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Research Article

Some Aspects on Fully Complete Domination in Picture Fuzzy Graphs Based on Strong Edges

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Abstract. Picture fuzzy graph is an efficient mathematical tool for dealing ambiguous real world problems where the fuzzy graph and intuitionistic fuzzy graph would not produce high accuracy. It can be very useful in situations in which there are multiple choices of such type: yes, no, abstain and refusal. The primary aim of this study is to define the fully complete domination in picture fuzzy graph based on strong edges. Due to the importance of the notion of domination and its applications in various situations, The fully complete picture fuzzy dominating set is introduced. In addition, many significant properties related to this parameter are obtained. Further, the relation between the fully complete picture fuzzy domination number and picture fuzzy domination number is discussed. Some theorems are proved with suitable examples. An algorithm is provided to compute the fully complete picture fuzzy dominating set and its domination number and verified through an example.

Keywords. Picture fuzzy graph, Fully complete picture fuzzy dominating set, Fully complete picture fuzzy domination number, Strong edge, Strong neighbors

Mathematics Subject Classification (2020). 03E72, 05C72, 05C90

1. Introduction

L A. Zadeh [14] introduced the concept of fuzzy sets in 1965, and it has been successfully utilized in a variety of real life situations that are often unsure. A fuzzy set is a generalized form of a crisp set in which elements have different degrees of membership values. As this crisp set has only two values 0 and 1 (no or yes), it cannot manage uncertain real-world problems. Instead of considering 0 or 1, a fuzzy set allows its elements to have a membership value between 0 and 1 for a better result. However, in some cases, those single membership degree values are unable to handle the vagueness. In order to deal this type of unknown situation, Atanassov [1] proposed the intuitionistic fuzzy set which contains an additional membership degree called the hesitation margin. Intuitionistic fuzzy set is the modified version of Zadeh's fuzzy set. Due to the presence of hesitation margin, it is more accessible and effective to work with uncertainty than a typical fuzzy set.

The intuitionistic fuzzy set is widely used in real world situations that involve human perception and knowledge which are entirely imprecise and unreliable. In recent years, scientists and analysts have been using the concept of intuitionistic fuzzy set successfully in various areas such as image processing, robotic system, social network, machine learning, decision making, medical diagnosis recognition etc. But in intuitionistic fuzzy set theory, the notion of neutrality degree is not considered. However, in everyday situations, such as democratic election station, medical diagnosis recognition, social network and decision making etc. the degree of neutrality must be considered.

Cuong [3] introduced the idea of picture fuzzy set which is a developed form of fuzzy set and intuitionistic fuzzy set to fulfill the neutrality degree. The picture fuzzy set (PFS) consists of the degree of positive membership $\mu : X \to [0,1]$, degree of neutral membership $\eta : X \to [0,1]$ and degree of negative membership $\gamma : X \to [0,1]$ under the condition $\mu(x)+\eta(x)+\gamma(x) = 1$, where $\pi(x) = 1-(\mu(x)+\eta(x)+\gamma(x))$ is the degree of refusal membership values of a vertex.

Graph theory has been shown to be one of the most powerful tools for modelling complex problems because of its simplicity and generality. Graph models have a broad range of application in research areas such as system analysis, operations research, and economics.

The fundamental concept of fuzzy graph was introduced by Rosenfeld [10], a decade after Zadeh's classic study on fuzzy set. Kaufmann and Magens [4] provides detailed information regarding fuzzy sets and fuzzy relations. When compared to the graph theory, the fuzzy graph is an ideal way to approach such problems since it is more effective, convenient, flexible, and compatible with uncertain situations. Intuitionistic fuzzy graph and several properties related to this concept have been introduced by Shannon and Atanassov [11].

