Communications in Mathematics and Applications
Vol. 14, No. 1, pp. 203–213, 2023
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v14i1.2040

http://www.rgnpublications.com

Research Article

A Goal Programming Approach to Solve Multi-objective Chance Constrained Programming in Fuzzy Environment

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Received: August 31, 2022 **Accepted:** January 10, 2023

Abstract. A new solution process is presented to solve multi-objective fuzzy chance constrained nonlinear decision making problems using goal programming techniques. The right sided parameters of probabilistic constraint are assumed to follow Rayleigh distribution with known parameters whereas the constraints coefficients are trapezoidal fuzzy numbers. The stochastic constraints are transformed into fuzzy constraints using CCP technique and α-cut techniques are applied to obtain the identical crisp nonlinear programming problem. The crisp MONLPP is solved by goal programming by means of membership and non-membership functions. The proposed solution methodology is validated by an example.

Keywords. Multi objective fuzzy chance constrained nonlinear programming problem, MOFCCNLPP, Trapezoidal fuzzy numbers, Rayleigh distribution, Goal programming

Mathematics Subject Classification (2020). 90C20, 03E72, 03F55

1. Introduction

Most of the managerial problems are defined and drafted by multiple and incompatible paradigms. Such circumstances are usually reckoned by multi-objective functions. Due to imprecise and ambiguous information of many industrial and engineering problems the fuzziness and randomness are consider under one roof by the decision makers. Barik and Biswal [2] formulated chance constrained quadratic programming problems where randomness is characterized by weibull distribution. Using goal programming approach, Masoud *et al*. [8] investigated a stochastic linear programming with multi-objective functions, in which the probabilistic parameters have been normally distributed. Dalman and Bayram [5] developed an interactive fuzzy goal programming based on Taylor's series to solve MONLPP. A new method is recommended to solve MOFCCNLPP in this study. At first probabilistic constraints are converted into fuzzy constraints using CCP technique then obtained MOFNLPP is solved and a goal programming approach is applied to get compromise solution.

2. Preliminaries

Definition 2.1 (Trapezoidal Fuzzy Number (TrFN) [4]). *A TrFN $\widetilde{A}$ can be characterized by $\widetilde{A} = \langle a', a'', a''', a''''\rangle$ where $a' < a'' < a''' < a''''$ whose membership function is specified by*

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{a'-x}{a'-a''}, & a' \le x \le a'', \\ 1, & a'' \le x \le a''', \\ \dfrac{a''''-x}{a''''-a'''}, & a''' \le x \le a'''', \\ 1, & elsewhere. \end{cases}$$

Definition 2.2 (α-Cut of Trapezoidal Fuzzy Number [1]). *Let $\widetilde{A} = \langle a', a'', a''', a''''\rangle$ be the TrFN. Its α-cut is described by*

$$\widetilde{A}(\alpha) = [\underline{\widetilde{A}}(\alpha), \overline{\widetilde{A}}(\alpha)] = [a' + \alpha(a''-a')a'''' - \alpha(a''''-a''')].$$

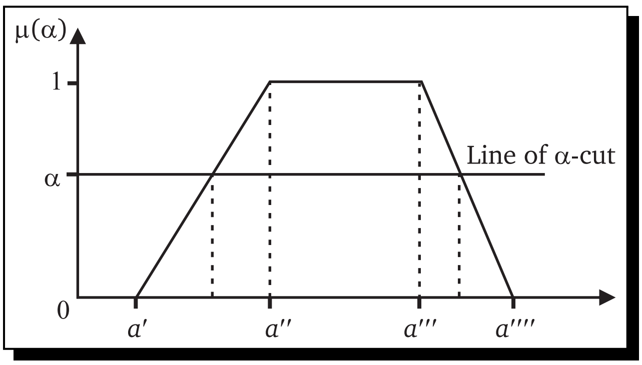

Figure 1

Definition 2.3 (Rayleigh Distribution-Statistical Preliminary). *A continuous random variable b is said to follow Rayleigh distribution if its probability density function is given by*

$$f(b_i) = \frac{b_i}{\sigma_i^2} e^{-\frac{b_i^2}{2\sigma_i^2}}, \quad i = 1,2,\dots m, \text{ where } \sigma \text{ is known parameter.}$$

