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Research Article

Some Outcomes on Fuzzy Menger Space via Common Property (E.A) and Absorbing Maps

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Abstract. The present idea of this paper is to generalize the result proved by Sing *et al*. (Fixed point results in Fuzzy Menger space, *Journal of Applied Mathematics and Bioinformatics* **5** (2015), 67 – 75) obtaining two results in fuzzy Menger space by employing absorbing maps, common property (E.A) and occasionally weakly compatible mappings. Moreover, these results are justified by suitable examples.

Keywords. Absorbing maps, Common property (E.A), Occasionally weakly compatible mappings, Fuzzy Menger space

Mathematics Subject Classification (2020). 54H25, 47H10

1. Introduction

The Menger space is hot spot for researchers because it has various applications in mathematics as well as other fields. Due to its novelty, expansion, continuity, contribution attracted many researchers to work on this field. The notion of fuzzy set is coined by Zadeh [12]. In continuation Kramosil and Michálek [2] initiated the Structure of fuzzy metric space. In Menger space the topology was given by Schweizer *et al*. [7] through t-norm. After words many researchers contributing their results by extending the concepts from metric space to Menger space and fuzzy metric space. In this aspect, Mishra [4] use the concept of compatible mappings this channelize to generate fixed points in Menger space. Mishra *et al*. [5] used the concept of weakly compatible mappings in fuzzy metric space and generated some results. Singh and Devi [10] generated coincidence points in Menger space by applying absorbing maps. By employing the CLR_s property Reddy and Srinivas [6] obtained some results in fuzzy metric space. Shrivastav *et al*. [9] introduced fuzzy Menger space by generalizing Menger space. Singh *et al*. [11] generated coincidence points in fuzzy Menger space by employing the concepts of common property (E.A) and weakly compatible mappings. Some more results can be witnessed by Sharma and Sharma [8]. The aim of the paper is to generate two common fixed point theorems by using common property (E.A), absorbing maps and occasionally weakly compatible mappings in fuzzy Menger space.

2. Preliminaries

Definition 2.1 ([11]). A non-empty set $\mathcal{Y}$ and a mapping F_α from $\mathcal{Y}\times\mathcal{Y}$ into the collection of all fuzzy distribution functions $F_\alpha \in \Re$ for all $\alpha\in[0,1]$ make up a fuzzy probabilistic metric space ($\mathcal{FPM}$-space) $(\mathcal{Y},F_\alpha)$ and the fuzzy distribution function is expressed by $F_\alpha(a,b)$ and $F_{\alpha(a,b)}(u)$ is the value of $F_{\alpha(a,b)}$ at u in R.
The function $F_{\alpha(a,b)}$ for all $\alpha\in[0,1]$ is to satisfy the following properties:

(a) $F_{\alpha(a,b)}(u)=1 \Longleftrightarrow a=b$,

(b) $F_{\alpha(a,b)}(0)=0$,

(c) $F_{\alpha(b,a)}=F_{\alpha(a,b)}$,

(d) if $F_{\alpha(a,b)}(u)=1$ and $F_{\alpha(b,c)}(v)=1 \implies F_{\alpha(a,c)}(u+v)=1$,

for all $a,b,c\in\mathcal{Y}$ and $u,v>0$.

Definition 2.2 ([1]). A binary operation $t_\phi\colon[0,1]\times[0,1]\to[0,1]$ is stated as t-norm if

(i) $t_\phi(a,1)=a$, $t_\phi(0,0)=0$,

(ii) $t_\phi(a,b)=t_\phi(b,a)$,

(iii) $t_\phi(a,t_\phi(b,c))=t_\phi(t_\phi(a,b),c)$,

(iv) if $c\ge a, d\ge b \implies t_\phi(c,d)\ge t_\phi(a,b)$,

for all $a,b,c,d\in[0,1]$.

Definition 2.3. A fuzzy Menger space $(\mathcal{Y},F_\alpha,t_\phi)$ is formed by $(\mathcal{Y},F_\alpha)\mathcal{FPM}$-space, t_ϕ where t_ϕ is t-norm and satisfy triangle inequality

$$F_{\alpha(a,c)}(u+v)\ge t_\phi(F_{\alpha(a,b)}(u),F_{\alpha(b,c)}(v)),$$

for all $a,b,c\in\mathcal{Y}$ and $u,v>0$ and $\alpha\in[0,1]$.

Example 2.4. If $(\mathcal{Y},d)$ is metric space then $F_\alpha\colon\mathcal{Y}\times\mathcal{Y}\to L$ given by

(i) $F_{\alpha(a,b)}=H_{\alpha(x-d(a,b))}$, $a,b\in\mathcal{Y}$, for all $\alpha\in[0,1]$, and if $t_\phi : [0,1]\times[0,1] \to [0,1]$ is given by

(ii) $t_\phi(c,d) = \min\{c,d\}$, then $(\mathcal{Y}, F_\alpha, t_\phi)$ forms fuzzy Menger space. Further, it is complete whenever $(\mathcal{Y}, d)$ is complete.