Zuo *et al*. [15] proposed the definition of picture fuzzy graph based on the picture fuzzy relations. Nagoorgani *et al*. [6] introduced the concept of strong and weak domination in picture fuzzy graph. Xiao *et al*. [13] investigated the regular picture fuzzy graphs and its properties. This motivated us to introduce the concept of fully complete domination in picture fuzzy graphs.

The definition of the fully complete dominating set and its domination number are introduced in this paper. Theorems and propositions of this dominating sets are stated and proved with examples. An algorithm is provided to compute the domination number and verified through an example.

1.1 Significance of the Study

In uncertainty modelling, the notion of domination and the picture fuzzy graph theory play a pivotal role. Somasundaram and Somasundaram [12] introduced domination in fuzzy graphs based on the effective edges. Nagoorgani and Chandrasekaran [7] later proposed the concept of strong edges to define domination in fuzzy graphs. Generally, the number of effective edges is low as the possibility of the effectiveness of an edge is less. This implies that the domination number becomes large. However, we have large number of edges, it is difficult to find the strength of the connectedness. Hence to get more accurate and appropriate solution, strong edges are used.

Some basic notations with their meanings

Notation	*Meaning*
$\mu_1(v_i)$	Degree of positive membership value of v_i
$\eta_1(v_i)$	Degree of neutral membership value of v_i
$\gamma_1(v_i)$	Degree of negative membership value of v_i
$\mu_2(v_i,v_j)$	Degree of positive membership value of (v_i,v_j)
$\eta_2(v_i,v_j)$	Degree of neutral membership value of (v_i,v_j)
$\gamma_2(v_i,v_j)$	Degree of negative membership value of (v_i,v_j)
Ns (v_i)	Strong neighborhood of v_i
Ns $[v_i]$	Closed strong neighborhood of v_i
$\delta_s(G)$	The minimum strong degree
$\Delta_s(G)$	The maximum strong degree
γ_{pf}	Picture fuzzy domination number
γ_{fpf}	Fully complete picture fuzzy domination number

2. Preliminaries

In this section, some basic definitions which are used to construct theorems and properties of the picture fuzzy graph are given.

Definition 2.1. A graph $G=(V,E)$ is said to be *Picture Fuzzy Graph* (PFG) if

(i) V is the set of vertices where $V=\{v_1,v_2\dots v_n\}$ such that $\mu_1:V\to[0,1]$, $\eta_1:V\to[0,1]$ and $\gamma_1:V\to[0,1]$.

(ii) E is the set of edges where $E\subseteq V\times V$ where $\mu_2:V\times V\to[0,1],\eta_2:V\times V\to[0,1]$ and $\gamma_2:V\times V\to[0,1]$ are such that

$$\mu_2(v_i,v_j)\le\min(\mu_1(v_i),\mu_1(v_j)),$$

$$\eta_2(v_i,v_j) \le \min(\eta_1(v_i),\eta_1(v_j)),$$
$$\gamma_2(v_i,v_j) \le \max(\gamma_1(v_i),\gamma_2(v_j)),$$

where μ_1, η_1, γ_1 are degrees of positive membership, neutral membership and negative membership of the vertex $v_i \in V$ respectively with the condition $0 \le \mu_1(v_i)+\eta_1(v_i)+\gamma_1(v_i) \le 1$ for every $v_i \in V$, $i = 1, 2\dots n$ and μ_2, η_2, γ_2 are degrees of positive membership, neutral membership, and negative membership of the edge $e_{ij} \in E$ respectively with the condition $0 \le \mu_2(v_i,v_j)+\eta_2(v_i,v_j)+\gamma_2(v_i,v_j) \le 1$ for every $(v_i,v_j) \in E$, $\forall\ i,j = 1,2\dots n$.

Example 2.2.