3. Mathematical Formulation of Problems

3.1 Multi-Objective Chance Constrained Nonlinear Programming (MOCCNLPP)

The formulation of MOSNLP can be written as

$$\max\ f^{(k)} = \sum_{j=1}^{n} c_j^{(k)} x_j^{(p)}\ ,\quad p = 2,3,4,\dots,\ k = 1,2,\dots,K$$

$$\text{subject to Prob}\left[\sum_{j=1}^{n} a_{ij}x_j \le b_i\right] \ge 1-\gamma_i, \quad i = 1,2,\dots,m,\ j = 1,2,\dots,n$$

$$x_j \ge 0.$$

3.2 Multi-Objective Fuzzy Chance Constrained Nonlinear Programming (MOFCCNLPP)

The formulation of FMOSNLP can be stated as

$$\max\ \widetilde{f}^{(k)} = \sum_{j=1}^{n} c_j^{(k)} x_j^{(p)},\quad p = 2,3,4\dots,\ k = 1,2,\dots,K$$

$$\text{subject to Prob}\left[\sum_{j=1}^{n} \widetilde{a}_{ij}x_j \le b_i\right] \ge 1-\gamma_i, \quad i = 1,2,\dots,m;\ j = 1,2,\dots,n$$

$$x_j \ge 0$$

where the constraint coefficients $\widetilde{a}_{ij}$ are to be trapezoidal fuzzy numbers and right hand of the constraint b_i $(i = 1,2,\dots,m)$ are independent chance variables which follows Rayleigh distribution with known parameters and $\widetilde{\sigma}_i$ which is also assumed as trapezoidal fuzzy number.

4. Algorithm to Solve MOFCCNLPP

The flowchart of proposed solution procedure of solving MOFCCNLPP is given below:

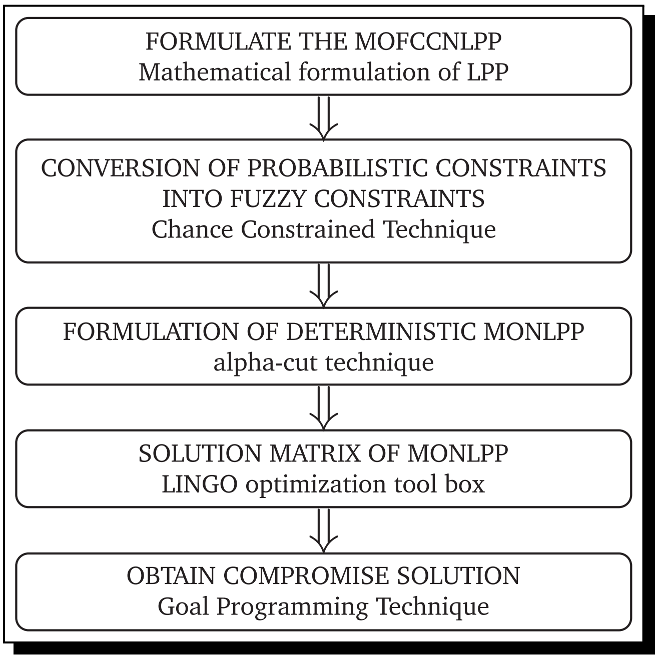

Figure 2

Step 1: Formulate the multi-objective fuzzy chance constrained nonlinear programming problem of the given real time problem or industrial problem using mathematical formulation of LPP/NLPP technique.

Step 2: The fuzzy probabilistic constraint is remodeled into equivalent fuzzy constraint using the following theorem.

