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Research Article

# Viscous Dissipation Impact on Hydromagnetic Flow on a Stretching Surface: A Numerical Study

V. V. L. Deepthi*[1], R. Srinivasa Raju[2] and V. K. Narla[2]

[1] *Department of Mathematics, CVR College of Engineering, Ibrahimpatnam (M), Hyderabad, Telangana State, India*
[2] *Department of Mathematics, GITAM (Deemed to be University), Hyderabad Campus, Rudraram, Sanga Reddy District 502329, Telangana State, India*
***Corresponding author:** deepthisri.cute@gmail.com

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**Abstract.** The influence of magnetic field and viscous dissipation on a non-Newtonian fluid flowing across a nonlinear stretching sheet is investigated in this investigation. Researchers use similarity transformations to make the governing nonlinear *partial differential equations* (PDE) into *ordinary differential equations* (ODE) and then solve them using the ND Solve code in Mathematica. In the process of enhance the values of Eckert number, the temperature profile gets enhanced, while the rise in magnetic parameter decreases the velocity *boundary layer* (BL) thickness. The applications of this investigation are found in several heating devices and industrial processes such as incandescent light bulbs, food production, and many more.

**Keywords.** Magnetic field, Eckert number, Nanofluid, Stretching surface, Viscous dissipation

**Mathematics Subject Classification (2020).** 76A05, 76A10, 76N20, 76W05



## 1. Introduction

*Nanofluid* (NF) contains "nanometre-sized particles", and it has colloidal discontinuities. Nanoparticles utilised in nanofluids are often comprised of oxides, carbon Nano tubes, metals, and carbides, among other materials. The flow and heat transfer study caused by a stretching medium is extremely essential in various industrialised developments. For example, in the process of rubber and plastic sheets manufacturing etc. In the circumstances mentioned above, flow and heat transfer investigations are critical since the ultimate product quality is decided at the bulk level by the coefficient of skin friction and heat transfer surface rate. Several researchers discussed the various characteristics of the stretched flow problem. Crane [10], Chaim [5], Liao and Pop [18], Khan and Sanjayanand [16], and Fang *et al*. [14] are a few of them.

Heat transfer and BL flow of NFs are the focus of current study in engineering and fluid science. Many studies looked at the flow of NF through the convective BL as it passed through a stretched sheet nanotechnology advancements are predicted to have a huge impact on our lives in the future. Choi [8] was the first to propose the concept of "nanofluids" and to present a paper on the heat transfer properties of NFs. Derjaguin and Yalamov [12] investigated the effect of thorough exposure on thermophoretic flux. Chamkha and Issa [7] investigated "the heat and mass transmission of an MHD thermophoretic stream above a planar surface". Tsai [25] examined the effect of thermophoresis on aerosol particles.

Without a doubt, viscous dissipation causes a significant jump up in fluid temperature. This would occur due to the kinetic motion of the fluid while being converted to thermal energy. In a flow environment with a high gravitational field, viscous dissipation is unavoidable. MHD heat and mass transport phenomena over a permeable stretching sheet under the influence of stree work and internal energy was investigated by Khan *et al*. [17]. Gebhart and Mollendroff [15] analised the impact of viscous dissipation on external natural convection flow over a stretching media, whereas Duwairi [13] discussed "the effects of Joule heating and viscous dissipation on forced convection flow with thermal radiation". Chamka and Khaled [6] studied Hiemenz flow across porous medium in the presence of a magnetic field. Sriramalu *et al*. [24] examined the impact of the heat transfer for an incompressible viscous fluid with porous type species over a stretching surface under steady conditions. The effects of magnetohydrodynamics on heat transfer and convective mass flow were investigated by Ali *et al*. [2]. They found that the temperature profile is unaffected by the thermophysical phenomena affecting a small number of particles. In [1, 3, 4, 20, 23], several other magnetohydrodynamic research are discussed. The findings of Ramya *et al*. [22] have been repeated and extended in this paper by considering viscous dissipation effect. A similar mutation is utilized to convert the acquired arrangement of partial differential equations into non-linear and connected ordinary differential equations. The numerical solution which is obtained by the ODE was brought up by using MATHEMATICA and the results obtained for heterogeneous controlling parameters are displayed in a detailed manner, inform of tables and graphs.

## 2. Mathematical Formulation

The impact of viscous dissipation and magnetic field on 2-D electrically conducting, incompressible laminar flow of a NF to a stretching surface are studied in this research work in the presence of slip conditions. The physical configuration of this research work is presented in Figure 1. For this investigation, the following assumptions are made.

(i) The sheet is stretched with a set source location at a velocity $uw = ax^n$, where $n$ is a non-linear stretching parameter, $a$ is a constant, and coordinate $x$ is the determined by the stretching shell's side [22]. The stretching surface must be held at the specified surface temperature,

$$T = T_w \ \ (= T_\infty + bx^r) \text{ at } y = 0,$$

where $b$ is a constant, $r$ is the surface temperature parameter in the defined boundary condition of the surface temperature along with the temperature of the fluid $T_\infty$ is set at a distance from the surface.

