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Research Article

Y-index of Different Corona Products of Graphs

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Abstract. For a molecular graph G, the Y-index is defined as the sum of fourth degree of all vertices of the graph. Among different products, corona product of two graphs is one of the most important. In this paper, we explore the explicit expressions of Y-index of different types of corona product of graphs.

Keywords. Zagreb index, F-index, Y-index, Corona product

Mathematics Subject Classification (2020). 0C07, 05C76, 92E10

1. Introduction

All graphs considered here are simple, connected, finite and undirected. Let $G = (V(G), E(G))$ be a connected graph of order n with $|E(G)| = m$ edges. The degree of a vertex $u \in V(G)$, denoted by $d_G(u)$, is the number of edges incident to u. The neighborhood of a vertex $u \in V(G)$ is defined as the set $N_G(u)$ consisting of all vertices v which are adjacent to u in G.

In the fields of chemical graph theory, molecular topology and mathematical chemistry, a topological index is a type of molecular descriptor that is calculated as degree based on the molecular graph of a chemical compound. Topological indices are numerical parameters of a graph which characterize its topology and are usually graph invariant. Topological indices are used for example in the development of *quantitative structure-activity relationships* (QSARs) in which the biological activity or other properties of molecules are correlated with their chemical structure.

Amid various degree-based topological indices, one of the most studied topological index is the Zagreb index, introduced by Gutman and Trinajstić in [11]. The Zagreb indices are defined as

$$M_1(G) = \sum_{v \in V(G)} d_G(u)^2 = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]$$

and

$$M_2(G) = \sum_{u,v \in V(G)} d_G(u) d_G(v).$$

In [10], Furtula and Gutman investigated the F-index or the forgotten topological index. The *F-index* of a graph G is defined as

$$F(G) = \sum_{v \in V(G)} d_G(v)^3 = \sum_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2].$$

The *Y-index* recently introduced by Alameri *et al*. [4] is defined as

$$Y(G) = \sum_{u \in V(G)} d_G(u)^4 = \sum_{uv \in E(G)} [d_G(u)^3 + d_G(v)^3].$$

They also showed that the predictive ability of this index is similar to that of first Zagreb index. There are various recent studies of Y-index (one can refer [2,4]).

Among the most well known products of graphs, the corona product of graphs is one of the most important graph operations as different important classes of graphs can be formed by taking corona product of some general and particular graphs. Also, by specializing the components of corona product of graphs different interesting classes of graph such as t-theory graph, sunlet graph, bottleneck graph, suspension of graphs and some classes of bridge graphs can be formed (see [3,5–8] and [14]).

Lu and Miao[1] has determined spectra of subdivision-vertex and subdivision-edge coronae. In [12], Liu and Lu has determined spectra of subdivision-vertex and subdivision-edge neighborhood corona. In [13], Malpashree has determined some degree and distance based topological indices of vertex-edge corona of two graphs.

We explore the explicit expressions of different types of corona product of graphs such as subdivision-vertex corona, subdivision-edge corona, subdivision-vertex neighborhood, subdivision-edge neighborhood corona and vertex-edge corona of two graphs in this paper.

2. Main Results

Let G_1 and G_2 be two simple connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. The corona product of $G_1 \circ G_2$ of these two graphs is obtained by taking one copy of G_1 and n_1 copies of G_2 and by joining each vertex of the i-th copy of G_2 to the i-th vertex of G_1, where $1 \le i \le n_1$. The corona product of G_1 and G_2 has total number of $(n_1 n_2 + n_1)$ vertices and $(m_1 + n_1 m_2 + n_1 n_2)$ edges.

Several authors defined other different versions of corona product of graphs such as subdivision-vertex corona, subdivision-edge corona, subdivision-vertex neighborhood corona, subdivision-edge neighborhood corona and vertex-edge corona (see [1],[12,13]). The *subdivision graph* $S(G)$ of a graph G is a graph obtained by inserting a new vertex onto each edge of G.

[1]P. Lu and Y. Miao, Spectra of the subdivision-vertex and subdivision-edge coronae, arXiv:1302.0457v2, URL: https://arxiv.org/pdf/1302.0457.pdf.

In this paper, we calculate the Y-index of subdivision-vertex corona, subdivision-edge corona, subdivision-vertex neighborhood corona, subdivision-edge neighborhood corona and vertex-edge corona of two connected simple graphs.

