



# MHD Stagnation-Point Flow with Viscous Dissipation and Chemical Reaction Effects: Numerical Study

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Received: August 27, 2022

Accepted: January 18, 2023

**Abstract.** In this article, a numerical study of the magnetohydrodynamics flow with heat and mass diffusion of an electrically conducting stagnation point flow past a shrinking/stretching sheet with chemical reaction of diffusing species and internal heat absorption/generation is analyzed. Flow equations are modified to a system of non-linear *Ordinary Differential Equations* (ODE) by using the similarity transformations. Numerical solution of the *Ordinary Differential Equations* (ODE) is found by using the shooting technique with Adam's Moulton method of order four. Finally, the results are discussed for different parameters affecting the flow and transfer of heat.

**Keywords.** MHD, Viscous dissipation, Chemical reaction, Stretching sheet, Adams-Moulton method

**Mathematics Subject Classification (2020).** 65L05, 76W05, 79D10

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## 1. Introduction

Fluid is serving as a source of life for human beings and human beings have always curiosity for discovering nature and fluid is an important factor of nature, so it attracts human. It also attracted many scientists to understand the patterns of flow of sea to the smallest pond.

Archimedes was first who investigated fluid statics and buoyancy, and formulated his famous law known as the Archimedes principle, which was published in his work “On floating bodies” generally considered to be the first major work on fluid mechanics. Rapid advancement in fluid mechanics began in fifteenth century. Leonardo Da Vinci responded to this attraction by observing and recording the phenomenon that we recognize today as a fundamental law of physics, namely the law of mass conservation. In this regard, Da Vinci was the first person who took the task of making sketches of different fields of flow. From the time of Da Vinci, there has been a remarkable change in the studies of fluid dynamics. The study of boundary layer flow of an incompressible viscous fluid over a shrinking sheet has many applications in manufacturing industries, such as the shrinking sheet flows happen in some applicable circumstances like, for glass fiber production, for paper production, also for metal and polymer processing. In both theory and practice, for all at rest, motion and linearly shrinking surfaces, heat transfer effects are important for boundary layer, MHD and stagnation point flows. For manufacturing process, these flows have applications in industries. Boundary layer applications include material handling along conveyers, the aerodynamics, blood flowing problems, extrusion of plastic sheets, in a bath the cooling of metallic plate, in paper and textile industries (Ashraf and Ashraf [2]). Boundary layer for incompressible, steady fluid of a viscous flow over a linearly fluctuating stretching sheet considered first time by Crane [10]. Using least square technique to minimize residual of a differentiable equation and the result for the problem approximated for laminar steady flow of electrically conducting a viscous incompressible flow over a stretching sheet analyzed by Chakraborty and Mazumdar [6]. Ishak *et al.* [17] studied electrical and an incompressible viscous fluid with magnetic field of two-dimensional stagnation-point over a stretching vertical sheet.

On a smooth plate, Hiemenz [14] considered the classical stagnation point flow of two dimension and the axisymmetric case was extended by Homann [15]. Viscous flow of magneto hydrodynamic fluid over a shrinking surface analyzed by Noor *et al.* [24] and the results obtained by Homotopy analysis method and Adomain decomposition was the same. The parametric influence of radiation while resolving the problematic cases of MHD stable and irregular flow of an electrical directing flow on infinite semi-inactive surface was discussed numerically by Raptis *et al.* [25]. The Homotopy analysis method to solve the problem of viscous fluid with stagnation point flow with a stretching sheet was used by Nadeem *et al.* [22]. The dual solution for stagnation-point of mixed convection flow over a vertical sheet represented by Ishak *et al.* [16, 18, 19]. On a vertical sheet in a porous medium, the mixed convection boundary layer stagnation-point flow with slip condition studied by Harris *et al.* [13]. Recently, Aziz [3] used local similarity with slip boundary condition over a at surface with constant heat flux. The MHD slip flow over a at plate and the steady slip flow with porous medium discussed by Bhattacharyya *et al.* [5], and Mukhopadhyay *et al.* [21]. Recently, Rohni *et al.* [26] reported mixed boundary-layer convection of unsteady flow with numerical investigations near stagnation point of two-dimensional flow on a porous surface vertical with thermal slip condition. The chemical reaction with the diffusion of species for the boundary layer fluid have numerous applications

