1)$ is less than or equal to 1 and the absolute difference of $e_f(0)$ and $e_f(1)$ is less than or equal to 1.

Thus g is a product cordial graph. □

3. Illustrations

Illustration 3.1.

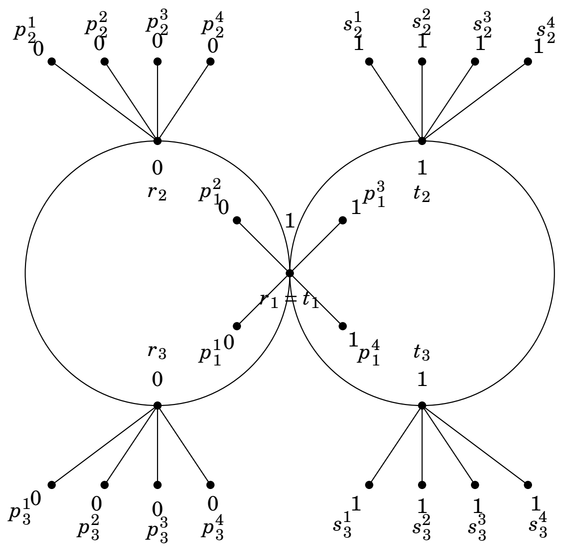

Figure 1. Product cordial labelling of $B[3,3]*S_4$

Illustration 3.2.

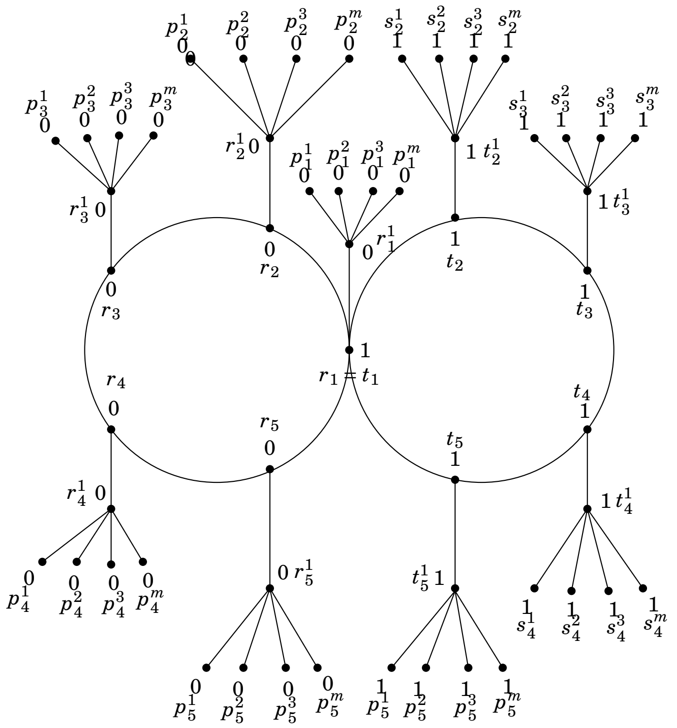

Figure 2. Product cordial labelling of $B[5,5]*P_2*S_4$

Illustration 3.3.

