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Research Article

Facilitation of Proactive Decision Making by Using Models Incorporating Stochastic Integrals

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Abstract. Stochastic integrals are generally recognized as very strong tools for several significant research areas of probability theory. Moreover, stochastic discounting models are suitable for the description of situations in various practical disciplines. The present paper concentrates on the establishment of theoretical properties for two types of stochastic integrals. These properties facilitate the study of the extremely useful family of infinitely divisible distributions. Moreover, the paper makes use of such properties for the formulation of stochastic discounting models. In addition, the paper provides interpretations of these stochastic models in strategic thinking, proactive global decision making, cindynics, systemics and other very significant practical disciplines.

Keywords. Stochastic integral, Model, Strategic thinking, Systemics

Mathematics Subject Classification (2020). 60E10, 60H05, 90B50, 91B70, 97K60

1. Introduction

In general, it is readily recognized that stochastic processes are extremely suitable analytical tools for establishing new results in wide varieties of theoretical or practical disciplines. Moreover, it is quite obvious the particular importance of undertaking research activities on the formulation, investigation, and interpretation of analytical concepts arising in stochastic processes. Theorists and practitioners in the discipline of probability theory strongly support that stochastic integrals constitute powerful concepts of stochastic processes [3, 5]. The main theoretical contributions of stochastic integrals in the discipline of probability theory are the investigation of connections between families of probability distributions [11]. Moreover, the main practical contributions of stochastic integrals in the discipline of probability theory are the developments of principal components for the formulations of stochastic models [2, 7].

It is easily understood the particular importance of a comment on the incorporations of stochastic integrals as principal components in the formulation of stochastic models. The incorporation of a stochastic integral in a stochastic model depends on the practical interpretation of the stochastic integral. It is known that some continuous discounting operations can be described by stochastic integrals. In consequence, it can be said that it is of some interest the formulation of stochastic models incorporating stochastic integrals describing continuous discounting operations [2]. In particular, the formulations of stochastic models incorporating various stochastic integrals arising in the description of continuous discounting operations, as principal components, are quite easily adopted by practitioners for the investigation of the behavior and evolution of systems under conditions of uncertainty [1, 8].

The present paper is mainly devoted to establishing interconnections between two stochastic integrals of different type. Such interconnections strongly facilitate research activities on the fundamental property of infinite divisibility. Moreover, the paper concentrates on the interpretations of these stochastic integrals as principal components of stochastic models. Such interpretations can be of some particular importance in strategic thinking, proactive decision making, and global risk governance. From a theoretical and a practical point of view the present paper concentrates on the introduction of a new research area by making use of three known research areas of probability theory. The new research area incorporates the research area of stochastic integrals, the research area of infinite divisible distributions, and the research area of stochastic models. The significance of these three research areas reveals the usefulness of the theoretical and practical results of the present paper [6, 10].

2. Stochastic Integrals and Equality in Distribution

The present section is mainly devoted to the establishment of an interconnection and two relationships between two types of stochastic integrals.

We consider the stochastic process $\{Y(t), t \geq 0\}$, the positive random variable $S = Y(t+1) - Y(t)$, the characteristic function $\varphi_S(u)$, the stochastic process $\{H(t), t \geq 0\}$, the positive random variable $C = H(t+1) - H(t)$, the characteristic function $\varphi_C(u)$, the stochastic process $\{X(t), t \geq 0\}$, the positive random variable $L = X(t+1) - X(t)$, the characteristic function $\varphi_L(u)$, and the stochastic integrals

$$\int_0^\infty e^{-t/a} dY(t), \ \int_0^\infty e^{-t/a} dH(t), \ \int_0^\infty e^{-t/a} dX(t), \quad a > 0. \tag{2.1}$$