Theorem 4.1. *If b_i, $i = 1,2,3,\dots,m$ are independent chance variables which follows Rayleigh distribution with known parameter $\tilde{\sigma}_i$ then* $\text{Prob}\left[\sum_{j=1}^{n} \tilde{a}_{ij}x_j \le b_i\right] \ge 1-\gamma_i$ *is equivalent to*

$$\sum_{j=1}^{n} \tilde{a}_{ij}x_j \le \left(2\tilde{\sigma}_i^2 \ln\left(\frac{1}{1-\gamma_i}\right)\right)^{\frac{1}{2}}.$$

Proof. The probability density function of chance variable b_i is given by

$$f(b_i) = \frac{b_i}{\tilde{\sigma}_i^2} e^{-\frac{b_i^2}{2\tilde{\sigma}_i{}^2}}, \quad i = 1,2,\dots,m.$$

Using chance constrained technique

$$\int_{y_i}^{\infty} f(b_i)db_i \ge 1-\gamma_i, \quad \text{where } y_i = \sum_{j=1}^{n} \tilde{a}_{ij}x_j$$

$$\cong \int_{y_i}^{\infty} \frac{b_i}{\tilde{\sigma}_i^2} e^{-\frac{b_i^2}{2\tilde{\sigma}_i^2}} db_i \ge 1-\gamma_i$$

$$\cong \int_{\frac{y_i^2}{2\tilde{\sigma}_i^2}}^{\infty} e^{-t}dt \ge 1-\gamma_i$$

$$y_i^2 \le 2\tilde{\sigma}_i^2 \ln\left(\frac{1}{1-\gamma_i}\right),$$

$$y_i \le \left(2\tilde{\sigma}_i^2 \ln\left(\frac{1}{1-\gamma_i}\right)\right)^{\frac{1}{2}},$$

$$\sum_{j=1}^{n} \tilde{a}_{ij}x_j \le \left(2\tilde{\sigma}_i^2 \ln\left(\frac{1}{1-\gamma_i}\right)\right)^{\frac{1}{2}}.$$

The equivalent mathematical form of MOFCCNLPP is formulated by

$$\max \tilde{f}^{(k)} = \sum_{j=1}^{n} c_j^{(k)} x_j^{(p)}, \quad p = 2,3,4\dots, \; k = 1,2,\dots,K$$

$$\text{subject to } \sum_{j=1}^{n} \tilde{a}_{ij}x_j \le \left(2\tilde{\sigma}_i^2 \ln\left(\frac{1}{1-\gamma_i}\right)\right)^{\frac{1}{2}}, \quad i = 1,2,\dots,m, \; j = 1,2,\dots,n$$

$$x_j \ge 0.$$

□

Step 3: To obtain the deterministic MONLPP using α-cut of trapezoidal fuzzy numbers
Make use of α-cut of trapezoidal fuzzy numbers, the MOFCCNLPP is modernized into its equivalent deterministic multi-objective nonlinear programming problem MONLPP, which is

stated below:

$$\max \widetilde{f}^{(k)} = \sum_{j=1}^{n} c_j^{(k)} x_j^p, \quad p = 2,3,4\ldots, \quad k = 1,2,\ldots,K$$

$$\text{subject to } \sum_{j=1}^{n} \underline{\widetilde{a}}_{ij}(\alpha)\overline{\widetilde{a}}_{ij}(\alpha)x_j \leq \underline{\widetilde{\sigma}}_{ii}(\alpha)\overline{\widetilde{\sigma}}_i(\alpha)\left(2\ln\left(\frac{1}{1-\gamma_i}\right)\right)^{\frac{1}{2}}, \quad i = 1,2,\ldots,m$$

$$x_j \geq 0$$

This model can be decomposed into

$$\max \widetilde{f}^{(k)} = \sum_{j=1}^{n} c_j^{(k)} x_j^p, \quad p = 2,3,4\ldots, \quad k = 1,2,\ldots,K$$

$$\text{subject to } \sum_{j=1}^{n} \underline{\widetilde{a}}_{ij}(\alpha)x_j \leq \underline{\widetilde{\sigma}}_{ii}(\alpha)\left(2\ln\left(\frac{1}{1-\gamma_i}\right)\right)^{\frac{1}{2}}, \quad i = 1,2,\ldots,m$$