in atmosphere pollution, water, fluids relevant to atmosphere and many other problems of chemical engineering. For boundary layer laminar flow of reactive chemically species with the diffusion which are used by a body over the surface considered by Chambré and Young [7]. For non-Newtonian fluids and their solution for the species of diffusion with chemical reactive in a flow over a stretching sheet with porous medium reported by Akyildiz *et al.* [1]. Cortell [9] also discuss the two types of viscoelastic fluid over a porous stretching sheet with the chemically reactive species. Hiemenz flow through porous media considered by Chamka and Khaled [8] with the presence of magnetic field. Heat transfer with steady condition considered by Sriramalu *et al.* [28] for incompressible viscous fluid with porous type species over a stretching surface. Khan *et al.* [20] discussed MHD viscoelastic fluid, transfer of mass and heat over a permeable stretching surface with stress work and energy dissipation. The fluid on stretching surface close with stagnation-point discussed by Tripathy *et al.* [29]. Seddeek and Salem [27] observed that the mass and heat transfer distribution on stretching type surface with thermal diffusivity and variable viscosity.

Recently, Bhattacharyya [4], Wang [30], Govardhan *et al.* [12], and Narender *et al.* [23] has studied mass transfer and chemical reaction past a stretching sheet with the dual solutions in boundary layer and he has deliberated that flow is to be nonconducting electrically. They did not consider all aspects of transfer of heat. In the boundary layer flow, we had measured the effect of first order diffusing species with chemical reaction.

In this article, we provide a review study of Dash *et al.* [11]. A numerical study of boundary layer stagnation point flow past a shrinking sheet in the presence of the magnetic field and Joule heating along with viscous dissipation is analyzed. The constitutive equations of the flow model are solved numerically and the impact of physical parameters concerning the flow model on the dimensionless temperature, velocity and concentration are presented through graphs and tables. Also, a comparison of the achieved numerical results by the Shooting method with the published results of Dash *et al.* [11] has been made and found both in excellent.

## 2. Mathematical Modeling

A steady two-dimensional laminar boundary layer stagnation point flow of viscous incompressible electrically conducting fluid towards a stretching/shrinking sheet with chemically reactive species undergoing first order chemical reaction is considered. The flow field is exposed to uniform transverse magnetic field  $\vec{B}_0 = (0, B_0, 0)$ . It is assumed that the flow is generated by stretching of nonconducting elastic boundary sheet by imposing two opposite and equal forces along  $x$ -axis in such a way that the velocity of the boundary sheet is of linear order in the flow diffraction and the origin remains fixed. A uniform magnetic field of strength  $B_0$  is assumed to be applied in the positive  $y$ -direction normal to the plate. The magnetic Reynolds number of the flow is taken small, therefore induced magnetic field is negligible in comparison with the applied one. The level of concentration of foreign mass assumes to be below, therefore Soret and Dufour effects are negligible.

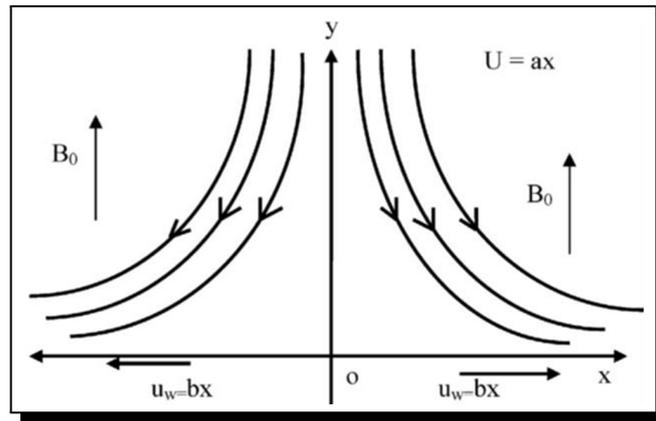


Figure 1. Flow geometry

The model of first order chemical reaction is considered. By usual boundary layer approximation, following Bhattacharyya [4], the governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{\partial U}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma B_0^2}{\rho} (u - U), \tag{2.2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{Q}{\rho C_p} [T - T_\infty] + \frac{\mu}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2, \tag{2.3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - R(C - C_\infty), \tag{2.4}$$

where  $u$  and  $v$  are the velocity components along the  $x$ - and  $y$ -axes,  $U$  is the straining velocity,  $C$  is the concentration,  $T$  is the fluid temperature,  $\nu$  is the kinematic viscosity of fluid,  $\rho$  is the density of fluid,  $C_p$  is the specific heat at constant pressure,  $D$  is the species diffusion coefficient,  $\sigma$  is the electric conductivity of fluid,  $B_0$  is the applied uniform magnetic field normal to the surface of the sheet,  $Q$  is the heat source parameter. The boundary conditions for eqs. (2.1)-(2.4) are

$$\left. \begin{aligned} u = bx : v = 0, T = T_w, C = C_w, \quad \text{at } y = 0, \\ u \rightarrow U(x) = ax, T \rightarrow T_\infty, C \rightarrow C_\infty, \quad \text{as } y \rightarrow \infty. \end{aligned} \right\} \tag{2.5}$$

We use similarity transformation [8, 28] to solve eqs. (2.1)-(2.4)

$$\psi(x, y) = \sqrt{av}xf(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \eta = y\sqrt{\frac{a}{\nu}}. \tag{2.6}$$

The velocity component of stream function which is defined as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \tag{2.7}$$

So, we have

$$u = axf'(\eta), \quad v = -\sqrt{av}f(\eta), \tag{2.8}$$

where prime shows differentiation with respect to  $\eta$ .