(ii) In other circumstances, $\gamma = 0$ is used to obtain a constant surface temperature.

(iii) The induced magnetic field with strength $B$ applied normally to the flow of the fluid compares favorably to applied magnetic fields.

The flow governing equations are [11, 19, 22]:

*Equation of Continuity:*

$$\left(\frac{\partial u}{\partial x}\right) + \left(\frac{\partial u}{\partial y}\right) = 0, \tag{2.1}$$

*Equation of Momentum:*

$$u\left(\frac{\partial u}{\partial x}\right) + v\left(\frac{\partial u}{\partial y}\right) = \nu\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma^* B}{\rho}u, \tag{2.2}$$

*Equation of Thermal Energy:*

$$u\left(\frac{\partial T}{\partial x}\right) + v\left(\frac{\partial T}{\partial y}\right) = \alpha\left(\frac{\partial^2 T}{\partial y^2}\right) + \tau\left\{D_B\left(\frac{\partial T}{\partial y}\right)\left(\frac{\partial \varphi_1}{\partial y}\right) + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right\} + D_{TC}\left(\frac{\partial^2 C_1}{\partial y^2}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.3}$$

*Equation of Species Concentration:*

$$u\left(\frac{\partial C_1}{\partial x}\right) + v\left(\frac{\partial C_1}{\partial y}\right) = D_S\left(\frac{\partial^2 C_1}{\partial y^2}\right) + D_{CT}\left(\frac{\partial^2 T}{\partial y^2}\right), \tag{2.4}$$

*Equation of Nanoparticles Concentration:*

$$u\left(\frac{\partial \varphi_1}{\partial x}\right) + v\left(\frac{\partial \varphi_1}{\partial y}\right) = D_B\left(\frac{\partial^2 \varphi_1}{\partial y^2}\right) + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial y^2}\right). \tag{2.5}$$

The boundary conditions associated with this flow are

$$\left.\begin{aligned} &u = u_w + u_s,\ v = \pm v_w,\ T = T_w,\ C_1 = C_{1w},\ \varphi_1 = \varphi_{1w} \text{ at } y = 0 \\ &u \to 0,\ T \to T_\infty,\ C_1 \to C_{1\infty},\ \varphi_1 \to \varphi_{1\infty} \text{ as } y \to \infty \end{aligned}\right\}. \tag{2.6}$$

Here $u_s$ is the speed of slipping and can be stated as

$$u_s = l\left(\frac{\partial u}{\partial y}\right)_{y=0}. \tag{2.7}$$

By using similarity transformations

$$\left.\begin{aligned} &\eta = y\left(\sqrt{\frac{a(n+1)}{2\nu}}\right)x^{\left(\frac{n-1}{2}\right)},\ u = ax^n f'(\eta),\ v = -\sqrt{\frac{a(n+1)\nu}{2}}x^{\left(\frac{n-1}{2}\right)}\left[f(\eta) + \left(\frac{n-1}{n+1}\right)\eta f'(\eta)\right], \\ &\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ C(\eta) = \frac{C_1 - C_{1\infty}}{C_{1w} - C_{1\infty}},\ \phi(\eta) = \frac{\varphi_1 - \varphi_{1\infty}}{\varphi_{1w} - \varphi_{1\infty}} \end{aligned}\right\} \tag{2.8}$$

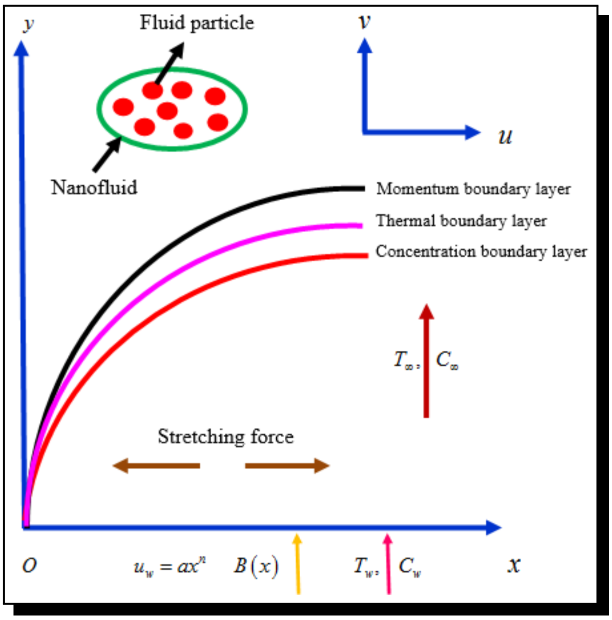


**Figure 1.** Physical model and system of coordinate (Narender *et al.* [19])

Using eq. (2.8), the fundamental eqs. (2.1) to (2.5) become

$$f''' + ff'' - \left(\frac{2n}{n+1}\right) f'^2 - Mf' = 0, \tag{2.9}$$

$$\frac{1}{P_r}\theta'' + f\theta' - \left(\left(\frac{2n}{n+1}\right) f'\theta + N_b\theta'\varphi' + N_t\theta'^2 + N_d C''\right) + E_c(f'')^2 = 0, \tag{2.10}$$