$$\sum_{j=1}^{n} \overline{\widetilde{a}}_{ij}(\alpha)x_j \leq \overline{\widetilde{\sigma}}_i(\alpha)\left(2\ln\left(\frac{1}{1-\gamma_i}\right)\right)^{\frac{1}{2}}, \quad i = 1,2,\ldots,m$$

$$x_j \geq 0; \ 0 \leq \alpha \leq 1.$$

Step 4: Construct the payoff matrix by finding ideal solutions.

Solve the MONLPP by considering single objective function at a time and ignoring others subject to the set of constraints. Obtain k-different solutions by repeating the process K-times for k-independent objective functions $\widetilde{f}^{(k)} = [\widetilde{f}^{(1)}, \widetilde{f}^{(2)}, \ldots, \widetilde{f}^{(k)}]$, $k = 1,2,\ldots,K$. Let $X^{(1)}, X^{(2)}, \ldots, X^{(k)}$ be the k ideal solutions for K different nonlinear programming problem. In accordance with each solution $X^{(i)}$, $i = 1,2,\ldots,k$ construct the solution payoff matrix of order k as follows:

$$\begin{array}{c} \\ X^{(1)} \\ X^{(2)} \\ \ldots \\ X^{(k)} \end{array} \begin{array}{c} \begin{array}{cccc} f^1(x) & f^2(x) & \ldots & f^k(x) \end{array} \\ \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1k} \\ f_{21} & f_{22} & \cdots & f_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ f_{k1} & f_{k2} & \cdots & f_{kk} \end{bmatrix} \end{array}$$

Step 5: Obtain optimal compromise solution using Zimmermann goal programming technique. Identify the lower bound L^k and upper bound U^k from the payoff matrix for each objective function f^k such that $L^k \leq f^k \leq U^k$ where $U^k = \max(f_{1k}, f_{2k}, \ldots, f_{kk})$; $k = 1,2,\ldots K$ and $L^k = f_{kk}$.

Define the nonlinear membership function for the kth objective function $f^k(x)$ as

$$\mu(f^k(x)) = \begin{cases} 0, & L^k \geq f^k, \\ 1 - \dfrac{f^k - U^k}{L^k - U^k}, & L^k \leq f^k \leq U^k, \\ 1, & U^k \geq f^k. \end{cases}$$

Using the membership function the identical crisp nonlinear programming problem is obtained by

Minimize λ

$$\text{subject to } \sum_{j=1}^{n} c_j^{(k)} x_j^p + (U_k - L_k)\lambda \geq U_k, \quad k = 1,2,\dots,K$$

$$\sum_{j=1}^{n} \underset{\sim}{\tilde{a}}_{ij}(\alpha) x_j \leq \underset{\sim}{\tilde{\sigma}}_{ii}(\alpha)\left(2\ln\left(\frac{1}{1-\gamma_i}\right)\right)^{\frac{1}{2}}, \quad i = 1,2,\dots,m$$

$$\sum_{j=1}^{n} \overline{\tilde{a}}_{ij}(\alpha) x_j \leq \overline{\tilde{\sigma}}_{i}(\alpha)\left(2\ln\left(\frac{1}{1-\gamma_i}\right)\right)^{\frac{1}{2}}, \quad i = 1,2,\dots,m, \ x_j \geq 0.$$

Solve the crisp nonlinear programming problem obtain the optimal compromise solution X^* and determine the values of all objective function at X^*.