Definition 2.5. In fuzzy Menger space $(\mathcal{Y}, F_\alpha, t_\phi)$ two mappings $\gamma, \delta : \mathcal{Y} \to \mathcal{Y}$ are compatible if

$$F_{\alpha(\gamma\delta a_n, \delta\gamma a_n)}(t_\phi) \to 1, \quad \text{for all } t_\phi > 0,$$

whenever $(a_n) \in \mathcal{Y}$ in order for $\gamma a_n, \delta a_n \to \mu$ for some $\mu \in \mathcal{Y}$.

Definition 2.6 ([11])**.** Let $(\mathcal{Y}, F_\alpha, t_\phi)$ is fuzzy Menger space. Two pairs (α,β), (γ,δ) of self mappings are said to satisfy the common property (E.A) if there exists two sequences (c_m), (d_m) in $\mathcal{Y}$ and for some $\mu \in \mathcal{Y}$ such that

$$\lim_{m\to\infty} \alpha(c_m) = \lim_{m\to\infty} \beta(c_m) = \lim_{m\to\infty} \gamma(d_m) = \lim_{m\to\infty} \delta(d_m) = \mu.$$

If $\alpha = \gamma$ and $\beta = \delta$ then we say the self mappings α, β are mentioned to hold the property (E.A).

Example 2.7. Let $(\mathcal{Y}, F_\alpha, t_\phi)$ be a fuzzy Menger space, where $\mathcal{Y} = \left[0, \frac{1}{2}\right]$ and define

$$F_{\alpha(a,b)}(t_\phi) = \begin{cases} e^{-\frac{|a-b|}{t_\phi}}, & \text{if } t_\phi > 0, \\ 0, & \text{if } t_\phi = 0, \end{cases} \tag{2.1}$$

for all $a, b \in \mathcal{Y}$.

Define the self mappings $\alpha, \beta, \gamma, \delta$ on $\mathcal{Y}$ as

$$\alpha(a) = \sin(\pi a),$$
$$\beta(a) = \cos(\pi a),$$
$$\gamma(a) = (\sqrt{2})\sin^2(\pi a),$$
$$\delta(a) = (\sqrt{2})\cos^2(\pi a), \quad \text{for all } a \in \mathcal{Y}.$$

Then, there exist sequences (c_m), (d_m) in $\mathcal{X}$ with

$$(c_m) = \frac{1}{4} + \frac{1}{m},$$
$$(d_m) = \frac{1}{4} + \frac{\pi}{m}, \quad \text{for all } m \geq 1$$

such that

$$\lim_{m\to\infty} \alpha(c_m) = \lim_{m\to\infty} \beta(c_m) = \lim_{m\to\infty} \gamma(d_m) = \lim_{m\to\infty} \delta(d_m) = \frac{1}{\sqrt{2}}, \quad \frac{1}{\sqrt{2}} \in \mathcal{Y}.$$

Thus, the four self mappings satisfied common property (E.A).

Definition 2.8 ([11])**.** Two mappings $\gamma, \delta : \mathcal{Y} \to \mathcal{Y}$ are weakly compatible in fuzzy metric space if these are commuting at their coincidence points.

Definition 2.9 ([10])**.** Two mappings $\gamma, \delta : \mathcal{Y} \to \mathcal{Y}$ are occasionally weakly compatible in fuzzy metric space if there is a coincidence point at which the mapping commutes.

The following example demonstrates that OWC not necessarily weakly compatible.

Example 2.10. Define the mappings are

$$\gamma(a) = 2\sin^2(\pi a), \quad \text{for all } a \in \left[0, \frac{3}{2}\right],$$

$$\delta(a) = \sin(\pi a), \qquad \text{for all } a \in \left[0, \frac{3}{2}\right].$$

Then the mappings coincide at $a = \frac{1}{6}, 1, 0$; however at $a = \frac{1}{6}$:

$$\gamma\left(\frac{1}{6}\right) = 2\sin^2\left(\pi\frac{1}{6}\right) = 2\frac{1}{4} = \frac{1}{2},$$

$$\delta\left(\frac{1}{6}\right) = \sin\left(\pi\frac{1}{6}\right) = \frac{1}{2},$$

$$\gamma\left(\frac{1}{6}\right) = \delta\left(\frac{1}{6}\right) = \frac{1}{2},$$

$$\gamma\delta\left(\frac{1}{6}\right) = \gamma\left(\frac{1}{2}\right) = 2\sin^2\left(\pi\frac{1}{2}\right) = 2,$$

$$\delta\gamma\left(\frac{1}{6}\right) = \delta\left(\frac{1}{2}\right) = \sin\left(\pi\frac{1}{2}\right) = 1.$$

At $a = 1$,

$$\gamma(1) = 2\sin^2(\pi 1) = 0,$$

$$\delta(1) = \sin(\pi 1) = 0,$$

$$\gamma(1) = \delta(1) = 0,$$

$$\gamma\delta(1) = \gamma(0) = 0,$$

$$\delta\gamma(1) = \delta(0) = \sin(\pi 0) = 0.$$

Resulting $\gamma\delta(1) = \delta\gamma(1)$ but $\gamma\delta\left(\frac{1}{6}\right) \neq \delta\gamma\left(\frac{1}{6}\right)$. Hence the result.