$$g'' + Lefg' + Ld\theta'' = 0, \tag{2.11}$$

$$\phi'' + Lnf\phi' + \frac{Nt}{Nb}\theta'' = 0, \tag{2.12}$$

and the related boundary conditions (2.6) become

$$\left.\begin{array}{l} f = f_w,\ f' = 1 + \lambda f'',\ \theta = 1,\ C = 1,\ \phi = 1 \text{ at } \eta = 0 \\ f' \to 0,\ \theta \to 0,\ C \to 0,\ \phi \to 0 \text{ as } \eta \to \infty \end{array}\right\}, \tag{2.13}$$

where primes indicate differentiation in relation to $\eta$, the physical parameters are

$$\left.\begin{array}{l} Pr = \frac{\nu}{\alpha},\ Ln = \frac{\nu}{D_B},\ Le = \frac{\alpha}{D_S},\ Nb = \frac{\tau D_B(\varphi_{1w} - \varphi_{1\infty})}{\nu}, Nt = \frac{\tau D_B(T_w - T_\infty)}{T_\infty \nu}, \\ Nd = \frac{D_{TC}(C_{1w} - C_{1\infty})}{\alpha(T_w - T_\infty)},\ f_w = -\frac{v_w}{\sqrt{\frac{ax^{n-1}\nu(n+1)}{2}}},\ \lambda = l\sqrt{\frac{ax^{n-1}(n+1)}{2\nu}}, \\ Ld = \frac{D_{CT}(T_w - T_\infty)}{\alpha(C_{1w} - C_{1\infty})},\ M = \frac{\sigma B}{\rho a},\ E_c = \frac{\mu_w^2}{\rho(T_w - T_\infty})\end{array}\right\} \tag{2.14}$$

It should be noticed that the boundary for non-straight extending ($n$) and the surface facilitate ($x$) which are found in the $f_w$ and $\lambda$ keep an eye on breakdown the likeness arrangement. Our conditions are compelled to be tackled locally by this boundary. Reconsidering $f_w$ and $\lambda$ depend on the non-linear term, $P_{nx}$ yields an independent $f_w$ and $\lambda$ from $x$ and $n$ as the following:

$$f_w = \frac{F_w}{\sqrt{P_{nx}}} \quad \text{and} \quad \lambda = \lambda_1\sqrt{P_{nx}} \tag{2.15}$$

where $F_w = \frac{v_w}{\sqrt{av}}$ and $\lambda_1 = l\sqrt{\frac{a}{v}}$ are parameters for pull/infusion and slip, in view of $P_{nx}$ which are free from $x$ and $n$. In this way, there is an acceptable opportunity to illustrate these boundaries by ($f_w$ and $\lambda$) avoiding challenges of the reliance of $F_w$ and $\lambda_1$ on $n$ and $x$. Consequently, for the various estimations of the tough boundaries mentioned, the answer to the issue of neighborhood comparability for fixed estimations of the neighborhood organize $x$, varying $n$, will be significant.

Nusselt number $N_{ux}$, skin friction coefficient $C_f$ and $Sh_x$ are expressed as:

$$Nu_x = \frac{xq_w}{k(T_{w-T_\infty})},\ C_f = \frac{\tau_w}{\rho {U_w}^2},\ P_{nx} = \frac{x^{n-1}(n+1)}{2} \text{ and } Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)}, \tag{2.16}$$

where $\tau_w$ is the shear stress, $q_m$ is the wall mass flux from the surface, and $q_w$ is the heat flux at the wall surface, given by:

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0},\quad q_m = -D_B\left(\frac{\partial C}{\partial y}\right)_{y-0}. \tag{2.17a}$$

Using the dimensionless variables, we get

$$\frac{Nu_x}{\sqrt{R_x}} = -\theta'(0),\quad \frac{Sh_x}{\sqrt{R_x}} = -\theta'(0). \tag{2.17b}$$

## 3. Solution Process

To identify the solution for ODE's (2.9)-(2.12) and related numerical beginning and boundary conditions (2.13), the domain $[0,\infty)$ has been replaced by the bounded domain $[0,\eta_\infty]$ where $\eta_\infty$ is the required finite real number which should be chosen in this fashion the solution fulfills the domain. (2.9)-(2.12) likewise builds up a firmly nonlinear coupled starting limit esteem issue of third and second order ODEs.

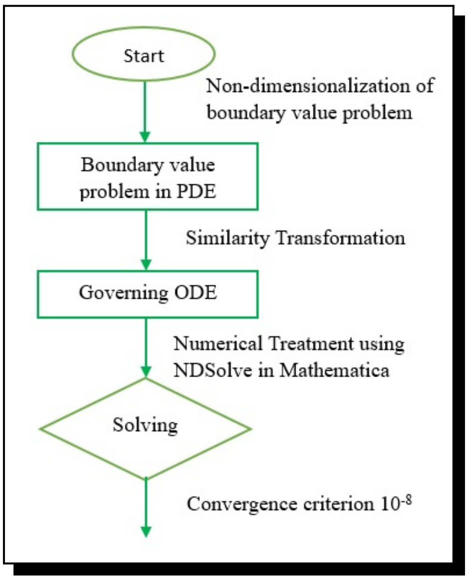


**Figure 2.** Flow chart of the numerical method

We are thus developing the most efficient numerical technique in accordance with the fourth order Runge-Kutta scheme. The symbolic programme Mathematica was used to arrive at the numerical solution.