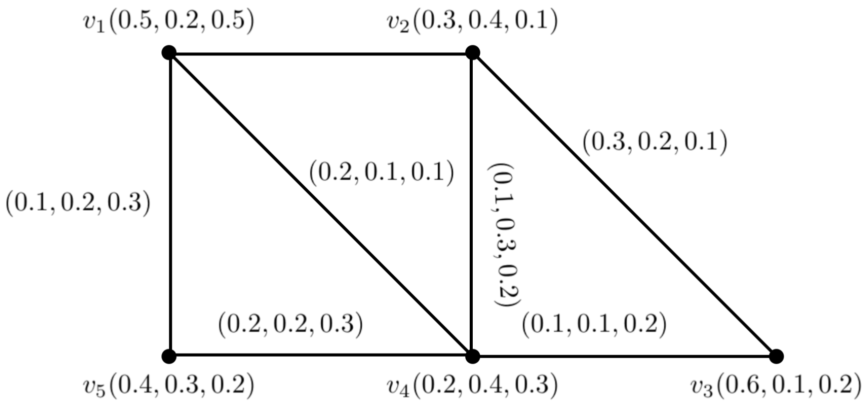

Figure 1

Definition 2.3. Let $G = (V,E)$ be the picture fuzzy graph, if $\mu_2(v_i,v_j) = \min(\mu_1(v_i),\mu_1(v_j))$, $\eta_2(v_i,v_j) = \min(\eta_1(v_i),\eta_1(v_j))$ and $\gamma_2(v_i,v_j) = \max(\gamma_1(v_i),\gamma_1(v_j))$ for every $v_i,v_j \in V$, then G is called the complete picture fuzzy graph.

Definition 2.4. An edge (v_i,v_j) is called a strong edge, if $\mu_2(v_i,v_j) \ge \mu'^{\infty}(v_i,v_j)$, $\eta_2(v_i,v_j) \ge \eta'^{\infty}(v_i,v_j)$ and $\gamma_2(v_i,v_j) \le \gamma'^{\infty}(v_i,v_j)$ for every $v_i,v_j \in V$, $\forall\ i,j = 1,2\dots n$.

Definition 2.5. Let $G = (V,E)$ be a picture fuzzy graph. Then $N_s(v_i)$ is said to be strong neighborhood of v_i, if $N_s(v_i) = \{v_j \in V : (v_i,v_j) \text{ is a strong edge}\}$.
The set $N_s[v_i]$ is said to be the closed strong neighborhood of v_i, if $N_s[v_i] = N_s(v_i) \cup \{v_i\}$.

3. Some Properties on Fully Complete Domination in Picture Fuzzy Graph

In this section, the fully complete picture fuzzy dominating set as well as fully complete picture fuzzy domination number are defined. Based on the definition, some theorems and propositions are stated and proved with examples.

Definition 3.1. Let $G = (V,E)$ be a PFG. Let $v_i,v_j \in V$ and $D_f \subset V$, then D_f is a fully complete picture fuzzy dominating set, if every $v_i \in D_f$ is adjacent to some vertices $v_j \in V - D_f$ such that the edge (v_i,v_j) is a strong edge.

Definition 3.2. The maximum fuzzy cardinality of all possible fully complete picture fuzzy dominating set is said to be fully complete picture fuzzy domination number of G and it is denoted by $\gamma_{fpf}(G)$.

Example 3.3.

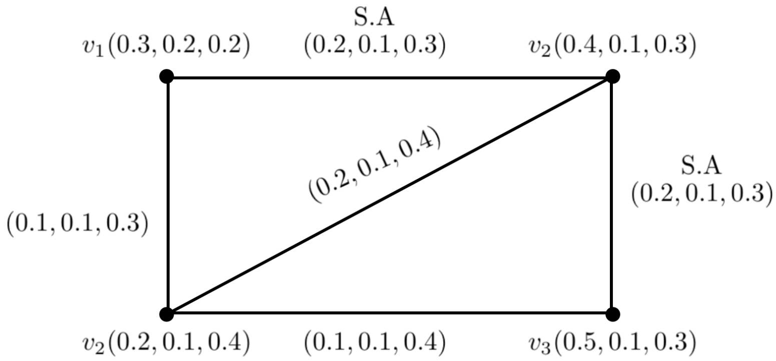

Figure 2

In the above PFG G, (v_1,v_2) and (v_2,v_3) are strong edges. The fully complete picture fuzzy dominating set $D_f = \{v_1,v_3\}$. The picture fuzzy domination number $\gamma_{pf} = 0.85$ and the fully complete picture fuzzy domination number $\gamma_{fpf} = 1.00$.