Definition 2.11 ([1])**.** Here we extend the notion of absorbing maps from Menger space to fuzzy Menger space as: Let γ, δ are two mappings on fuzzy Menger space $(\mathcal{Y}, F_\alpha, t_\phi)$ if

(i) γ is δ-absorbing if $\exists\ R \in Z^+$ such that

$$F_{\alpha(\delta a, \delta\gamma a)}(t_\phi) \geq F_{\alpha(\delta a, \gamma a)}\left(\frac{t_\phi}{R}\right), \quad \text{for all } a \in \mathcal{Y}.$$

(ii) δ is γ-absorbing if $\exists\ R \in Z^+$ such that

$$F_{\alpha(\gamma a, \gamma\delta a)}(t_\phi) \geq F_{\alpha(\gamma a, \delta a)}\left(\frac{t_\phi}{R}\right), \quad \text{for all } a \in \mathcal{Y}.$$

(iii) γ is called point wise δ-absorbing if for given $a \in X$, $\exists\ R \in Z^+$ such that

$$F_{\alpha(\delta a, \delta\gamma a)}(t_\phi) \geq F_{\alpha(\delta a, \gamma a)}\left(\frac{t_\phi}{R}\right), \quad \text{for all } a \in \mathcal{Y}.$$

(iv) δ is point wise γ-absorbing if for given $a \in \mathcal{Y}$, $\exists\ R \in Z^+$ such that

$$F_{\alpha(\gamma a, \gamma\delta a)}(t_\phi) \geq F_{\alpha(\gamma a, \delta a)}\left(\frac{t_\phi}{R}\right), \quad \text{for all } a \in \mathcal{Y}.$$

The following example explains that point wise absorbing maps are not necessarily weakly compatible.

Example 2.12. By defining $\gamma(a) = 3^a$, for all $a \in \Re$ as well as $\delta(a) = 3^{3a}$, for all $a \in \Re$ then the mapping are point wise absorbing but not commutes at $a = 3$, i.e., $\delta\gamma(3) \neq \gamma\delta(3)$. Further

$$F_{\alpha(\delta 0,\delta\gamma 0)}(t) \geq F_{\alpha(\delta 0,\gamma 0)}\left(\frac{t}{R}\right).$$

Definition 2.13 (Implicit relation [3])**.** Take Φ be the collection of all continuous real valued functions

$$\phi_o((\Re)^5) : [0,1] \to \Re,$$

non-decreasing in the argument satisfies as under:

(i) for $a, b \geq 0$, $\phi_o(a,b,a,b,1) \geq 0 \implies a \geq b$.

(ii) $\phi_o(a,1,1,a,1) \geq 0$ or $\phi_o(a,a,1,1,a) \geq 0$ or $\phi_o(a,1,a,1,a) \geq 0 \implies a \geq 1$.

Example 2.14. Defying

$$\phi_o(t_a,t_b,t_c,t_d,t_e) = \frac{4}{5}t_a - \frac{3}{5}t_b - \frac{2}{5}t_c + \frac{1}{5}t_d + \frac{1}{5}t_e - \frac{1}{5}.$$

Then

(i) $\phi_o(a,b,b,a,1) \geq 0$ implies $\frac{4}{5}a - \frac{3}{5}b - \frac{2}{5}b + \frac{1}{5}a + \frac{1}{5}1 - \frac{1}{5} \geq 0$ $a - b \geq 0$ implies $a \geq b$.

(ii) $\phi_o(a,1,1,a,1) \geq 0$ implies $a - 1 \geq 0$ implies $a \geq 1$
or
$\phi_o(a,1,a,1,a) \geq 0$ implies $\frac{3a-3}{5} \geq 0$ implies $a \geq 1$
or
$\phi_o(a,a,1,1,a) \geq 0$ implies $\frac{2a-2}{5} \geq 0$ implies $a \geq 1$.

Thus $\phi_o \in \Phi$.

The following theorem proved by Singh *et al.* [11].

Theorem 2.15. *Let p, q, f and g be four self-mappings on a fuzzy PM space $(\mathcal{X}, F_\alpha, t_\phi)$ satisfying the following:*

(i) *the pair (p, f) and (q, g) shares the common property (E.A),*

(ii) *for any $p, q \in \mathcal{X}$, $\phi \in \Phi$, for all $t_\phi > 0$,*

$$\phi(F_{\alpha(pa,qb)}(t_\phi), F_{\alpha(fa,gb)}(t_\phi), F_{\alpha(fa,pa)}(t_\phi), F_{\alpha(gb,qb)}(t_\phi), F_{\alpha(gb,pa)}(t_\phi)) \geq 0,$$

(iii) *$f(\mathcal{X}) and g(\mathcal{X})$ are closed subsets of $\mathcal{X}$,*

(iv) *the pairs (p, f) and (q, g) are weakly compatible.*

Then the pairs (p, f) and (q, g) have a point of coincidence each. Moreover p, q, f and g have a unique common fixed point.