Throughout the statistical simulation, the step size is to be $\Delta\eta = 0.001$ to acquire the result. The convergence criterion is $10^{-6}$. We choose $\eta_{\max} = 4$ such that on all the parameters, our solution converges. The following method is visualized by means of Figure 2.

## 4. Program Code Validation

To guarantee that the current code is correct, the said values of $\theta'(0)$ using the Runge–Kutta-method in NDSolve (Mathematica) are given in Table 1 for different non-linear stretching parameter values ($n$). From this Table 1, The information provided by the current code and those of Cortell [9] have been found to be well-organized, and the mathematical code's use is justified.

**Table 1.** Comparison of the values $\theta'(0)$ for different $n$ values

| $Pr$ | $n$ | Cortell [9] | Present study |
|---|---|---|---|
| 1.0 | 0.2 | 0.610262 | 0.6095521 |
| | 0.5 | 0.595277 | 0.5862013 |
| | 1.5 | 0.574537 | 0.5601232 |
| 5.0 | 0.2 | 1.607175 | 0.5901223 |
| | 0.5 | 1.586744 | 1.5782003 |
| | 1.5 | 1.557463 | 1.5460233 |

## 5. Results and Discussion

The structure of ODE (2.9)-(2.12) subjected to the boundary conditions (2.13) for the different values of physical emerging strictures such as "velocity slip parameter" ($\lambda$), "Magnetic parameter" ($M$), "Brownian motion parameter" ($Nb$), Eckert number ($Ec$), "thermophoresis parameter" ($Nt$), "Lewis number" ($Le$), "Thermal diffusion parameter" ($Ld$) and Diffusion thermo parameter ($Nd$) on Streamwise velocity, temperature, concentration and nanoparticles concentration profiles discussed through graphs from Figure 3 to Figure 12, after solving numerically using Mathematica.