Definition 3.4. A vertex cover of a picture fuzzy graph G is a set of vertices such that each strong edge of G is incident to at least one vertex of the set.

Definition 3.5. In a PFG $G = (V,E)$, a fully complete picture fuzzy dominating set D_f is called maximal fully complete picture fuzzy dominating set, if for every vertex $v_i \in V - D_f$, the set $D_f U\{v_i\}$ is not a fully complete picture fuzzy dominating set of G.

Proposition 3.6. For any picture fuzzy graph $G = (V,E)$, The set $D_f \subset V$ is a fully complete picture fuzzy dominating set iff $V - D_f$ is a picture fuzzy dominating set.

Proof. Let $G = (V,E)$ be the picture fuzzy graph. Let D_f be a fully complete picture fuzzy dominating set. By the definition, every vertex $v_i \in D_f$ is adjacent to some vertices $v_j \in V - D_f$, $\forall\ v_i, v_j \in V$, i.e., every vertex $v_i \in D_f$ dominates some $v_i \in V - D_f$. This implies that there exist some vertices $v_j \in V - D_f$ is adjacent to every vertex $v_i \in D_f$ which satisfies the definition of dominating set as domination is symmetric. Hence $V - D_f$ is a picture fuzzy dominating set.
Conversely, suppose $V - D_f$ is a picture fuzzy dominating set. To prove, D_f is a fully complete picture fuzzy dominating set. Suppose D_f is not a fully complete picture fuzzy dominating set, there exists at least one vertices $v_i \in D_f$ does not dominates any vertices $V - D_f$. Since $V - D_f$ is a picture fuzzy dominating set, every vertex must be dominated by some vertices in $V - D_f$ which contradicts to our assumption. Hence D_f is a fully complete picture fuzzy dominating set. □

Proposition 3.7. Let $G = (V,E)$ be a picture fuzzy graph of order p. If there exists a fully complete picture fuzzy dominating set, then $\gamma_{fpf} \le p/2$.

Proof. If D_f is a fully complete picture fuzzy dominating set, then $V - D_f$ is a picture fuzzy dominating set, by Proposition 3.6.
Therefore,

$$\gamma_{fpf} \le |D_f| \Rightarrow \gamma_{fpf} \le p - |D_f| \gamma_{fpf} \le p - p/2.$$

Hence $\gamma_{fpf} \le p/2$. □

Proposition 3.8. Let $G = (V,E)$ be a picture fuzzy graph of order p. Then $\gamma_{fpf}(G) + \gamma_{pf}(G) = p$.

Proof. If G is a fully complete picture fuzzy dominating set, then by Proposition 3.6, $V - D_f$ is a dominating set

$$\gamma_{pf} \le |V| - |D_f|,$$
$$\gamma_{pf} \le p - |D_f|,$$
$$\gamma_{pf} \le p - \gamma_{fpf},$$
$$\gamma_{fpf} + \gamma_{pf} \le p. \tag{3.1}$$

Now, let D_f be the picture fuzzy dominating set. Then $V - D_f$ is a fully complete picture fuzzy dominating set.

$$\gamma_{fpf} \ge |V| - |D_f|,$$
$$\gamma_{fpf} + \gamma_{pf} \ge p, \tag{3.2}$$

from (3.1) and (3.2), we get $\gamma_{fpf} + \gamma_{pf} = p$. □

Example 3.9. In Figure 2, $p = 1.85$, $\gamma_{fpf} = 1.00$, $\gamma_{pf} = 0.85$. The given picture fuzzy graph G satisfies the equation $\gamma_{fpf} + \gamma_{pf} = p$.