Now we generalize above theorem as under.

3. Main Results

Theorem 3.1. *Let d, e, f and g be four self-mappings on a fuzzy PM space $(\mho, F_\alpha, t_\phi)$ having*

(i) *the pairs (d, f) and (e, g) are sharing the common property (E.A),*

(ii) *for any $a, b \in \mho$, $\phi \in \Phi$, for all $t_\phi > 0$,*

$$\phi(F_{\alpha(da,eb)}(t_\phi), F_{\alpha(fa,gb)}(t_\phi), F_{\alpha(fa,da)}(t_\phi), F_{\alpha(gb,eb)}(t_\phi), F_{\alpha(gb,da)}(t_\phi)) \geq 0, \tag{3.1}$$

(iii) *$f(\mho)$ and $g(\mho)$ are having closed property in $\mho$.*

(iv) *d is point wise f-absorbing and the pair (e, g) is occasionally weakly compatible.*

Then the four mappings d, e, f and g have a unique common fixed point in $\mho$.

Proof. By (i) the pairs (d, f) and (e, g) are sharing the common property (E.A) implies $\exists$ (c_m) and $(d_m) \in \mho$ such that

$$\lim_{m\to\infty} dc_m = \lim_{m\to\infty} fc_m = \lim_{m\to\infty} ed_m = \lim_{m\to\infty} gd_m = \mu, \tag{3.2}$$

for some $\mu \in \mho$.

By (iii), $f(\mho)$ and $g(\mho)$ closed subsets of $\mho$ resulting

$$\mu = \lim_{m\to\infty} fc_m \in f(\mho) \text{ and } \mu = \lim_{\mu\to\infty} gd_m \in g(\mho)$$

implies

$$\mu \in f(\mho) \text{ and } \mu \in g(\mho).$$

So that there exist u and v in $\mho$ such that $\mu = fu = gv$.

Claim $\mu = du$.

By assigning $a = u$, $b = d_m$ in (3.1),

$$\phi(F_{\alpha(du,ed_m)}(t_\phi), F_{\alpha(fu,gd_m)}(t_\phi), F_{\alpha(fu,du)}(t_\phi), F_{\alpha(gd_m,ed_m)}(t_\phi), F_{\alpha(gd_m,du)}(t_\phi)) \geq 0$$

as $m \to \infty$.

Using (3.2) and $\mu = fu$,

$$\phi(F_{\alpha(du,\mu)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi), F_{\alpha(\mu,du)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi), F_{\alpha(\mu,du)}(t_\phi)) \geq 0$$

$$\phi(F_{\alpha(du,\mu)}(t_\phi), 1, F_{\alpha(\mu,du)}(t_\phi), 1, F_{\alpha(\mu,du)}(t_\phi)) \geq 0.$$

From Example 2.14(ii) implies

$$F_{\alpha(du,\mu)}(t_\phi) \geq 1 \text{ implies } F_{\alpha(du,\mu)}(t_\phi) = 1.$$

Thus $\mu = du$, implies

$$\mu = du = fu = gv. \tag{3.3}$$

Claim $ev = \mu$.

By assigning the values $a = u$, $b = v$ in (3.1) and using (3.3),

$$\phi(F_{\alpha(du,ev)}(t_\phi), F_{\alpha(fu,gv)}(t_\phi), F_{\alpha(fu,du)}(t_\phi), F_{\alpha(gv,ev)}(t_\phi), F_{\alpha(gv,du)}(t_\phi)) \geq 0,$$

$$\phi(F_{\alpha(\mu,ev)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi), F_{\alpha(\mu,ev)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi)) \geq 0,$$

$$\phi(F_{\alpha(\mu,ev)}(t_\phi), 1, 1, F_{\alpha(\mu,ev)}(t_\phi), 1) \geq 0.$$

From Example 2.14(ii) implies

$$F_{\alpha(\mu,ev)}(t_\phi) \geq 1 \implies \mu = ev.$$

Therefore from (3.3), we have

$$\mu = du = fu = ev = gv. \tag{3.4}$$

The pair (e,g) is OWC resulting $ev = gv \implies egv = gev \implies e\mu = g\mu$.