Sufficient conditions for establishing the functions

$$\exp\left\{a \int_0^u \frac{\log \varphi_s(w)}{w} dw\right\}, \ \exp\left\{a \int_0^u \frac{\log \varphi_c(w)}{w} dw\right\}, \ \exp\left\{a \int_o^u \frac{\log \varphi_L(w)}{w} dw\right\}$$

as the characteristic functions of the stochastic integrals (2.1) are known [2]. Moreover, sufficient conditions for establishing

$$\exp\left\{\frac{a}{u^a}\int_0^u \log\varphi_s(w)w^{a-1}dw\right\},\ \exp\left\{\frac{a}{u^a}\int_0^u \log\varphi_c(w)w^{a-1}dw\right\},\ \exp\left\{\frac{a}{u^a}\int_0^u \log\varphi_L(w)w^{a-1}dw\right\}$$

as characteristic functions of the stochastic integrals

$$\int_0^1 t^{1/a}dY(t),\quad \int_0^1 t^{1/a}dH(t),\quad \int_0^1 t^{1/a}dX(t) \tag{2.2}$$

are also known [4].

Theorem. *We consider the independent stochastic integrals*

$$\int_0^\infty e^{-t/a}dH(t),\quad \int_0^\infty e^{-t/a}dX(t),$$

the positive random variable

$$C$$

and the stochastic integrals

$$\int_0^1 t^{1/a}dY(t),\quad \int_0^1 t^{1/a}dX(t).$$

We suppose that the positive random variable

$$C$$

is independent of the stochastic integrals

$$\int_0^1 t^{1/a}dX(t),\quad \int_0^\infty e^{-t/a}dH(t),\quad \int_0^\infty e^{-t/a}dX(t),$$

then

$$\int_0^\infty e^{-t/a}dY(t)\overset{d}{=}C+\int_0^\infty e^{-t/a}dH(t)+\int_0^\infty e^{-t/a}dX(t) \tag{2.3}$$

if and only if,

$$\int_0^1 t^{1/a}dY(t)\overset{d}{=}C+\int_0^1 t^{1/a}dX(t), \tag{2.4}$$

where $\overset{d}{=}$ *denotes equality in distribution.*

Proof. It is readily understood that only the sufficiency condition will be established since the necessity condition can be established by reversing the argument. The introduction of characteristic functions in the equality in distribution (2.4) implies the integral equation

$$\exp\left\{\frac{a}{u^a}\int_0^u \log\varphi_s(w)w^{a-1}dw\right\}=\varphi_c(u)\exp\left\{\frac{a}{u^a}\int_0^u \log\varphi_L(w)w^{a-1}dw\right\}. \tag{2.5}$$

Moreover, it is obvious that the introduction of logarithms in the integral equation on equation (2.5) results in the integral equation

$$\frac{a}{u^a}\int_0^u \log\varphi_s(w)w^{a-1}dw=\log\varphi_c(u)+\frac{a}{u^a}\int_0^u \log\varphi_L(w)w^{a-1}dw. \tag{2.6}$$

Multiplying both sides of the integral equation (2.6) by u^a we obtain the integral equation

$$a\int_0^u \log\varphi_s(w)w^{a-1}dw=u^a\log\varphi_c(u)+a\int_0^u \log\varphi_L(w)w^{a-1}dw. \tag{2.7}$$

It is easily seen that the differentiation of the integral equation (2.7) implies the differential equation

$$au^{a-1}\log\varphi_s(u) = au^{a-1}\log\varphi_c(u) + u^a\frac{d}{du}\log\varphi_c(u) + au^{a-1}\log\varphi_L(u). \tag{2.8}$$

In addition, it is obvious that the differential equation (2.8) takes the form

$$a\frac{\log\varphi_s(u)}{u} = a\frac{\log\varphi_c(u)}{u} + \frac{d}{du}\log\varphi_c(u) + a\frac{\log\varphi_L(u)}{u} \tag{2.9}$$

for $u \neq 0$. Integration of the differential equation (2.9) results to the integral equation

$$a\int_0^u \frac{\log\varphi_s(w)}{w}dw = \log\varphi_c(u) + a\int_0^u \frac{\log\varphi_c(w)}{w}dw + a\int_0^u \frac{\log\varphi_L(w)}{w}dw. \tag{2.10}$$