5. Numerical Example

A manufacturing industry produces two machines parts P_1 and P_2 in a period. The production of the parts is processed by three machines such as lathe machine, milling machine and grinding machine. The required machining times for the machine parts are shown below:

Table 1

Type of machine	Machining time	
	P_1	P_2
Lathe machine	$\widetilde{10}$	$\tilde{5}$
Milling machine	$\tilde{4}$	$\widetilde{10}$
Grinding machine	$\tilde{1}$	$\tilde{2}$

The manufacturer has agreed with two dealers to sell his produced machine parts in the market. The cost functions and selling price of two dealers are2 as follows:

Table 2

Dealers	Selling price (in \$)		Cost function	
	P_1	P_2	P_1	P_2
D I	6	8	2	3
D II	6	8	3	4

The problem is to find how much of each product should be produced per month such that to maximize the cost functions of both dealers. Due to uncertainty of real situation it is assumed that all machining times are trapezoidal fuzzy parameters. Also, the constraints have to satisfy with a probability 0.95, 0.94 and 0.92, respectively. Using the proposed solution algorithm this problem can be solved as follows:

Step 1 - *Model* (i): The mathematical formulation of given problem can be stated as:

$$\max\ f^1(x) = 6\tilde{x}_1 + 8\tilde{x}_2 - 2\tilde{x}_1^2 - 3\tilde{x}_2^2$$

$$\max\ f^2(x) = 6\widetilde{x}_1 + 8\widetilde{x}_2 - 3\widetilde{x}_1^2 - 4\widetilde{x}_2^2$$

$$\text{subject to}\ \ \mathrm{Prob}[\widetilde{10}\widetilde{x}_1 + \widetilde{5}\widetilde{x}_2 \le b_1] \le 0.95$$

$$\mathrm{Prob}[\widetilde{4}\widetilde{x}_1 + \widetilde{10}\widetilde{x}_2 \le b_2] \le 0.94$$

$$\mathrm{Prob}[\widetilde{1}\widetilde{x}_1 + \widetilde{2}\widetilde{x}_2 \le b_3] \le 0.92$$

$$x_1, x_2 \ge 0$$

where b_i follows Rayleigh distribution with known trapezoidal fuzzy parameter $\widetilde{\sigma}_i$.

The constraints coefficients are given in Table 3.

Table 3

$\widetilde{10}$	$\langle 9.5, 10, 11, 12, 5\rangle$	$\widetilde{5}$	$\langle 8, 9, 10, 11\rangle$
$\widetilde{4}$	$\langle 4, 5, 6, 7\rangle$	$\widetilde{10}$	$\langle 0.5, 1, 1.5, 2\rangle$
$\widetilde{1}$	$\langle 3.6, 4.2, 4.5, 5.6\rangle$	$\widetilde{2}$	$\langle 1.4, 1.6, 2, 2.2\rangle$

Step 2 - *Model* (ii): Using Theorem 4.1, the equivalent MOFNLPP of Model 1 is given as below:

$$\max\ f^1(x) = 6\widetilde{x}_1 + 8\widetilde{x}_2 - 2\widetilde{x}_1^2 - 3\widetilde{x}_2^2$$

$$\max\ f^2(x) = 6\widetilde{x}_1 + 8\widetilde{x}_2 - 3\widetilde{x}_1^2 - 4\widetilde{x}_2^2$$

$$\text{subject to}\ \ \widetilde{10}\widetilde{x}_1 + \widetilde{5}\widetilde{x}_2 \le \widetilde{\sigma}_1(0.3203)$$

$$\widetilde{4}\widetilde{x}_1 + \widetilde{10}\widetilde{x}_2 \le \widetilde{\sigma}_2(0.3518)$$

$$\widetilde{1}\widetilde{x}_1 + \widetilde{2}\widetilde{x}_2 \le \widetilde{\sigma}_3(0.4084)$$

$$x_1, x_2 \ge 0$$

where the parameters $\widetilde{\sigma}_i$ are given in Table 4.