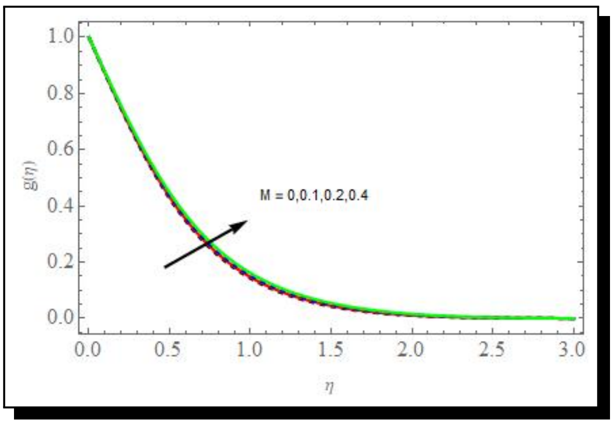


**Figure 3.** Influence of $M$ on $f'(\eta)$

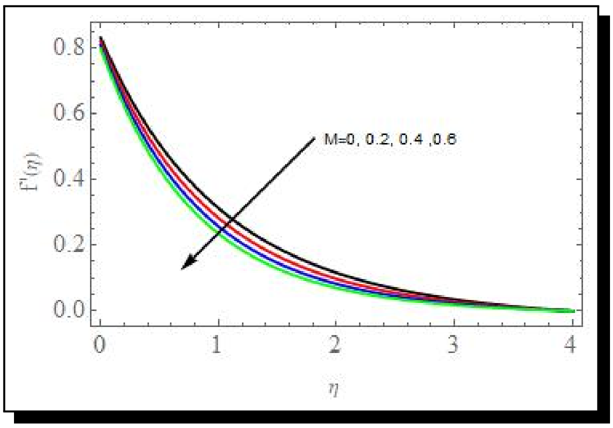


**Figure 4.** Influence of $M$ on $g(\eta)$

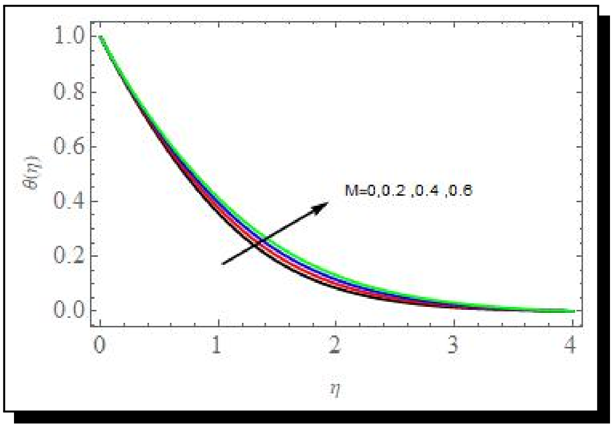


**Figure 5.** Influence of $M$ on $\theta(\eta)$

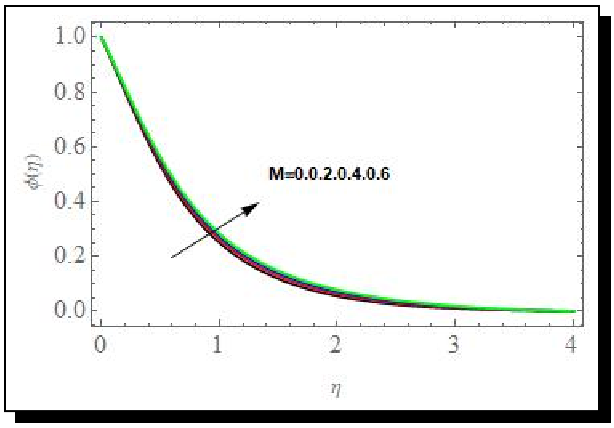


**Figure 6.** Influence of $M$ on $\theta(\eta)$

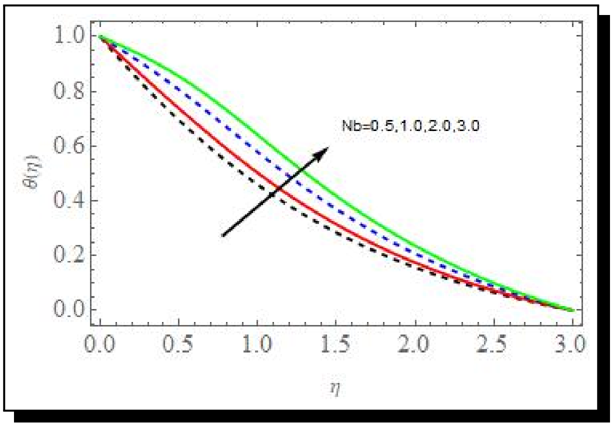


**Figure 7.** Influence of $Nb$ on $\phi(\eta)$

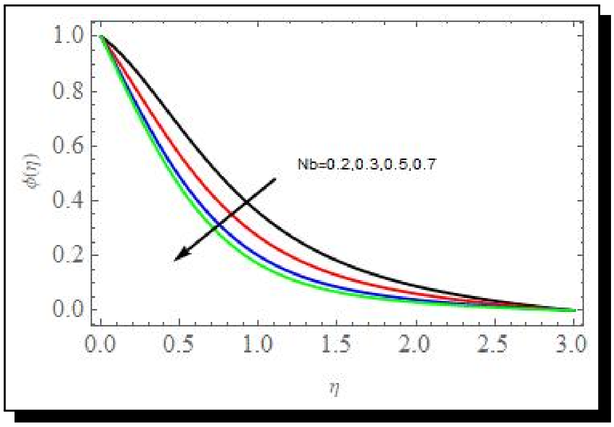


**Figure 8.** Influence of $Nb$ on $\theta(\eta)$

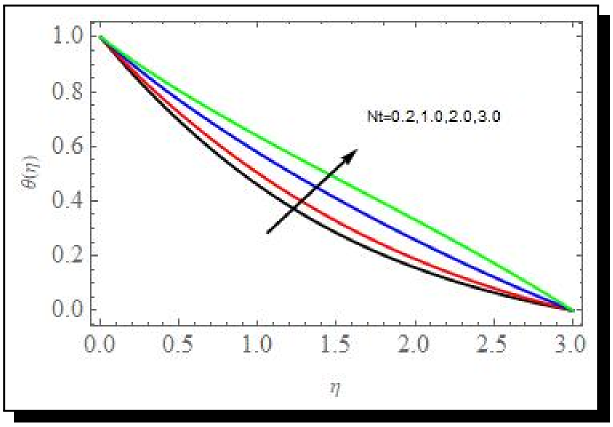


**Figure 9.** Influence of $Nt$ on $\phi(\eta)$

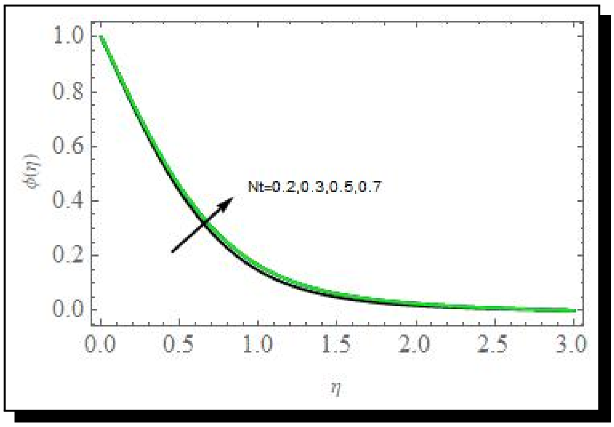