Claim $e\mu = \mu$.
By employing $a = u$, $b = \mu$ in (3.1),

$$\phi(F_{\alpha(du,e\mu)}(t_\phi), F_{\alpha(fu,g\mu)}(t_\phi), F_{\alpha(fu,du)}(t_\phi), F_{\alpha(g\mu,e\mu)}(t_\phi), F_{\alpha(g\mu,du)}(t_\phi)) \geq 0.$$

Using (3.4), $g\mu = e\mu$,

$$\phi(F_{\alpha(\mu,e\mu)}(t_\phi), F_{\alpha(\mu,e\mu)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi), F_{\alpha(e\mu,e\mu)}(t_\phi), F_{\alpha(e\mu,\mu)}(t_\phi)) \geq 0,$$

$$\phi(F_{\alpha(\mu,e\mu)}(t_\phi), F_{\alpha(\mu,e\mu)}(t_\phi), 1, 1, F_{\alpha(e\mu,\mu)}(t_\phi)) \geq 0.$$

By Example 2.14(ii),

$$F_{\alpha(e\mu,\mu)}(t_\phi) \geq 1$$

implies $e\mu = \mu$.
Therefore,

$$g\mu = e\mu = \mu. \tag{3.5}$$

Since $du = fu = gv = \mu$. d is point wise f-absorbing implies $\exists\, R \in Z^+$ such that

$$F_{\alpha(fu,fdu)}(t_\phi) \geq F_{\alpha(fu,du)}\left(\frac{t_\phi}{R}\right).$$

$$fu = fdu = ffu \implies \mu = f\mu.$$

Claim $\mu = d\mu$.
By employing $a = \mu$, $b = \mu$ in in (3.1),

$$\phi(F_{\alpha(d\mu,e\mu)}(t_\phi), F_{\alpha(f\mu,g\mu)}(t_\phi), F_{\alpha(f\mu,d\mu)}(t_\phi), F_{\alpha(g\mu,e\mu)}(t_\phi), F_{\alpha(g\mu,d\mu)}(t_\phi)) \geq 0.$$

Using (3.5) and $\mu = f\mu$,

$$\phi(F_{\alpha(d\mu,\mu)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi), F_{\alpha(\mu,d\mu)}(t_\phi), F_{\alpha(\mu,\mu)}(t), F_{\alpha(\mu,d\mu)}(t_\phi)) \geq 0,$$

$$\phi(F_{\alpha(d\mu,\mu)}(t_\phi), 1, F_{\alpha(\mu,d\mu)}(t_\phi), 1, F_{\alpha(\mu,d\mu)}(t_\phi)) \geq 0.$$

By Example 2.14(ii),

$$F_{\alpha(d\mu,\mu)}(t_\phi) \geq 1.$$

This implies $d\mu = \mu$.

Therefore,

$$f\mu = d\mu = g\mu = e\mu = \mu. \tag{3.6}$$

Hence μ is common fixed of four mappings.

Uniqueness: Assume θ be another such point of mappings d, f, e and g.
By (3.1),

$$\phi(F_{\alpha(d\mu,e\theta)}(t_\phi), F_{\alpha(f\mu,g\theta)}(t_\phi), F_{\alpha(f\mu,d\mu)}(t_\phi), (F_{\alpha(g\theta,e\theta)}(t_\phi), F_{\alpha(g\theta,d\mu)}(t_\phi)) \geq 0.$$

Using (3.6),

$$\phi(F_{\alpha(\mu,\theta)}(t_\phi), F_{\alpha(\mu,\theta)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi), (F_{\alpha(\theta,\theta)}(t_\phi), F_{\alpha(\theta,\mu)}(t_\phi)) \geq 0,$$

$$\phi(F_{\alpha(\mu,\theta)}(t_\phi), F_{\alpha(\mu,\theta)}(t_\phi), 1, 1, F_{\alpha(\theta,\mu)}(t_\phi)) \geq 0.$$

By Example 2.14(ii),

$$F_{\alpha(\theta,\mu)}(t_\phi) \geq 1,$$

$$F_{\alpha(\theta,\mu)}(t_\phi)) = 1 \implies \mu = \theta$$

Hence the theorem. □

Now our theory substantiated by following example.

Example 3.2. Let $(\mathcal{Y}, F_\alpha, t_\phi)$ be a induced fuzzy Menger space, where $\mathcal{Y} = [0,2]$. Define the mappings as

$$d(a) = e(a) = \begin{cases} \sqrt{2}\cos(\pi a), & \text{if } a \in \left[0, \frac{1}{2}\right], \\ 0, & \text{if } a \in \left(\frac{1}{2}, 1\right], \\ a, & \text{if } a \in (1,2], \end{cases} \tag{3.7}$$

$$f(a) = g(a) = \begin{cases} \sqrt{2}\sin(\pi a), & \text{if } a \in \left[0, \frac{1}{2}\right], \\ 1, & \text{if } a \in \left(\frac{1}{2}, 1\right], \\ \frac{a^2}{2}, & \text{if } a \in (1,2]. \end{cases} \tag{3.8}$$

Choose $(a_m) = 2 - \frac{\sqrt{2}}{m}$, for all $m \geq 1$.
Then, from (3.7) and (3.8),

$$\lim_{m\to\infty} da_m = \lim_{m\to\infty} \left(2 - \frac{\sqrt{2}}{m}\right) = 2, \tag{3.9}$$

$$\lim_{m\to\infty} fa_m = \lim_{m\to\infty} \frac{1}{2}\left(2 - \frac{\sqrt{2}}{m}^2\right) = 2. \tag{3.10}$$