Proposition 3.10. Let $G = (V,E)$ be a picture fuzzy graph. Then $\gamma_{fpf} = 0$ iff all vertices are isolated.

Proof. If all vertices isolated, then obviously they do not dominate any other vertices. This implies that the picture fuzzy dominating set need not be a fully complete picture fuzzy dominating set. Therefore, $\gamma_{fpf} = 0$.
Conversely, suppose $\gamma_{fpf} = 0$. If possible, let a vertex $v_1 \in V$ such that v_1 is not an isolated vertex. Then there exists a vertex v_j such that (v_i, v_j) is a strong edge. Hence, every fully complete picture fuzzy dominating set consists of at least one vertex, i.e., $\gamma_{fpf} = 1 \ne 0$ which is a contradiction to our assumption. Hence all vertices are isolated. □

Proposition 3.11. Let $G = (V,E)$ be the constant picture fuzzy graph without isolated vertices of degree (k_i, k_j). Then $V - D_f$ is the picture fuzzy dominating set, where D_f is the fully complex picture fuzzy dominating set.

Proof. Let $G = (V,E)$ be the constant picture fuzzy graph without isolated vertices. Let D_f be a fully complete picture fuzzy dominating set of G. Consider v_i be any vertex of D_f. There exists a vertex $v_j \in N(v_i)$, then at least one vertex in the dominating set $D_f - \{v_i\}$ dominates v_i. Therefore, all vertices in D_f are dominated by at least one vertex in $V - D_f$ and hence the set $V - D_f$ is a picture fuzzy dominating set. □

Proposition 3.12. If $G = (V, E)$ be any picture fuzzy graph of order P, the following inequalities holds:

(i) $\gamma_{pf}(G) \le \gamma_{fpf}(G) \le p - \Delta_s(G)$,

(ii) $\gamma_{pf}(G) \le \gamma_{fpf}(G) \le p - \delta_s(G)$,

where $\Delta_s(G)$ denotes the maximum strong degree and $\delta_s(G)$ denotes the minimum strong degree of G.

Proof. Let D_f be a fully complete picture fuzzy dominating set. By the definition,

$$\gamma_{pf}(G) \le \gamma_{fpf}(G). \tag{3.3}$$

Let $v_i, v_j \in V$ of G. Then $p - \Delta_s(G)$ is the difference between the order of G and maximum strong degree of G. This implies that

$$\gamma_{fpf}(G) \le p - \Delta_s(G). \tag{3.4}$$

Similarly, $p - \delta_s(G)$ is the difference between the order of G and the minimum strong degree of G. It is also clear that

$$\gamma_{fpf}(G) \le p - \delta_s(G). \tag{3.5}$$

From (3.4) and (3.5), we have

$$\gamma_{pf}(G) \le \gamma_{fpf}(G) \le p - \Delta_s(G).$$

From (3.3) and (3.5), we have

$$\gamma_{pf}(G) \le \gamma_{fpf}(G) \le p - \delta_s(G).$$

Hence inequalities hold. □

Theorem 3.13. *In a PFG $G = (V, E)$, the fully complete picture fuzzy dominating set exists only if any vertex in $V - D_f$ has at least one strong neighbor.*

Proof. Let D_f be a fully complete picture fuzzy dominating set. Then by the definition, every vertex in D_f dominates at least one vertex in $V - D_f$. This implies that at least one strong edge exists between D_f and $V - D_f$. Therefore, the vertex in $V - D_f$ has a strong neighbor in D_f. □

Theorem 3.14. *Let $G = (V, E)$ be the complete PFG. Then complement $V - D_f$ is a fully complete picture fuzzy dominating set, where D_f is the maximal fully complete picture fuzzy dominating set.*

Proof. Let $G = (V, E)$ be the complete picture fuzzy graph. By Definition 2.3, each vertex is adjacent to all other vertices and all edges are strong edges. This implies that each vertex dominates all other vertices. Hence maximal fully complete picture fuzzy dominating set exists. Since G is complete, the complement of this set is also fully complete picture fuzzy dominating set. □

Example 3.15.