**Figure 10.** Influence of $Nt$ on $\theta(\eta)$

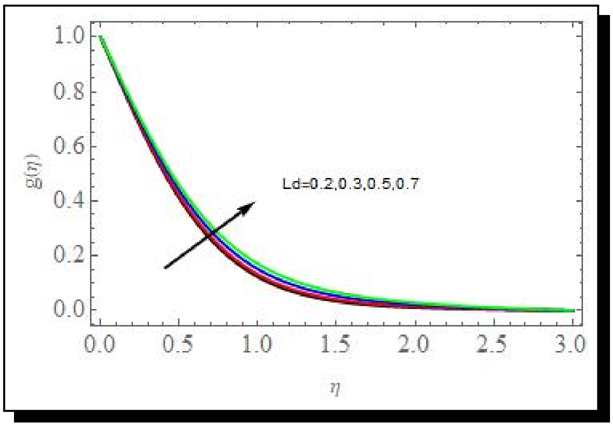


**Figure 11.** $Nd$ impact on $\theta(\eta)$

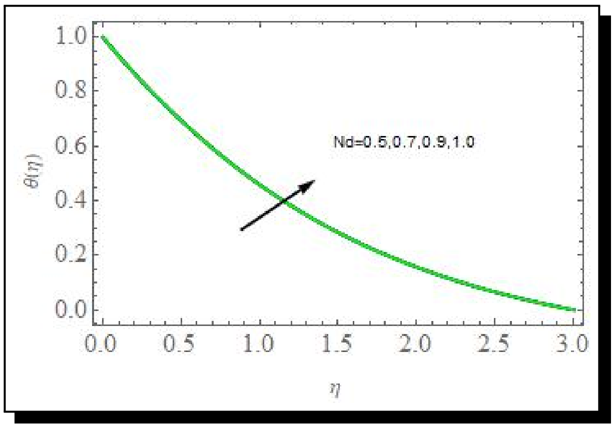


**Figure 12.** $Ld$ impact on $g(\eta)$

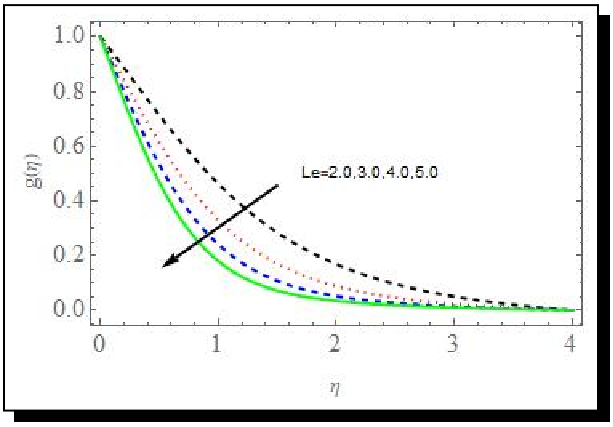


**Figure 13.** $Le$ impact on $g(\eta)$

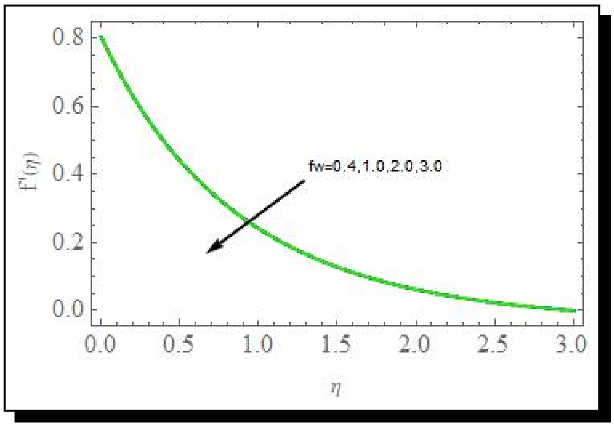


**Figure 14.** $fw$ impact on $f'(\eta)$

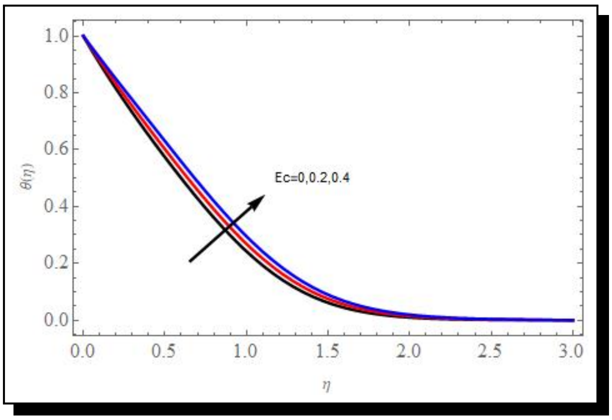


**Figure 15.** $Ec$ impact on $\theta(\eta)$

- Figure 3 depicts the behaviour of $M$ on a velocity profile. It can be observed that the effects of the *velocity profile* (VP) decase with raising $M$ value, due to the returned "Lorentz force".