From (3.9) and (3.10),

$$\lim_{m\to\infty} da_m = \lim_{m\to\infty} fa_m = 2. \tag{3.11}$$

Thus, the pairs (d, f) and (e, g) satisfy the common property (E.A).
If $a \in \left[0, \frac{1}{2}\right]$,

$$d(f(a), fd(a)) \leq Rd(f(a), d(a))$$

$$\implies \quad d(\sqrt{2}\sin(\pi a), \sqrt{2}\sin(\pi(\sqrt{2}\cos(\pi a))) \leq Rd(\sqrt{2}\sin(\pi a), \sqrt{2}\cos(\pi a)).$$

It is true for $2 \leq R$.

If $a \in \left(\frac{1}{2},1\right]$,

$$d(f(a),fd(a)) \le Rd(f(a),d(a))$$
$$\implies d(1,0) \le Rd(1,0)$$
$$\implies 1 \le R.$$

If $a \in (1,2]$,

$$d(f(a),fd(a)) \le Rd(f(a),d(a))$$
$$\implies d\left(\frac{a^2}{2},\frac{a^2}{2}\right) \le Rd\left(\frac{a^2}{2},a\right)$$
$$\implies 0 \le R.$$

Hence we can choose $R \ge 2$.
Moreover, from (3.7) and (3.8), $f(\mathcal{Y}) = g(\mathcal{Y}) = [0,2]$ are closed subsets and the mapping d is point wise f-absorbing and the pair (e,g) is occasionally weakly compatible but not weakly compatible. Because $\exists$ $a = 2$ with $e(2) = g(2)$ implies $eg(2) = ge(2)$, where as at $a = \frac{1}{4}$, $e\left(\frac{1}{4}\right) = g\left(\frac{1}{4}\right) = 1$, $eg\left(\frac{1}{4}\right) = e(1) = 0$, $ge\left(\frac{1}{4}\right) = g(1) = 1 \implies eg\left(\frac{1}{4}\right) \ne ge\left(\frac{1}{4}\right)$.
Moreover at $a = 2$, $d(2) = f(2) = e(2) = g(2) = 2$.
Therefore, $a = 2$ is the unique common fixed point mappings d,f,e and g and fulfilled all conditions of Theorem 3.1.

Now we discuss another theorem.

Theorem 3.3. *Let d,e,f and g be four self-mappings on a fuzzy PM space $(\mathcal{Y},F_\alpha,t_\phi)$ having*

(i) *the pairs (d,f) and (e,g) are sharing the common property (E.A),*

(ii) *for any $a,b \in \mathcal{Y}$, $\phi \in \Phi$, for all $t_\phi > 0$,*
$$\phi(F_{\alpha(da,eb)}(t_\phi),F_{\alpha(fa,gb)}(t_\phi),F_{\alpha(fa,da)}(t_\phi),F_{\alpha(gb,eb)}(t_\phi),F_{\alpha(gb,da)}(t_\phi)) \ge 0,$$

(iii) *$f(\mathcal{Y})$ and $g(\mathcal{Y})$ are having closed property in $\mathcal{Y}$,*

(iv) *the mappings d,e are point wise f-absorbing, g-absorbing, respectively.*

Then, the four mappings d,e,f and g have a unique common fixed point in $\mathcal{Y}$.

Proof. By (i) the pairs (d,f) and (e,g) are sharing the common property (E.A) implies $\exists$ (c_m) and $(d_m) \in \mathcal{Y}$ such that

$$\lim_{m\to\infty} dc_m = \lim_{m\to\infty} fc_m = \lim_{m\to\infty} ed_m = \lim_{m\to\infty} gd_m = \mu, \tag{3.12}$$

for some $\mu \in \mathcal{Y}$.
By (iii) $f(\mathcal{Y})$ and $g(\mathcal{Y})$ closed subsets of $\mathcal{Y}$ resulting $\mu = \lim_{m\to\infty} fc_m \in f(\mathcal{Y})$ and $\mu = \lim_{\mu\to\infty} gd_m \in g(\mathcal{Y})$ implies $\mu \in f(\mathcal{Y})$ and $\mu \in g(\mathcal{Y})$.
So that there exist u and v in $\mathcal{Y}$ such that $\mu = fu = gv$.