Using eq. (2.6) in eq. (2.1) that will be satisfied, also using eqs. (2.6)-(2.8) in eqs. (2.2)-(2.4), we will get the following ordinary differential equations.

$$f'''(\eta) + f(\eta)f''(\eta) - (f'(\eta))^2 - M(f'(\eta) - 1) + 1 = 0, \tag{2.9}$$

$$\theta''(\eta) + Pr f(\eta)\theta'(\eta) + Pr S\theta + Pr Ec(f''(\eta))^2 = 0, \tag{2.10}$$

$$\phi''(\eta) + Sc f(\eta)\phi'(\eta) - Sc \beta\phi = 0, \tag{2.11}$$

with the boundary conditions

$$\left. \begin{aligned} f(\eta) = 0, \quad f'(\eta) = \frac{b}{a}, \quad \theta(\eta) = 1, \quad \phi(\eta) = 1, \quad \text{at } \eta = 0, \\ f'(\eta) \rightarrow 1, \quad \theta(\eta) \rightarrow 0, \quad \phi(\eta) \rightarrow 0, \quad \text{as } \eta \rightarrow \infty. \end{aligned} \right\} \tag{2.12}$$

The dimensionless constants  $Pr, Sc, M, S, Ec$ , represent the Prandtl number, the Schmidt number, the magnetic parameter, the heat source parameter, the reaction rate parameter and Eckert number which are defined as

$$Pr = \frac{\mu C_p}{\kappa}, \quad Sc = \frac{\nu}{D}, \quad M = \frac{\sigma B_0^2}{\rho a}, \quad S = \frac{Q}{a\rho C_p}, \quad \beta = \frac{R}{a}, \quad Ec = \frac{u_w^2}{C_p(T_f - T_\infty)}. \tag{2.13}$$

In this problem the quantities of physical interest are the local Nusselt number  $Nu$ , the skin friction coefficient  $C_f$  and the local Sherwood number  $Sh$ , which are defined as

$$Nu = \frac{xq_w}{k(T_w - T_\infty)}, \quad Sh = \frac{xh_m}{D(C_w - C_\infty)}, \quad C_f = \frac{\tau_w}{\rho \frac{U^2}{2}}, \tag{2.14}$$

where  $h_m, q_w$  and  $\tau_w$  are mass flux from the sheet, heat flux and the skin friction or shear stress, which are given by

$$h_m = -D \left[ \frac{\partial C}{\partial y} \right]_{y=0}, \quad q_w = -k \left[ \frac{\partial T}{\partial y} \right]_{y=0}, \quad \tau_w = \mu \left[ \frac{\partial u}{\partial y} \right]_{y=0}, \tag{2.15}$$

where  $\mu$  is the dynamic viscosity of the fluid and  $k$  is thermal diffusivity. Using the similarity variables eq. (2.6), we get

$$\frac{Nu}{\sqrt{Re_x}} = -\theta'(0), \quad \frac{Sh}{\sqrt{Re_x}} = -\phi'(0), \quad \frac{1}{2}C_f Re_x^{1/2} = f''(0), \tag{2.16}$$

where the local Reynolds number is defined as,

$$Re_x = ux/v.$$

### 3. Method for Solution

Eqs. (2.9)-(2.11) are non-linear and coupled. We opted to solve the non-linear system consisting of eqs. (2.9)-(2.11) with boundary conditions eq. (2.12) by using the shooting iteration technique together with the fourth order Adams-Moulton method. While using this technique, boundary value problem is converted into the initial value problem. In this method we have to choose a suitable finite value of  $\eta \rightarrow \infty$ . Let's convert eqs. (2.9)-(2.11) by using following substitution:

$$f = y_1, \quad f' = y_2, \quad f'' = y_3, \quad f''' = y_3', \tag{3.1}$$

$$\theta = y_4, \quad \theta' = y_5, \quad \theta'' = y_5', \tag{3.2}$$

$$\phi = y_6, \quad \phi' = y_7, \tag{3.3}$$

$$\phi'' = y_7'. \tag{3.4}$$

Using above notations as a result we get seven first order non-linear coupled ODEs with the boundary conditions are also adjust according to the above supposition, written below