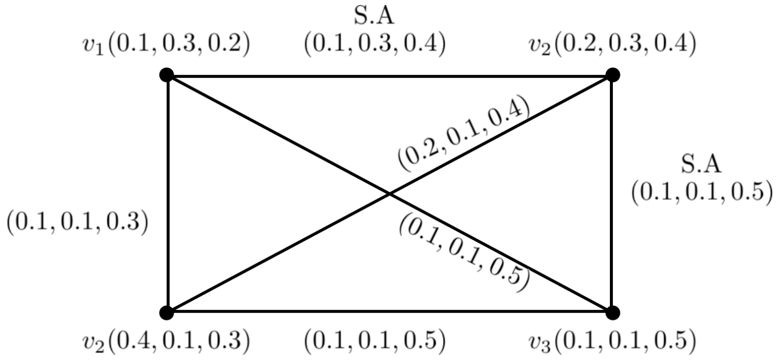

Figure 3

In the above PFG, each vertex is adjacent to all other vertices.

Therefore, fully complete picture fuzzy dominating set $D_f = \{v_1, v_2, v_3\}$, $\gamma_{fpf} = 0.3 + 0.25 + 0.5 = 1.05$.

Theorem 3.16. *Let $G = (V, E)$ be a complete picture fuzzy graph, then the fully complete picture fuzzy domination for any complement $\bar{\gamma}_{fpf}$ of the complete picture fuzzy graph is equal to zero.*

Proof. Since $G = (V, E)$ be a complete picture fuzzy graph, every edge is a strong edge. By Theorem 3.14, the fully complete picture fuzzy dominating set exists for the PFG G. By the definition of complement of a complete picture fuzzy graph, it has only isolated vertices, since isolate vertex does not dominate any other vertex, the fully complete picture fuzzy dominating set does not exists. Hence $\bar{\gamma}_{fpf} = 0$. □

Example 3.17.