Table 4

$\widetilde{\sigma}_1$	$\widetilde{25}$	$\langle 21, 23, 25, 26\rangle$
$\widetilde{\sigma}_2$	$\widetilde{27}$	$\langle 26, 28, 30, 33\rangle$
$\widetilde{\sigma}_3$	$\widetilde{23}$	$\langle 21, 24, 26, 30\rangle$

Step 3 - Model (iii): Using α-cut of trapezoidal fuzzy numbers model(ii) is reformulated into MONLPP.

$$\max\ f^1(x) = 6\widetilde{x}_1 + 8\widetilde{x}_2 - 2\widetilde{x}_1^2 - 3\widetilde{x}_2^2$$

$$\max\ f^2(x) = 6\widetilde{x}_1 + 8\widetilde{x}_2 - 3\widetilde{x}_1^2 - 4\widetilde{x}_2^2$$

$$\text{subject to}\ \ \left[\underline{\widetilde{10}}(\alpha), \overline{\widetilde{10}}(\alpha)\right]\widetilde{x}_1 + \left[\underline{\widetilde{5}}(\alpha), \overline{\widetilde{5}}(\alpha)\right]\widetilde{x}_2 \le \left[\underline{\widetilde{\sigma}}_1(\alpha), \overline{\widetilde{\sigma}}_1(\alpha)\right](0.3203)$$

$$\left[\underline{\widetilde{4}}(\alpha), \overline{\widetilde{4}}(\alpha)\right]\widetilde{x}_1 + \left[\underline{\widetilde{10}}(\alpha), \overline{\widetilde{10}}(\alpha)\right]\widetilde{x}_2 \le \left[\underline{\widetilde{\sigma}}_2(\alpha), \overline{\widetilde{\sigma}}_2(\alpha)\right](0.3518)$$

$$\left[\underline{\widetilde{1}}(\alpha), \overline{\widetilde{1}}(\alpha)\right]\widetilde{x}_1 + \left[\underline{\widetilde{2}}(\alpha), \overline{\widetilde{2}}(\alpha)\right]\widetilde{x}_2 \le \left[\underline{\widetilde{\sigma}}_3(\alpha), \overline{\widetilde{\sigma}}_3(\alpha)\right](0.4084)$$

$$x_1, x_2 \ge 0,\ 0 \le \alpha \le 1$$

Step 4 - *Model* (iv): Model (iii) is decomposed and solved by taking single objective at a time and repeat the process to obtain the payoff matrix of ideal solutions.

$$\max\ f^1(x) = 6\tilde{x}_1 + 8\tilde{x}_2 - 2\tilde{x}_1^2 - 3\tilde{x}_2^2$$
$$\max\ f^2(x) = 6\tilde{x}_1 + 8\tilde{x}_2 - 3\tilde{x}_1^2 - 4\tilde{x}_2^2$$
$$\text{subject to}\ \ \underline{\widetilde{10}}(\alpha)\tilde{x}_1 + \underline{\tilde{5}}(\alpha)\tilde{x}_2 \le \underline{\tilde{\sigma}}_1(\alpha)(0.3203)$$
$$\overline{\widetilde{10}}(\alpha)\tilde{x}_1 + \overline{\tilde{5}}(\alpha)\tilde{x}_2 \le \overline{\tilde{\sigma}}_1(\alpha)(0.3203)$$
$$\underline{\tilde{4}}(\alpha)\tilde{x}_1 + \underline{\widetilde{10}}(\alpha)(\alpha)\tilde{x}_2 \le \underline{\tilde{\sigma}}_2(\alpha)(0.3518)$$
$$\overline{\tilde{4}}(\alpha)\tilde{x}_1 + \overline{\widetilde{10}}(\alpha)\tilde{x}_2 \le \overline{\tilde{\sigma}}_2(\alpha)(0.3518)$$
$$\underline{\tilde{1}}(\alpha)\tilde{x}_1 + \underline{\tilde{2}}(\alpha)\tilde{x}_2 \le \underline{\tilde{\sigma}}_3(\alpha)(0.4084)$$
$$\overline{\tilde{1}}(\alpha)\tilde{x}_1 + \overline{\tilde{2}}(\alpha)\tilde{x}_2 \le \overline{\tilde{\sigma}}_3(\alpha)(0.4084)$$
$$x_1, x_2 \ge 0,\ 0 \le \alpha \le 1.$$