$$\left. \begin{aligned} y_1' &= y_2, \\ y_2' &= y_3, \\ y_3^1 &= [y_2^2 - y_1 y_3 + M(y_2 - 1) - 1], \\ y_4' &= y_5, \\ y_5^1 &= Pr[-y_1 y_5 - S y_4 - Ec(y_3)^2], \\ y_6' &= y_7, \\ y_7' &= Sc\beta y_6 - Sc y_1 y_7. \end{aligned} \right\} \tag{3.5}$$

The associated initial conditions are

$$y_1(0) = 0, \quad y_2(0) = \frac{b}{a}, \quad y_3(0) = t, \quad y_4(0) = 1, \quad y_5(0) = q, \quad y_6(0) = 1, \quad y_7(0) = -w. \tag{3.6}$$

In eq. (3.6)  $t$ ,  $q$  and  $w$  are the three initial guesses. Adams-Moulton method of order four is used to solve the intermediate initial value problem with some suitable initial guess  $t = t_0$ ,  $q = q_0$  and  $w = w_0$ . For the next iteration, the values of  $t$ ,  $q$  and  $w$  are updated by the Newton's method as follows

$$\begin{pmatrix} t^{(n+1)} \\ q^{(n+1)} \\ w^{(n+1)} \end{pmatrix} = \begin{pmatrix} t^{(n)} \\ q^{(n)} \\ w^{(n)} \end{pmatrix} - \begin{pmatrix} y_9 & y_{16} & y_{23} \\ y_{11} & y_{18} & y_{25} \\ y_{13} & y_{20} & y_{27} \end{pmatrix}_{(\eta_\infty, t^{(n)}, q^{(n)}, w^{(n)})}^{-1} \begin{pmatrix} y_2^{(n)} \\ y_4^{(n)} \\ y_6^{(n)} \end{pmatrix}_{(\eta_\infty, t^{(n)}, q^{(n)}, w^{(n)})} \tag{3.7}$$

where  $n = 0, 1, 2, 3, \dots$

### 4. Results and Discussion

The main objective to study the effect of different parameters on velocity profile  $f'(0)$ , temperature profile  $\theta(\eta)$  and concentration profile  $\phi(\eta)$ . For the conformation of the result, compared with Bhattacharyya [4]. The skin friction, rate of heat transfer and rate of mass diffusion at the surface represented in Table 1-3. Further, Bhattacharyya [4] discussed only mass diffusion and momentum cases, but in the present work thermal diffusion also involved. So, we have restrained our conversation to a particular solution dependent up on both the parameters  $\frac{b}{a}$  and  $M$  for the energy, momentum, and mass diffusion equations. In Table 1, we observed that the magnetic field intensity had direct relation with skin friction with constant shrinking rate. Skin friction coefficient is decreasing with an increase in the shrinking rate ( $\frac{b}{a} < 0$ ) in both absence and presence of the magnetic field. The conclusion is that the skin friction is enhanced with the presence of magnetic field. The skin friction negative when stretching rate more than 1. Its mean that ( $\frac{b}{a} > 0$ ), in case of stretching, the stretching rate ( $b$ ) greater than the straining rate ( $a$ ), that may lead to flow instability. Further, it is observed that keeping the shrinking rate constant the skin friction increases with an increase in magnetic field strength. Also, it is noted that when ( $\frac{b}{a} = 0$ ) the skin friction disappear. There is no relative motion between plate velocity and free stream velocity and hence shearing stress also disappears. It is observed that skin friction increased due to the increase of magnetic field. It is concluded that presence of magnetic field enhances the skin friction but the rate of shrinking/stretching decreases it in