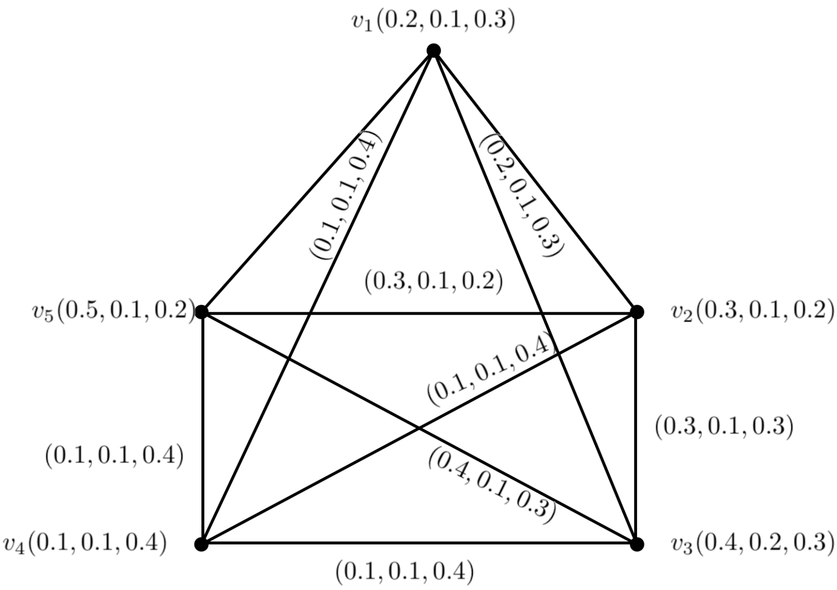

Figure 4

4. Algorithm to Compute the Fully Complete Picture Fuzzy Domination Number

1. Input the vertices $V = \{v_1, v_2, \cdots, v_n\}$ with their membership values μ_1, η_1 and γ_1 for any $v_i \in V$
2. Input their edges $E = \{(v_i, v_j)/v_i, v_j \in V\}$ with their membership values $\mu_2(v_i, v_j)$, $\eta_2(v_i, v_j)$, $\gamma_2(v_i, v_j)$ for any $(v_i, v_j) \in E$
3. Initialize all possible paths P_i, $i = 1, 2, \cdots, n$
4. $S\mu_2(v_i, v_j) = \min\{\mu_2(v_i, v_j)/v_i, v_j \in V\}$
5. $S\eta_2(v_i, v_j) = \min\{\eta_2(v_i, v_j)/v_i, v_j \in V\}$
6. $S\gamma_2(v_i, v_j) = \max\{\gamma_2(v_i, v_j)/v_i, v_j \in V\}$
7. $\mu_2'^{\infty}(v_i, v_j) = \max\{S_2(v_i, v_j)/v_i, V_j \in V\}$
8. $\eta_2'^{\infty}(v_i, v_j) = \max\{S_{\eta_2}(v_i, v_j)/v_i, v_j \in V\}$
9. $\gamma_2'^{\infty}(v_{\mathbf{i}}, v_{\mathbf{j}}) = \min\{S\gamma_2(v_{\mathbf{i}}, v_{\mathbf{j}})/v_{\mathbf{i}}, v_{\mathbf{j}} \in V\}$
10. Verify the conditions

$$\mu_2(v_{\boldsymbol{i}}, v_{\boldsymbol{j}}) \geq \mu_2'^{\infty}(v_{\boldsymbol{i}}, v_{\boldsymbol{j}})$$
$$\eta_2(v_{\boldsymbol{i}}, v_{\boldsymbol{j}}) \geq \eta_2'^{\infty}(v_{\boldsymbol{i}}, v_{\boldsymbol{j}})$$
$$\gamma_2(v_{\boldsymbol{i}}, v_{\boldsymbol{j}}) \leq \gamma_2'^{\infty}(v_{\boldsymbol{i}}, v_{\boldsymbol{j}}) \text{ holds}$$

11. If the condition holds, determine the edge (v_i, v_j) is a strong edge
12. If not, determine the edge (v_i, v_j) is not a strong edge
13. Repeat the same with remaining pair of vertices $v_i, v_j \in V$
14. Form the minimal picture fuzzy dominating set D_{fpf1} which contains vertices determined from *Step* 3 to *Step* 13
15. Find all remaining possible minimal picture fuzzy dominating sets $D_{fpf2}, D_{fpf3} \dots D_{fpfn}$
16. Compute the cardinality $|D_{fpf1}|, |D_{fpf2}|, \dots, |D_{fpfn}|$ of all minimal picture fuzzy dominating sets
17. Determine the picture fuzzy domination number

$$\gamma_{pf} = \min\{|D_{fpf1}|, |D_{fpf2}|, \dots |D_{fpfn}|\}$$

Example 4.1.