- Figures 4, 5 and 6 shows the variations in $g(\eta)$, $\theta(\eta)$ and $\theta(\eta)$ for different approximations of $M$. It is analyzed that the $\theta(\eta)$, $g(\eta)$ and thermal BL thickens are expanding functions of $M$.
- Figures 7-10 show the $Nb$ and $Nt$ results on dimensionless temperature and nanoparticle concentration profiles. The temperature profile increases as the $Nb$ values grow, whereas the nanoparticle concentration profile drops. The similar result was found by Ramya et al. [22].
- In Figure 11 The behavior of the Dufour number $(Nd)$ on the temperature profile is seen as the amount of Dufour increases, with a fall in temperature and thermal boundary layer thickness recorded. A greater gradient of concentration is produced by higher Dufour number values. As a result, mass diffusion occurs at a faster pace, and energy transfer occurs at a faster rate as well. As a result, the temperature profile rises.
- The effect of the thermal diffusion parameter $(Ld)$ on the concentration profile $g(\eta)$ is seen in Figure 12. The concentration of the thickness and boundary layer is increased when the quantity of Soret increases. Soret is a phenomenon in which the temperature gradient affects the concentration distribution. Physically higher Soret number values result in higher temperature gradients and thus higher convective flow, and the concentration distribution widens.
- The effect of the Lewis number on dimensionless concentration is discussed in Figure 13. It is observed that the fraction of the volume of nanoparticles decreases significantly at higher $Le$ values. $Le$ is defined as "the thermal and mass diffusivity ratio". Therefore, the raising value of $Le$ improves the thickness of the thermal BL and decreases the thickness of the concentration boundary layer.
- Figure 14 depicts the effects of the suction parameter $f_w$ on the Stream wise velocity profile. It is clear from Figure 14 that as the suction parameter is increased, the velocity profiles decrease monotonically, reflecting the normal fact that suction stabilises the growth of the boundary layer, which also reduces the development of velocity profile peaks.
- The impact of $Ec$ on the energy profile is depicted in Figure 15. As the Eckert number rises, so does the energy profile. Because of the accelerating values of $Ec$, heat energy is retained in due to friction, resulting in a temperature profile enhancement.

## 6. Conclusions

In this current work, the BL viscous flow of a NF over a non-linear stretching sheet has been studied numerically with the effect of velocity slip and thermal diffusion. The effect on the profiles of Streamwise velocity, temperature, concentration and nanoparticle concentration of the various governing parameters. The findings are as follows:

(i) The Streamwise velocity profile declines when the velocity slip parameter and the value of $M$ increases.

(ii) The rise in the Eckert number leads to the improvement in the temperature profile due to the internal energy.

(iii) Rise in the diffusion thermo parameter, the temperature gets improved and as the thermal flow parameter and the concentration Rises.

(iv) The temperature rises and the concentration of nanoparticles decreases by improving the $Nb$.

(v) As $Le$ increases, the concentration decreases because the Lewi's number is inversely proportional to diffusion coefficient.

## Competing Interests

The authors declare that they have no competing interests.

## Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

## References

**[1]** M. S. Alam, M. Ali, M. A. Alim and M. J. H. Munshi, Unsteady boundary layer nanofluid flow and heat transfer along a porous stretching surface with magnetic field, *AIP Conference Proceedings* **1851** (2017), 020023, DOI: 10.1063/1.4984652.

**[2]** M. Ali, M. A. Alim, R. Nasrin and M. S. Alam, Numerical analysis of heat and mass transfer along a stretching wedge surface, *Journal of Naval Architecture and Marine Engineering* **14**(2) (2017), 135 – 144, DOI: 10.3329/jname.v14i2.30633.

**[3]** M. Ali, M. A. Alim, R. Nasrin and M. S. Alam, Study the effect of chemical reaction and variable viscosity on free convection MHD radiating flow over an inclined plate bounded by porous medium, *AIP Conference Proceedings* **1754** (2016), 040009, DOI: 10.1063/1.4958369.

**[4]** M. Ali, M. A. Alim, R. Nasrin, M. S. Alam and M. J. H. Munshi, Similarity solution of unsteady MHD boundary layer flow and heat transfer past a moving wedge in a nanofluid using the buongiorno model, *Procedia Engineering* **194** (2017), 407 – 413, DOI: 10.1016/j.proeng.2017.08.164.

**[5]** T. C. Chaim, Hydromagnetic flow over a surface stretching with a power-law velocity, *International Journal of Engineering Science* **33**(3) (1995), 429 – 435, DOI: 10.1016/0020-7225(94)00066-S.

**[6]** A. J. Chamkha and A.-R. A. Khaled, Similarity solutions for hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media, *International Journal of Numerical Methods for Heat & Fluid Flow* **10**(1) (2000), 94 – 115, DOI: 10.1108/09615530010306939.

**[7]** J. Chamkha and C. Issa, Effects of heat generation/absorption and thermophoresis on hydromagnetic flow with heat and mass transfer over a flat surface, *International Journal of Numerical Methods for Heat & Fluid Flow* **10**(4) (2000), 432 – 449, DOI: 10.1108/09615530010327404.