both stretching and shrinking of the sheet, and the change of sign is a sign of the flow reversal when the velocity of the stretching sheet exceeds the freestream velocity. Table 2 shows the rate of heat transfer for stretching/shrinking of the plate surface. It is noted that the Nusselt number  $Nu$  decreases with an increase in the value of  $Pr$  without magnetic field and heat source ( $M = 0; S = 0$ ), but it is increasing with the presence of magnetic field but opposite impact is observed with an increase in the value of source parameter. Further, it is observed that due to an increase in the magnitude of shrinking rate with the presence of both heat source and magnetic field,  $Nu$  decreases. Also, with the presence of sink ( $S = 1$ ), the Nusselt number remains positive. For high value of  $Pr = 7$ , the Nusselt number  $Nu$  takes negative value for shrinking case. It is observed that an increase in Prandtl number and stretching rate both contribute to increase the rate of heat transfer but Prandtl number  $Pr$  contributes significantly. Further, in case of the shrinking sheet, high Prandtl number fluid causes a thermal instability at the surface (negative value of  $Nu$ ) whereas no such case arises for stretching sheet. Table 3 shows the value of Sherwood number denoted by  $Sh$ . For constructive reaction ( $Kr < 0$ ), the Sherwood number  $Sh$  decreases and takes negative value, for destructive reaction ( $Kr > 0$ ) Sherwood number  $Sh$  is positive. As Schmidt number  $Sc$  increases, Sherwood number  $Sh$  decreases in the absence of  $Kc$  and  $M$  but due to the chemical reaction  $Kc$  as well as the magnetic field  $M$ ,  $Sh$  increases. It is also noted that the effect of ( $M = 0; 1$ ) with ( $Kc = 0$ ) and ( $Sc = 100$ ) is same as that of shrinking sheet.

**Table 1.** The skin friction  $f''(0)$  for different values of  $b/a$  and  $M$

$b/a$	$M$	Bhattacharyya [4]	Dash <i>et al.</i> [11]	Present
-1.25	0		0.5971	0.5866554
-1.15	0	1.0822	1.0822	1.0822310
-2.14	1		1.5629	1.2048410
-1.0	0	1.3288	1.3288	1.3288170
-1.0	1		2.4299	2.4299610
-1.0	2		3.1526	3.1526780
-0.5	0	1.4956	1.4956	1.4956690
-0.5	1		2.1201	2.1201910
-0.5	2		2.5975	2.5975980
1.0	0		0.0001	0.0000030
1.0	1		0.0001	0.0000030
1.0	2		0.0001	0.0000030
0	0		1.2325	1.2325870
0	1		1.5853	1.5853310
0	2		1.8735	1.8735270
0.5	0		0.7132	0.7132949
0.5	1		0.8696	0.8696240
0.5	2		1.0024	1.0024120
2.0	1		-2.1326	-2.1326760
2.0	0		-1.8873	-1.8873070

**Table 2.** Rate of heat transfer  $-\theta'(0)$  for different values of  $b/a$ 

$b/a$	$Pr$	$M$	$S$	Bhattacharyya [4]	Dash <i>et al.</i> [11]	Present
-1.24	0.1	0	0	0.1282	0.1281	0.1211692
-1.24	0.5	0	0	0.0983	0.0958	0.0982834
-1.24	0.5	1	-1	0.6537	0.5981	0.5981584
-1	0.71	0	0		0.2282	0.2283487
-1	0.71	1	0		0.3249	0.3249776
-1	0.71	1	0.2		0.1782	0.1783237
-0.5	0.71	1	0.2		0.2997	0.2997978
0	0.71	1	0.2		0.4028	0.4028432
1	0.71	1	0.2		0.5740	0.5740881
-1	7	1	0.2		-0.6851	-0.6851493
-1	0.71	1	-0.2		0.4483	0.4483280
-0.5	0.71	1	-0.2		0.5397	0.5397812
-0.5	0.71	2	0.2		0.3296	0.3296987
0.5	0.71	1	0.2		0.4931	0.4931413
0.5	7	1	0.2		-1.3393	-1.3393760

**Table 3.** Rate of mass transfer  $-\phi'(0)$  for different values of  $b/a$ 

$b/a$	$Sc$	$M$	$Kc$	Bhattacharyya [4]	Dash <i>et al.</i> [11]	Present
-1.24	0.1	0	0	0.1282	0.1281	0.1295685
-1.24	0.5	0	0	0.0983	0.0958	0.0941111
-1.24	0.5	1	1	0.6537	0.5981	0.6738980
-1	0.22	0	0		0.2432	0.1992645
-1	0.22	1	1		0.4895	0.4893764
-1	0.6	1	1		0.7603	0.7603806
0	0.22	0	0		0.3173	0.3076355
0	0.22	1	1		0.5440	0.5439008
0	0.6	1	1		0.8741	0.8741919
0.5	0.22	0	0		0.3472	0.3429694
0.5	0.22	1	1		0.5668	0.5667440
0.5	0.6	1	1		0.9241	0.9241082
0.5	100	1	1		11.3770	11.3770600
0.5	100	0	1		11.3606	11.3616500
-1.0	100	1	1		8.1333	8.1340560
0.5	0.22	1	-1		0.0521	-0.0677538
-1	0.6	1	-1		-1.1561	-1.1586570
-1.24	0.6	1	1		0.7288	0.7288487
0.5	0.22	1	-1		-0.1994	0.2027713