The integral equation (2.10) is equivalently written in the form

$$\exp\left\{a\int_0^u \frac{\log\varphi_s(w)}{w}dw\right\} = \varphi_c(u)\exp\left\{a\int_0^u \frac{\log\varphi_c(w)}{w}dw\right\}\exp\left\{a\int_0^u \frac{\log\varphi_L(w)}{w}dw\right\}. \tag{2.11}$$

Consequently, the introduction of random variables in the integral equation (2.11) results in the equality in distribution

$$\int_0^\infty e^{-t/a}dY(t) \stackrel{d}{=} C + \int_0^\infty e^{-t/a}dH(t) + \int_0^\infty e^{-t/a}dX(t). \tag{2.12}$$

From a theoretical point of view, it can be said that the present section of the paper establishes a relationship between stochastic integrals of the form (2.1), a relationship between of stochastic integrals of the form (2.2), and an interconnection between these relationships.

3. Interpretations

It is easily understood that the theoretical and practical interpretations of the results incorporated in the previous section constitute contributions of some importance for two research areas of probability theory. More precisely, the family of infinitely divisible distributions and the formulation of stochastic discounting models are the two research areas of probability theory strongly facilitating the understanding of the importance of the results of the previous section.

The family of infinitely divisible distributions constitutes an extremely useful tool for the establishment of very strong probabilistic results [9]. The infinitely divisible characteristic functions are generally as very suitable and strong tools for revealing the structure of the family of infinitely divisible distributions [11]. The infinite divisibility of the characteristic functions of the paper constitutes the main factor supporting the establishment of the two theoretical results of the second section. The first theoretical result consists of the formulations of two functional equations incorporating three infinitely divisible characteristic functions of the form (2.1) and two infinitely divisible characteristic functions of the form (2.2). Moreover, the second theoretical result consists of the establishment of an interconnection between the above mentioned functional equations incorporating characteristic functions of the form (2.1) and the form (2.2).

It is obvious that the formulations of these functional equations strongly contribute to the construction of categories of infinitely divisible characteristic function facilitating the study of the family of infinitely divisible characteristic functions.

It is known a stochastic integral of the form (2.1) denotes the present value of income, as viewed from the time point 0, produced by an asset during its indefinite lifetime, where $1/a$ is the force of interest [10]. Hence, the equality in distribution in (2.3) is readily recognized as a direct result of the research area of stochastic discounting modeling. Moreover, the equality in distribution in (2.12) is readily recognized as an indirect result of the research area of stochastic discounting modeling.

4. Conclusion

It constitutes a quite significant structural factor the incorporation of stochastic integrals of different type for undertaking research activities in two areas of probability theory. The structure of the family of infinitely divisible characteristic functions constitutes the first area of probability theory. Moreover, the stochastic discounting modeling constitutes the second area of probability theory.

The paper makes quite clear that various stochastic integrals can be of particular theoretical and practical importance. More precisely, the paper reveals that stochastic integration constitutes a probabilistic operation extremely useful for investigating the structure of the very important family of infinitely divisible characteristic functions. In addition, the paper provides evidence that stochastic integration substantially supports the formulation of stochastic discounting models.

In consequence, it is readily recognized that the main contribution of the present paper constitutes of the incorporation of two stochastic integrals for the formulation, investigation, and interpretation of stochastic discounting models. It is also readily recognized that the formulated stochastic discounting models constitute extremely strong analytical tools for treatment of practical situations arising in strategic thinking, cindynics, proactive global risk governance, systemic, strategic management, computational intelligence, econometrics, logistics, operational research, and other practical disciplines.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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