2.1 Subdivision-vertex Corona

Definition 2.1. Let G_1 and G_2 be two simple connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. The subdivision-vertex corona of G_1 and G_2, denoted by $G_1 \odot G_2$, is obtained from $S(G_1)$ and n_1 copies of G_2, all vertex-disjoint, by joining the i-th vertex of $V(G_1)$ to every vertex in the i-th copy of G_2.

Let $V(G_1) = \{u_1, u_2, \dots, u_{n_1}\}$ and $V(G_2) = \{v_1, v_2, \dots, v_{n_2}\}$. Denote by G_2^i, the ith copy of G_2 in $G_1 \odot G_2$. Let $V(G_2^i) = \{v_{i1}, v_{i2}, \dots, v_{in_2}\}$, $1 \le i \le n_1$. Let $W(G_1) = \{w_1, w_2, \dots, w_{m_1}\}$ be the set of new vertices inserted on the edges of G_1.

From definition it is clear that the subdivision-vertex corona $G_1 \odot G_2$ has $n_1(1+n_2)+m_1$ vertices and $2m_1 + n_1(n_2+m_2)$ edges.

The degree of a vertex $w \in G_1 \odot G_2$ is given in the following lemma.

Lemma 2.1 ([1]). *Let G_1 and G_2 be two vertex disjoint graphs. Then the degree of $w \in V(G_1 \odot G_2)$ is*

$$d_{G_1\odot G_2}(w) = \begin{cases} d_{G_1}(w) + n_2, & \text{if } w \in V(G_1),\\ 2, & \text{if } w \in W(G_1),\\ d_{G_2}(w)+1, & \text{if } w \in V(G_2^i) \text{ for some } i. \end{cases}$$

Theorem 2.2. *The Y-index of the subdivision-vertex corona $G_1 \odot G_2$ is given by*

$$\begin{aligned} Y(G_1 \odot G_2) &= Y(G_1) + 4n_2F(G_1) + 6n_2^2M_1(G_1) + 8n_2^3m_1 + n_1n_2^4 + 16m_1 \\ &\quad + n_1Y(G_2) + 4n_1F(G_2) + 6n_1M_1(G_2) + 8n_1m_2 + n_1n_2. \end{aligned}$$

Proof. From definition of subdivision-vertex corona $G_1 \odot G_2$, we get

$$\begin{aligned} Y(G_1 \odot G_2) &= \sum_{w \in V(G_1\odot G_2)} d_G(w)^4 \\ &= \sum_{w\in V(G_1)} d_G(w)^4 + \sum_{w \in W(G_1)} d_G(w)^4 + \sum_{w\in V(G_2^i)} d_G(w)^4 \\ &= \sum_{i=1}^{n_1} \left(d_{G_1}(u_i)+n_2\right)^4 + \sum_{i=1}^{m_1} 2^4 + \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} (d_{G_2}(v_j)+1)^4 \\ &= \sum_{i=1}^{n_1} (d_{G_1}(u_i)^4 + 4n_2 d_{G_1}(u_i)^3 + 6n_2^2 d_{G_1}(u_i)^2 + 4n_2^3 d_{G_1}(u_i) + n_2^4) \\ &\quad + 16m_1 + \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} (d_{G_2}(v_j)^4 + 4d_{G_2}(v_j)^3 + 6d_{G_2}(v_j)^2 + 4d_{G_2}(v_j) + 1) \\ &= Y(G_1) + 4n_2F(G_1) + 6n_2^2M_1(G_1) + 8n_2^3m_1 + n_1n_2^4 + 16m_1 \\ &\quad + n_1(Y(G_2) + 4F(G_2) + 6M_1(G_2) + 8m_2 + n_2) \end{aligned}$$

from where the desired result follows. □

Example 2.3. Let C_n,P_n,K_n be the cycle, the path and the complete graph, respectively, on n vertices. Then by Theorem 2.2, we obtain the Y-index of the following graphs.

(1) $Y(C_n \odot C_m) = nm^4 + 8nm^3 + 24nm^2 + 113nm + 32n$, $n,m \geq 3$.

(2) $Y(C_n \odot P_m) = nm^4 + 8nm^3 + 24nm^2 + 113nm - 98n$, $n \geq 3$, $m \geq 2$.

(3) $Y(C_n \odot K_m) = nm^5 + nm^4 + 8nm^3 + 24nm^2 + 32nm + 32n$, $n \geq 3$, $m \geq 1$.