In Figure 2, we observe the effect of the velocity ratio ( $\frac{b}{a}$ ), i.e., stretching rate of the bounding surface ( $b$ ) and straining rate of the stagnation point flow ( $a > 0$ ). For ( $\frac{b}{a} < 1$ ) i.e., stretching rate at the plate is less than straining of potential flow, the boundary layer thickness decreases. Further, it is observed that in case of shrinking sheet ( $\frac{b}{a} = -0.5$ ), there is a limited back flow for a few layers near the plate. It is also observed that the opposing force due to the magnetic field reduces the boundary layer thickness in the flow region. Hence, it is concluded that the shrinking of the boundary surface is to be controlled to avoid the back flow.

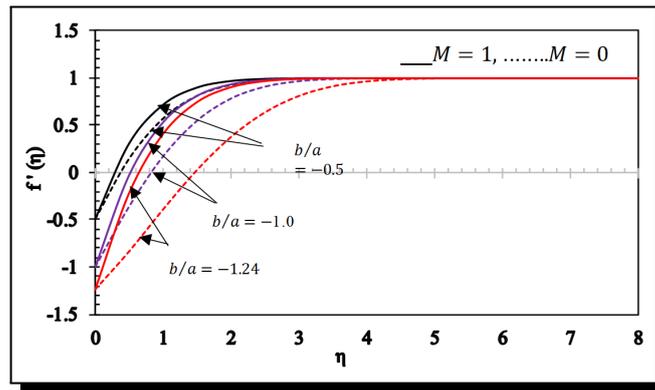


Figure 2. Velocity profile for different values of  $b/a$

In Figure 3, without magnetic field ( $M = 0$ ) as the shrinking rate ( $\frac{b}{a}$ ) increases, the velocity decreases within boundary layer, but velocity increases with the presence of magnetic field  $M$ . When boundary layer decreases then electromagnetic force is high and causes a back flow. If ( $\frac{b}{a} < 1$ ), then the magnetic field increases the velocity. Therefore, it is concluded that the stretching ratio ( $\frac{b}{a}$ ) and the shrinking of the bounding surface change the flow field overriding the effect of magnetic field.

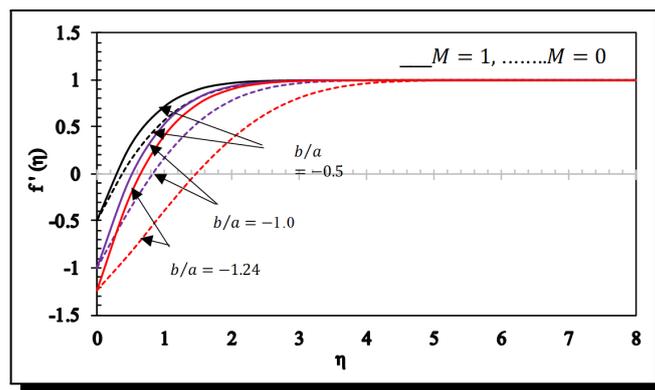


Figure 3. Velocity profile for shrinking sheet with  $S = 0.2$

The temperature profile is discussed through the Figures 4 to 7 in case of shrinking sheet. In Figure 4, the Curves I, II, III, IV, and V indicate that the temperature increases with source strength and increment in temperature is noted near the plate when the viscosity and conductivity of the fluid have value ( $Pr = 1.0$ ). In case of sink ( $S < 0$ ), the opposite effect is observed (curves VI and VII).

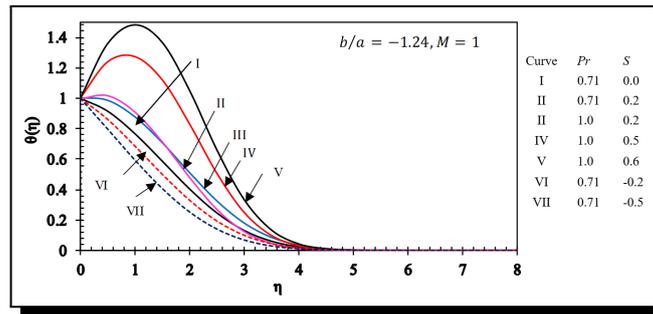


Figure 4. Impact of  $Pr$  and  $S$  on the temperature profile

In Figure 5, the temperature gradient is discussed. It is observed that the negative temperature gradient is mostly seen in the flow domain except two profiles which show positive rate of heat transfer at the plate.