Claim $\mu = du$.
By assigning $a = u, b = d_m$ in (3.1),

$$\phi(F_{\alpha(du,ed_m)}(t_\phi),F_{\alpha(fu,gd_m)}(t_\phi),F_{\alpha(fu,du)}(t_\phi),F_{\alpha(gd_m,ed_m)}(t_\phi),F_{\alpha(gd_m,du)}(t_\phi)) \ge 0 \text{ as } m \to \infty.$$

Using (3.12) and $\mu = fu$,

$$\phi(F_{\alpha(du,\mu)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi), F_{\alpha(\mu,du)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi), F_{\alpha(\mu,du)}(t_\phi)) \geq 0,$$

$$\phi(F_{\alpha(du,\mu)}(t_\phi), 1, F_{\alpha(\mu,du)}(t_\phi), 1, F_{\alpha(\mu,du)}(t_\phi)) \geq 0.$$

From 2.4(ii) implies

$$F_{\alpha(du,\mu)}(t_\phi) \geq 1 \text{ implies } F_{\alpha(du,\mu)}(t_\phi) = 1.$$

Thus $\mu = du$ implies

$$\mu = du = fu = gv. \tag{3.13}$$

Claim $ev = \mu$.
By assigning the values $a = u, b = v$ in (3.1) and using (3.13),

$$\phi(F_{\alpha(du,ev)}(t_\phi), F_{\alpha(fu,gv)}(t_\phi), F_{\alpha(fu,du)}(t_\phi), F_{\alpha(gv,ev)}(t_\phi), F_{\alpha(gv,du)}(t_\phi)) \geq 0,$$

$$\phi(F_{\alpha(\mu,ev)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi), F_{\alpha(\mu,ev)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi)) \geq 0,$$

$$\phi(F_{\alpha(\mu,ev)}(t_\phi), 1, 1, F_{\alpha(\mu,ev)}(t_\phi), 1) \geq 0.$$

From Example 2.14(ii) implies

$$F_{\alpha(\mu,ev)}(t_\phi) \geq 1 \implies \mu = ev.$$

Therefore, from (3.13), we have

$$\mu = du = fu = ev = gv. \tag{3.14}$$

The mapping d is point wise f-absorbing implies $\exists\, R \in Z^+$ such that

$$F_{\alpha(fu,fdu)}(t_\phi) \geq F_{\alpha(fu,du)}\left(\frac{t_\phi}{R}\right). \tag{3.15}$$

Eq. (3.14) implies

$$fu = fdu = ffu. \tag{3.16}$$

This gives

$$\mu = f\mu. \tag{3.17}$$

Claim $\mu = d\mu$,
By employing $a = \mu$, $b = v$ in (3.1),

$$\phi(F_{\alpha(d\mu,ev)}(t_\phi), F_{\alpha(f\mu,gv)}(t_\phi), F_{\alpha(f\mu,d\mu)}(t_\phi), F_{\alpha(gv,ev)}(t_\phi), F_{\alpha(gv,d\mu)}(t_\phi)) \geq 0.$$

Using (3.17),

$$\phi(F_{\alpha(d\mu,\mu)}(t_\phi), F_{\alpha(f\mu,\mu)}(t_\phi), F_{\alpha(f\mu,d\mu)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi), F_{\alpha(\mu,d\mu)}(t_\phi)) \geq 0.$$

This consequences

$$\phi(F_{\alpha(d\mu,\mu)}(t_\phi), 1, F_{\alpha(\mu,d\mu)}(t_\phi), 1, F_{\alpha(\mu,d\mu)}(t_\phi)) \geq 0,$$

$$F_{\alpha(d\mu,\mu)}(t_\phi) \geq 1.$$

This implies $d\mu = \mu$.

Hence

$$d\mu = f\mu = \mu. \tag{3.18}$$

The mapping e is point wise g-absorbing implies $\exists\, R \in Z^+$ such that

$$F_{\alpha(gv,gev)}(t_\phi) \geq F_{\alpha(gv,ev)}\left(\frac{t_\phi}{R}\right). \tag{3.19}$$

Then, from (3.19),

$$gv = gev = ggv. \tag{3.20}$$

This implies

$$\mu = g\mu. \tag{3.21}$$

Claim $e\mu = \mu$.
By assigning $a = \mu$, $b = \mu$ in (3.1), using (3.21)

$$\begin{aligned}
&\phi(F_{\alpha(d\mu,e\mu)}(t_\phi), F_{\alpha(f\mu,g\mu)}(t_\phi), F_{\alpha(f\mu,d\mu)}(t_\phi), F_{\alpha(g\mu,e\mu)}(t_\phi), F_{\alpha(g\mu,d\mu)}(t_\phi)) \geq 0,\\
&\phi(F_{\alpha(\mu,e\mu)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi), F_{\alpha(\mu,e\mu)}(t_\phi), F_{\alpha(\mu,\mu)}(t_\phi)) \geq 0,\\
&\phi(F_{\alpha(\mu,e\mu)}(t_\phi), 1, 1, F_{\alpha(\mu,e\mu)}(t_\phi), 1) \geq 0,\\
&F_{\alpha(\mu,e\mu)}(t_\phi) \geq 1.
\end{aligned}$$

Hence

$$\mu = e\mu. \tag{3.22}$$

From (3.18), (3.20), (3.21) implies we get

$$d\mu = f\mu = e\mu = g\mu = \mu.$$

Uniqueness follows from Theorem 3.1. □

Now our theorem is validated by discussing suitable example.