Using Definition 2.2 Model (iv) can be rewritten as

Model (v):

$$\max\ f^1(x) = 6\tilde{x}_1 + 8\tilde{x}_2 - 2\tilde{x}_1^2 - 3\tilde{x}_2^2$$
$$\max\ f^2(x) = 6\tilde{x}_1 + 8\tilde{x}_2 - 3\tilde{x}_1^2 - 4\tilde{x}_2^2$$
$$(12.5 - 1.5\alpha)\tilde{x}_1 + (7-\alpha)\tilde{x}_2 \le (26-\alpha)(0.3203)$$
$$(9.5 + 0.5\alpha)\tilde{x}_1 + (8+\alpha)\tilde{x}_2 \le (26+2\alpha)(0.3203)$$
$$(5.6 - 1.1\alpha)\tilde{x}_1 + (11-\alpha)\tilde{x}_2 \le (33-3\alpha)(0.3518)$$
$$(3.6 + 0.6\alpha)\tilde{x}_1 + (8+\alpha)\tilde{x}_2 \le (26+2\alpha)(0.3518)$$
$$(1.2 - 0.3\alpha)\tilde{x}_1 + (2.2-0.2\alpha)\tilde{x}_2 \le (30-4\alpha)(0.4084)$$
$$(0.5 + 0.3\alpha)\tilde{x}_1 + (1.4+0.2\alpha)\tilde{x}_2 \le (21+3\alpha)(0.4084)$$
$$x_1, x_2 \ge 0,\ 0 \le \alpha \le 1$$

By considering the objective function f^1 and f^2 separately, subject to the same set of constraints and solving for different values of α, the solution obtained is tabulated below.

Table 5

α	x_1	x_2	f^1
0	0.1926756	0.845622	5.701552
0.2	0.2028263	0.8513558	5.771107
0.4	0.2133678	0.8576701	5.843722
0.6	0.2245397	0.8642093	5.919503
0.8	0.2363481	0.8711075	5.998743
1	0.2488463	0.8783651	6.081574

Table 6

α	x_1	x_2	f^2
0	0.27641	0.69609	5.05982
0.2	0.28627	0.70166	5.11571
0.4	0.29668	0.70746	5.17369
0.6	0.30767	0.71354	5.23379
0.8	0.31931	0.7199	5.29617
1	0.33163	0.72659	5.36084

Step 5: Using proposed goal programming using nonlinear membership function the optimal compromise solution is obtained by solving Model (vi).

Minimize λ

subject to $6\tilde{x}_1+8\tilde{x}_2-2\tilde{x}_1^2-3\tilde{x}_2^2(U_1-L_1)\lambda \ge U_1$

$$6\tilde{x}_1+8\tilde{x}_2-3\tilde{x}_1^2-4\tilde{x}_2^2(U_2-L_2)\lambda \ge U_2$$
$$(12.5-1.5\alpha)\tilde{x}_1+(7-\alpha)\tilde{x}_2 \le (26-\alpha)(0.3203)$$
$$(9.5+0.5\alpha)\tilde{x}_1+(8+\alpha)\tilde{x}_2 \le (26+2\alpha)(0.3203)$$
$$(5.6-1.1\alpha)\tilde{x}_1+(11-\alpha)\tilde{x}_2 \le (33-3\alpha)(0.3518)$$
$$(3.6+0.6\alpha)\tilde{x}_1+(8+\alpha)\tilde{x}_2 \le (26+2\alpha)(0.3518)$$
$$(1.2-0.3\alpha)\tilde{x}_1+(2.2-0.2\alpha)\tilde{x}_2 \le (30-4\alpha)(0.4084)$$
$$(0.5+0.3\alpha)\tilde{x}_1+(1.4+0.2\alpha)\tilde{x}_2 \le (21+3\alpha)(0.4084)$$
$$x_1,x_2 \ge 0,\ 0 \le \alpha \le 1$$