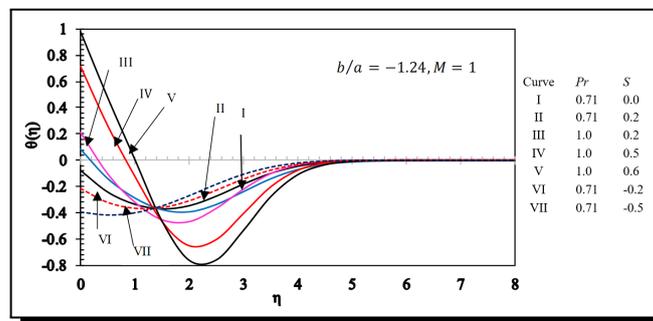


Figure 5. Impact of  $Pr$  and  $S$  on the temperature gradient profile

Figures 6 and 7, show the impact of magnetic parameter on temperature and on its gradient. The resistive force generated due to interaction of conducting fluid and magnetic field reduces the temperature at all points with a transverse compression of profiles (curves for  $M = 1$ ) reducing the thermal boundary layer thickness due to transverse magnetic field. Figure 3-7, indicate the fluctuation of the temperature gradient near the plate. We can observe that the presence of magnetic field achieve maximum of temperature in layers near to the bounding surface.

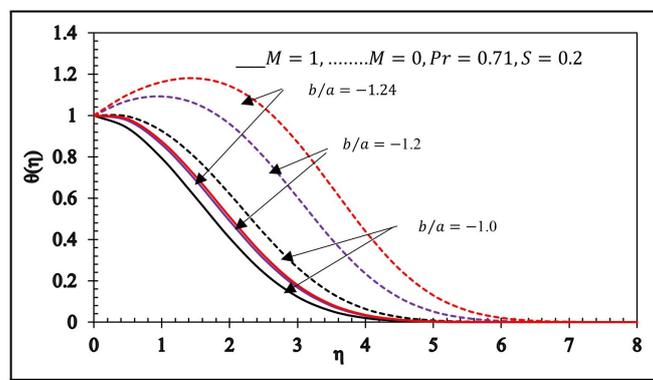


Figure 6. Impact of  $b/a$  and  $M$  on the temperature profile

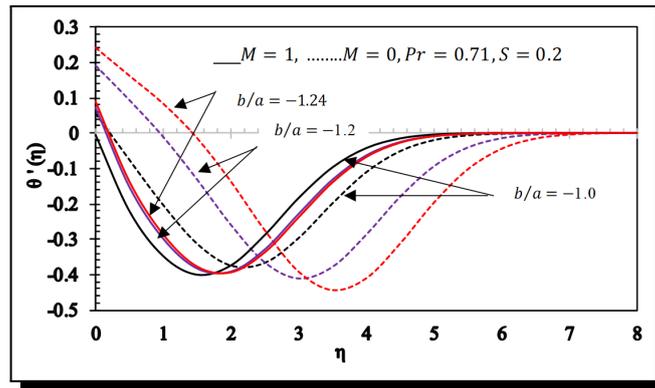


Figure 7. Impact of  $b/a$  and  $M$  on the temperature gradient profile

Figures 8 and 9, show the profiles of concentration and its gradient without reactive species with impact of shrinking of the sheet and magnetic field. The magnetic field decreases the concentration level of the species near the plate. The high value of  $Sc$  increases the concentration level near the plate, but later opposite effect is observed. In the absence of magnetic field ( $M = 0$ ), the concentration profile increases. Figure 9, represents the concentration gradient without chemical reaction. The concentration gradient increases near the plate without magnetic field.