Example 3.4. Let $(\mathcal{Y}, F_\alpha, t_\phi)$ be a induced fuzzy Menger space, where $\mathcal{Y} = [0, \sqrt{2}]$. Define the mappings as

$$d(a) = e(a) = \begin{cases} \sqrt{2}\cos(2\pi a), & \text{if } a \in [0, \frac{1}{4}],\\ 0, & \text{if } a \in (\frac{1}{4}, \frac{1}{2}],\\ a, & \text{if } a \in (\frac{1}{2}, \sqrt{2}], \end{cases} \tag{3.23}$$

$$f(a) = g(a) = \begin{cases} \sqrt{2}\sin(2\pi a), & \text{if } a \in [0, \frac{1}{4}],\\ 1, & \text{if } a \in (\frac{1}{4}, \frac{1}{2}],\\ \frac{a^2}{\sqrt{2}}, & \text{if } a \in (\frac{1}{2}, \sqrt{2}]. \end{cases} \tag{3.24}$$

Choose $(a_m) = \sqrt{2} - \frac{1}{m}$, for all $m \geq 1$.
Then from (3.23) and (3.24),

$$\lim_{m\to\infty} da_m = \lim_{m\to\infty}\left(\sqrt{2} - \frac{1}{m}\right) = \sqrt{2}, \tag{3.25}$$

$$\lim_{m\to\infty} fa_m = \lim_{m\to\infty}\frac{1}{\sqrt{2}}\left(\sqrt{2} - \frac{1}{m}^2\right) = \sqrt{2}. \tag{3.26}$$

From (3.25) and (3.26),

$$\lim_{m\to\infty} da_m = \lim_{m\to\infty} fa_m = \sqrt{2}. \tag{3.27}$$

Thus the pairs (d,f) and (e,g) satisfy the common property (E.A).

If $a \in [0, \frac{1}{4}]$,

$$\begin{aligned} & d(f(a), fd(a)) \le Rd(f(a), d(a)) \\ \Longrightarrow \quad & d(\sqrt{2}\sin(2\pi a), \sqrt{2}\sin(2\pi(\sqrt{2}\cos(2\pi a))) \le Rd(\sqrt{2}\sin(2\pi a), \sqrt{2}\cos(2\pi a)). \end{aligned}$$

It is true for $2 \le R$.

If $a \in (\frac{1}{4}, \frac{1}{2}]$,

$$\begin{aligned} & d(f(a), fd(a)) \le Rd(f(a), d(a)) \\ \Longrightarrow \quad & d(1,0) \le Rd(1,0) \\ \Longrightarrow \quad & 1 \le R. \end{aligned}$$

If $a \in (\frac{1}{2}, \sqrt{2}]$,

$$\begin{aligned} & d(f(a), fd(a)) \le Rd(f(a), d(a)) \\ \Longrightarrow \quad & d\left(\frac{a^2}{\sqrt{2}}, \frac{a^2}{\sqrt{2}}\right) \le Rd\left(\frac{a^2}{\sqrt{2}}, a\right) \\ \Longrightarrow \quad & 0 \le R. \end{aligned}$$

Hence, we can choose $R \ge 2$.

Moreover from (3.23) and (3.24), $f(\mho) = g(\mho) = [0, \sqrt{2}]$ are closed subsets, the mappings d, e are point wise f-absorbing and point wise g-absorbing. Further, the pairs (d,f), (e,g) are occasionally weakly compatible but not weakly compatible.

Because $\exists\ a = 2$ with $e(2) = g(2)$ implies $eg(2) = ge(2)$, where as at $a = \frac{1}{8}$, $e\left(\frac{1}{8}\right) = g\left(\frac{1}{8}\right) = 1$, $eg\left(\frac{1}{8}\right) = e(1) = 1$, $ge\left(\frac{1}{8}\right) = g(1) = \frac{1}{\sqrt{2}} \Longrightarrow eg\left(\frac{1}{4}\right) \ne ge\left(\frac{1}{4}\right)$.

Moreover at $a = \sqrt{2}$, $d(\sqrt{2}) = f(\sqrt{2}) = e(\sqrt{2}) = g(\sqrt{2}) = \sqrt{2}$.

Therefore, $a = \sqrt{2}$ is the unique common fixed point mappings d, f, e and g and fulfilled all conditions of Theorem 3.3.

4. Conclusion

We generalized Theorem 2.15 in fuzzy Menger space by using

(i) the pairs (d,f), (e,g) are sharing the common property (E.A) along with the map d is f-absorbing and the second pair (e,g) is occasionally weakly compatible mapping in Theorem 3.1,

(ii) the pairs (d,f), (e,g) are sharing the common property (E.A) along with the mappings d, e are assumed to be point wise f-absorbing and point wise g-absorbing respectively in Theorem 3.3.

Further, these two theorems are justified by a suitable examples.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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