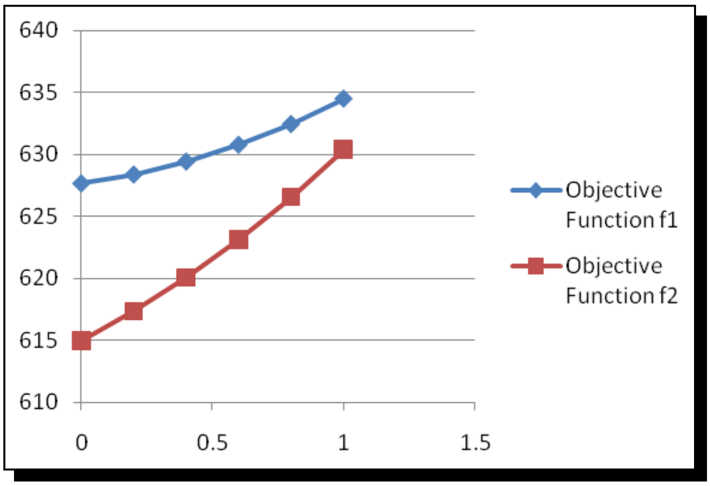

Figure 3

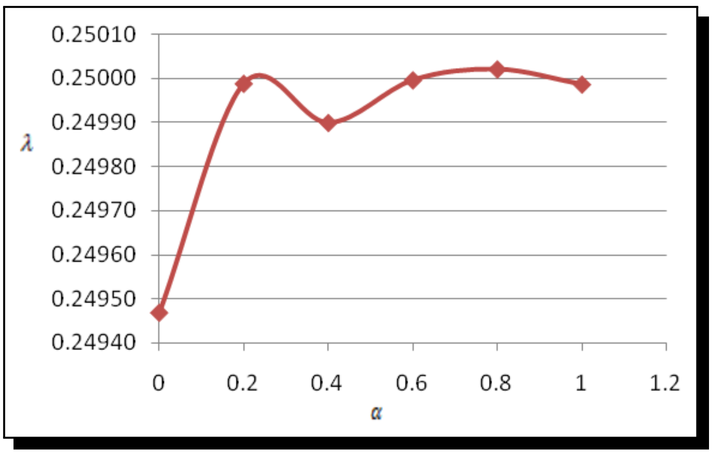

Figure 4. Optimal compromise solution

The optimal compromise solution for different values of α are given in Tables 7 and 8.

Table 7

α	U_1	U_2	L_1	L_2	$U_1 - L_1$	$U_2 - L_2$
0	5.70155	5.05982	5.62077	4.94935	0.08079	0.11047
0.2	5.77111	5.11571	5.68998	5.00516	0.08113	0.11055
0.4	5.84372	5.17369	5.76221	5.06260	0.08151	0.11110
0.6	5.91950	5.23379	5.83759	5.12223	0.08191	0.11156
0.8	5.99874	5.29617	5.91638	5.18405	0.08236	0.11211
1	6.08157	5.36084	5.99876	5.24812	0.08282	0.11272

Table 8

α	x_1	x_2	λ	f^1	f^2
0	0.23459	0.77078	0.24947	5.68140	5.03226
0.2	0.24455	0.77650	0.24999	5.75083	5.08807
0.4	0.25503	0.78255	0.24990	5.82335	5.14593
0.6	0.26611	0.78886	0.25000	5.89902	5.20590
0.8	0.27783	0.79550	0.25002	5.97815	5.26814
1	0.29024	0.80247	0.24999	6.06087	5.33266

6. Conclusion

A new solution technique has been developed to obtain the optimal compromise solution for a multi-objective fuzzy chance constrained nonlinear programming problem in which the constraint coefficients are supposed to be trapezoidal fuzzy parameters and the probabilistic constraints follow Rayleigh distribution. This technique is very helpful to solve various industrial and decision-making problems in which fuzziness and randomness are altogether with multiple objectives. This work can be extended to solve bi-objective geometric programming where the random variables follow different types of distribution and with other types of fuzzy parameters.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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