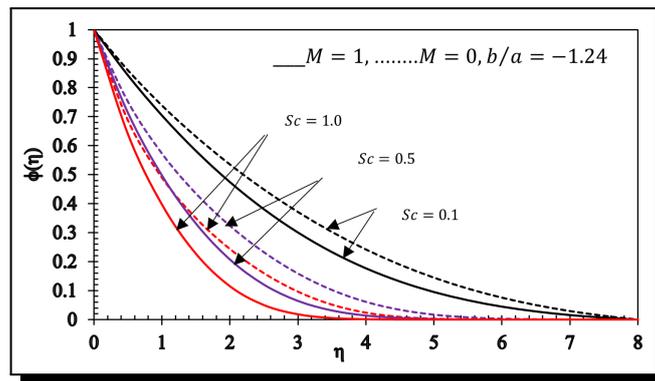


Figure 8. Impact of  $Sc$  and  $M$  on the concentration profile

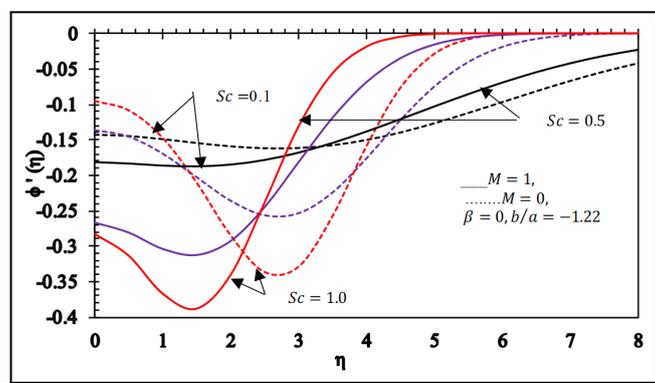


Figure 9. Impact of  $Sc$  and  $M$  on the concentration gradient profile

In Figure 10, it is noted that the chemical reaction parameter has a different impact in decreasing concentration level with magnetic field, a transverse compression resulting the thinner concentration boundary layer. The destructive reaction is responsible to decrease the concentration level at all the layers.

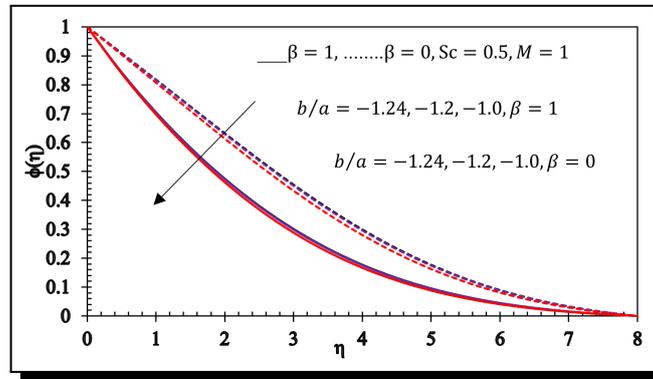


Figure 10. Impact of  $b/a$  and  $\beta$  on the concentration profile

Figure 11 show that as shrinking rate increases with and without reaction parameter, the concentration gradient level decreases.

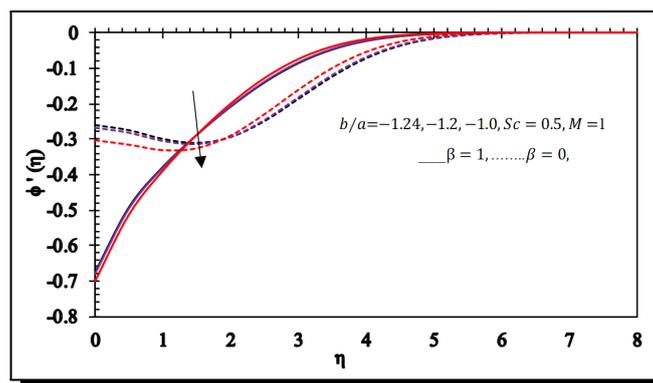


Figure 11. Impact of  $b/a$  and  $\beta$  on the concentration profile

In Figure 12, it indicate that increasing the value of Eckert number  $Ec$  has the enhancing effect on temperature profile and increases the thermal boundary layer thickness in the flow field. The temperature increases due to increasing the Eckert number  $Ec$  that generate heat in fluid.

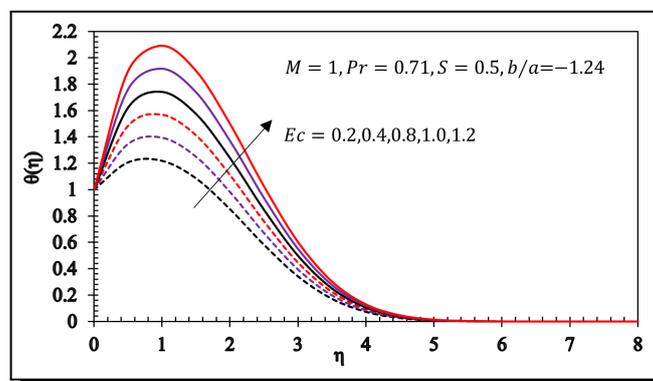


Figure 12. Impact of  $Ec$  on temperature profile

## 5. Conclusion

Conclusions which are obtained:

- When the chemical reaction parameter  $\beta$  increases, the boundary layer thickness decreases.
- Magnetic field increase velocity and temperature but reduces the concentration.
- The skin friction decreases as the shrinking velocity increases but the magnetic field  $M$  increases the skin friction and decreases the rate of heat transfer at the plate.
- The temperature profile increases as the Eckert number increases.
- The effect of chemical reaction  $\beta$  and the magnetic parameter  $M$  is the same for both stretching /shrinking sheet.

### Competing Interests

The authors declare that they have no competing interests.

### Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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