[8] S. U. S. Choi, Enhancing thermal conductivity of fluids with nanoparticles, in: D. A. Siginer and H. P. Wang (eds.), *Developments and Applications of Non-Newtonian Flows*, ASME, New York, Vol. **66** (1995), 99 – 105, DOI: https://ecotert.com/pdf/196525_From_unt-edu.pdf.

[9] R. Cortell, Viscous flow and heat transfer over a nonlinearly stretching sheet, *Applied Mathematics and Computation* **184**(2) (2007), 864 – 873, DOI: 10.1016/j.amc.2006.06.077.

[10] L. J. Crane, Flow past a stretching plate, *Zeitschrift für angewandte Mathematik und Physik ZAMP* **21**(4) (1970), 645 – 647, DOI: 10.1007/BF01587695.

[11] K. Das, Nanofluid flow over a non-linear permeable stretching sheet with partial slip, *Journal of the Egyptian Mathematical Society* **23**(2) (2015), 451 – 456, DOI: 10.1016/j.joems.2014.06.014.

[12] B. V. Derjaguin and Y. Yalamov, Theory of thermophoresis of large aerosol particles, *Journal of Colloid Science* **20**(6) (1965), 555 – 570, DOI: 10.1016/0095-8522(65)90035-8.

[13] H. M. Duwairi, Viscous and Joule heating effects on forced convection flow from radiate isothermal porous surfaces, *International Journal of Numerical Methods for Heat & Fluid Flow* **15**(5) (2005), 429 – 440, DOI: 10.1108/09615530510593620.

[14] T. Fang, C.-F. F. Lee and J. Zhang, The boundary layers of an unsteady incompressible stagnation-point flow with mass transfer, *International Journal of Non-Linear Mechanics* **46**(7) (2011), 942 – 948, DOI: 10.1016/j.ijnonlinmec.2011.04.007.

[15] B. Gebhart and J. Mollendorf, Viscous dissipation in external natural convection flows, *Journal of Fluid Mechanics* **38**(1) (1969), 97 – 107, DOI: 10.1017/S0022112069000061.

[16] S. K. Khan and E. Sanjayanand, Viscoelastic boundary layer flow and heat transfer over an exponential stretching sheet, *International Journal of Heat and Mass Transfer* **48**(8) (2005), 1534 – 1542, DOI: 10.1016/j.ijheatmasstransfer.2004.10.032.

[17] S. K. Khan, M. S. Abel and R. M. Sonth, Visco-elastic MHD flow, heat and mass transfer over a porous stretching sheet with dissipation of energy and stress work, *Heat and Mass Transfer* **40** (2003), 47 – 57, DOI: 10.1007/s00231-003-0428-x.

[18] S.-J. Liao and I. Pop, Explicit analytic solution for similarity boundary layer equations, *International Journal of Heat and Mass Transfer* **47**(1) (2004), 75 – 85, DOI: 10.1016/S0017-9310(03)00405-8.

[19] G. Narender, S. Misra and K. Govardhan, Numerical solution of MHD nanofluid over a stretching surface with chemical reaction and viscous dissipation, *Chemical Engineering Research Bulletin* **21**(1) (2020), 36 – 45, DOI: 10.3329/cerb.v21i1.47370.

[20] A. Prathiba and A. V. Lakshimi, Exploration of the internal energy effect on 3d-casson fluid embedded by porous media over a rotating sheet, *EJ-MATH - European Journal of Mathematics and Statistics* **3**(3) (2022), 1 – 12, DOI: 10.24018/ejmath.2022.3.3.111.

[21] R. S. Raju, B. M. Reddy, M. M. Rashidi and R. S. R. Gorla, Application of finite element method to unsteady magnetohydrodynamic free-convection flow past a vertically inclined porous plate including thermal diffusion and diffusion thermo effects, *Journal of Porous Media* **19**(8) (2016), 701 – 722, DOI: 10.1615/JPorMedia.v19.i8.40.

[22] D. Ramya, R. S. Raju, J. A. Rao and A. J. Chamkha, Effects of velocity and thermal wall slip on magnetohydrodynamics (MHD) boundary layer viscous flow and heat transfer of a nanofluid over a non-linearly-stretching sheet: A numerical study, *Propulsion and Power Research* **7**(2) (2018), 182 – 195, DOI: 10.1016/j.jppr.2018.04.003.

[23] S. Sindhu and B. J. Gireesha, Flow of colloidal suspension and irreversibility analysis with aggregation kinematics of nanoparticles in a microchannel, *Applied Mathematics and Mechanics* **41** (2020), 1671 – 1684, DOI: 10.1007/s10483-020-2669-9.

**[24]** A. Sriramalu, N. Kishan and A. J. Anand, Steady flow and heat transfer of a viscous incompressible fluid flow through porous medium over a stretching sheet, *Journal of Energy, Heat and Mass Transfer* **23** (2001), 483 – 495.

**[25]** R. Tsai, A simple approach for evaluating the effect of wall suction and thermophoresis on aerosol particle deposition from a laminar flow over a flat plate, *International Communications in Heat and Mass Transfer* **26**(2) (1999), 249 – 257, DOI: 10.1016/S0735-1933(99)00011-1.