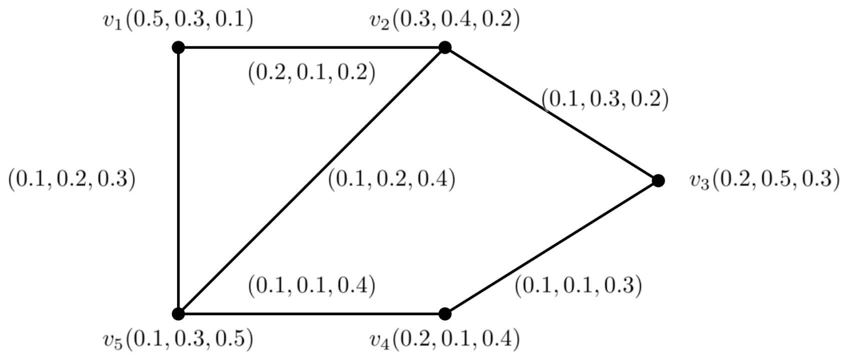

Figure 5

1. Initialize the vertex set $V = \{v_1, v_2, v_3, v_4\}$ with their positive, neutral and negative membership values.

   $v_1 \to (0.5, 0.3, 0.1)$

   $v_2 \to (0.3, 0.4, 0.2)$

   $v_3 \to (0.2, 0.5, 0.3)$

   $v_4 \to (0.2, 0.1, 0.4)$

   $v_5 \to (0.1, 0.3, 0.5)$

2. Initialize the edge set $E = \{(v_1,v_2),(v_2,v_3),(v_3,v_4),(v_4,v_5),(v_2,v_5)\}$ with their positive, neutral and negative membership values.

   $(v_1, v_2) \to (0.2, 0.1, 0.2)$

   $(v_2, v_3) \to (0.1, 0.3, 0.2)$

   $(v_3, v_4) \to (0.1, 0.1, 0.3)$

   $(v_4, v_5) \to (0.1, 0.1, 0.4)$

   $(v_5, v_1) \to (0.1, 0.2, 0.3)$

   $(v_2, v_5) \to (0.1, 0.2, 0.4)$

3. Computation of possible paths
   Initialize the p possible paths between the pair of vertices (v_i, v_j)
4. $S\mu(v_1, v_2) =$ minimum of $\mu_2(v_1, v_2)$ of the edge (v_1, v_2)

   $S\mu(v_1, v_2) = \{0.2, 0.1, 0.1\}$

5. $S\eta(v_1, v_2) =$ minimum of $\eta_2(v_1, v_2)$ of the edge (v_1, v_2)

   $S\eta(v_1, v_2) = \{0.1, 0.1, 0.1\}$

6. $S\gamma(v_1, v_2) =$ maximum of $\gamma_2(v_1, v_2)$ of the edge (v_1, v_2)

   $S_\gamma(v_1, v_2) = \{0.2, 0.2, 0.3\}$

7. $\mu_2'^{\infty}(v_1, v_2) =$ maximum of $S_\mu(v_1, v_2) = 0.2$
8. $\eta'^{\infty}(v_1, v_2) =$ maximum of $S_\eta(v_1, v_2) = 0.1$
9. $\gamma_2'^{\infty}(v_1, v_2) =$ minimum of $S'_\gamma(v_1, v_2) = 0.2$
10. Verify

    $\mu_2(v_1, v_2) \geq \mu_2'^{\infty}(v_1, v_2)$

    $\eta_2(v_1, v_2) \geq \eta_2'^{\infty}(v_1, v_2)$

    $\gamma_2(v_1, v_2) \leq \gamma_2'^{\infty}(v_1, v_2)$

11. Determine the edge (v_1, v_2)
12. Output the edge (v_1, v_2) is a strong edge
13. Repeat the same with remaining pair of vertices $(v_i, v_j) \in V$
14. Form the minimal picture fuzzy dominating set $D_{fpf1} = \{v_2, v_4\}$
15. Compute the cardinality $|D_{fpf1}| = |v_2| + |v_4| = 0.6 + 0.4 = 2.3$

5. Conclusion

The fully complete dominating set and fully complete domination number have been defined in picture fuzzy graph by using strong edges. The relationship between the domination number and the fully complete domination number in picture fuzzy graph have been discussed. Bounds of some picture fuzzy graphs have been obtained. Some theorems and properties have been proved with examples. To compute the fully complete dominating set and its domination number in the picture fuzzy graph, an algorithm has been developed and verified through an example.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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