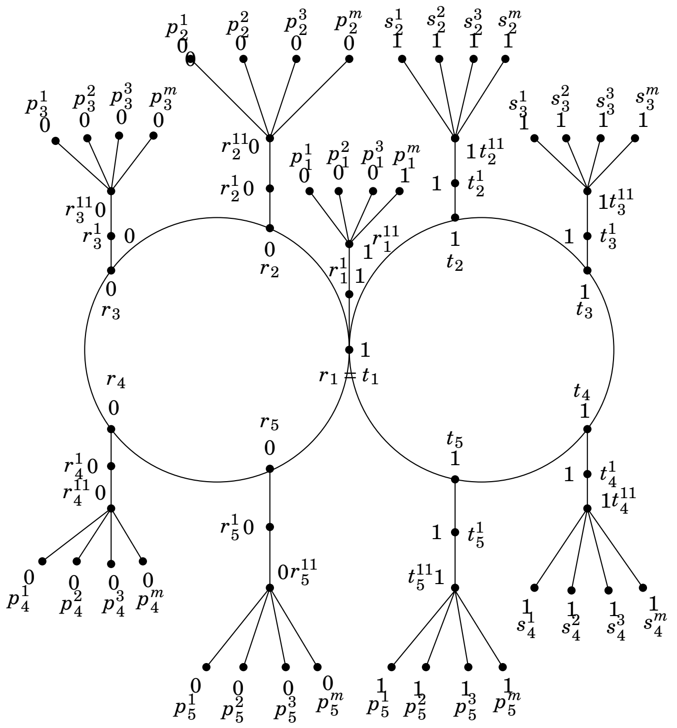

Figure 3. Product cordial labelling of $B[5,5]*P_3*S_4$

Illustration 3.4.

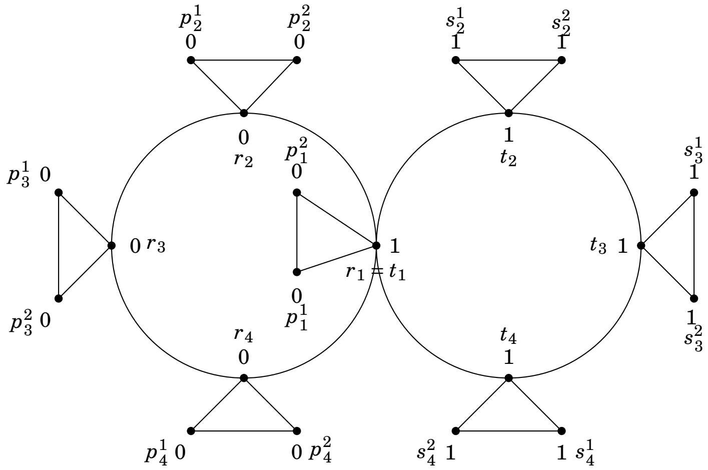

Figure 4. Product cordial labelling of $B[4,4] \odot K_2$

Illustration 3.5.

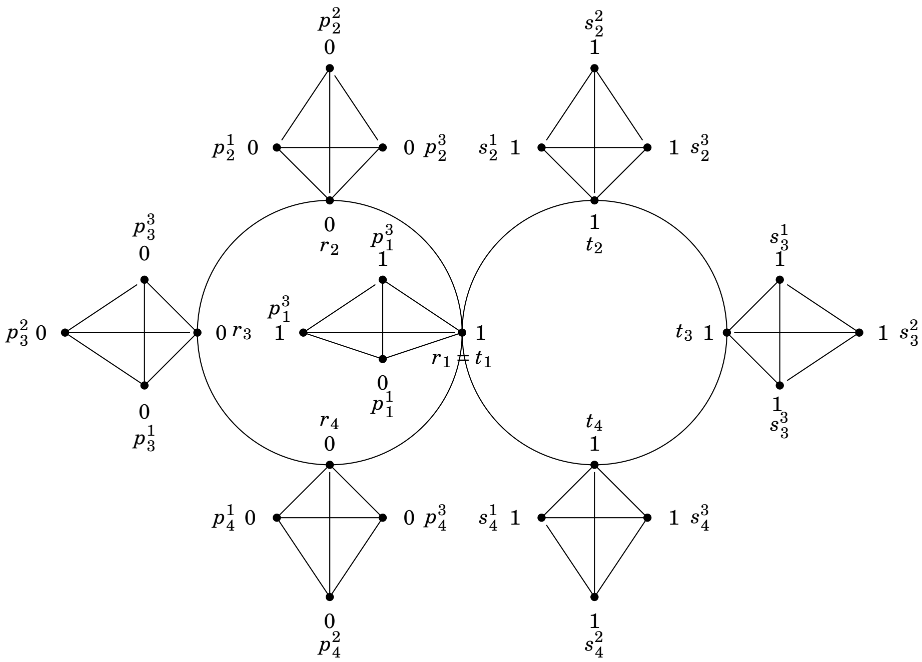

Figure 5. Product cordial labelling of $B[4,4] \odot K_3$

4. Conclusion

We provide five new theorems on product cordial labelling. It is terribly interesting to examine whether or not a graph family admits product cordial labelling. We try to link bicyclic graphs and graph operations. Similar results are often derivative for alternative graph families.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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