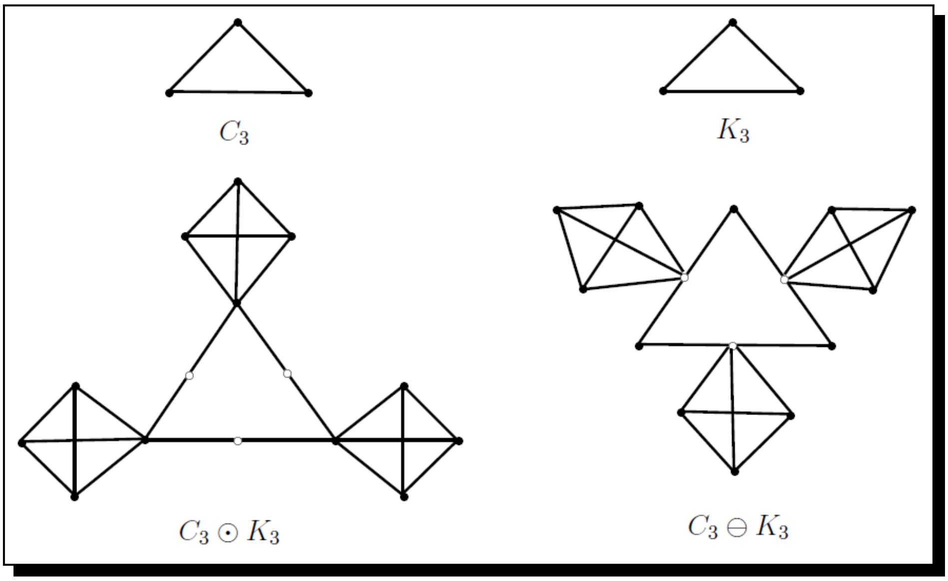

Figure 1. Subdivision-vertex and subdivision-edge corona products of C_3 and K_3

2.2 Subdivision-edge Corona

Definition 2.2. Let G_1 and G_2 be two simple connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. The subdivision-edge corona of G_1 and G_2, denoted by $G_1 \ominus G_2$, is obtained from $S(G_1)$ and m_1 copies of G_2, all vertex-disjoint, by joining the i-th new vertex of $S(G_1)$, obtained by subdividing each edge of G_1, to every vertex in the i-th copy of G_2.

Let $V(G_1) = \{u_1,u_2,\dots,u_{n_1}\}$ and $V(G_2) = \{v_1,v_2,\dots,v_{n_2}\}$. Denote by G_2^i, the ith copy of G_2 in $G_1 \ominus G_2$. Let $V(G_2^i) = \{v_{i1},v_{i2},\dots,v_{in_2}\}$, $1 \leq i \leq n_1$. Let $W(G_1) = \{w_1,w_2,\dots,w_{m_1}\}$ be the set of new vertices inserted on the edges of G_1.

From definition, we have the subdivision-edge corona $G_1 \ominus G_2$ has $m_1(1+n_2)+n_1$ vertices and $m_1(n_2+m_2+2)$ edges.

The degree of a vertex $w \in G_1 \ominus G_2$ is given in the following lemma.

Lemma 2.4 ([1]). *Let G_1 and G_2 be two vertex disjoint graphs. Then the degree of $w \in V(G_1 \ominus G_2)$ is*

$$d_{G_1 \ominus G_2}(w) = \begin{cases} d_{G_1}(w), & \text{if } w \in V(G_1), \\ 2 + n_2, & \text{if } w \in W(G_1), \\ d_{G_2}(w) + 1, & \text{if } w \in V(G_2^i) \text{ for some } i. \end{cases}$$

Theorem 2.5. *The Y-index of the subdivision-edge corona $G_1 \ominus G_2$ is given by*

$$Y(G_1 \ominus G_2) = Y(G_1) + m_1(n_2+2)^4 + m_1 Y(G_2) + 4m_1 F(G_2) + 6m_1 M_1(G_2) + 8m_1 m_2 + m_1 n_2.$$

Proof. From definition of subdivision-edge corona $G_1 \ominus G_2$, we get

$$\begin{aligned}
Y(G_1 \ominus G_2) &= \sum_{w \in V(G_1 \ominus G_2)} d_G(w)^4 \\
&= \sum_{w \in V(G_1)} d_G(w)^4 + \sum_{w \in W(G_1)} d_G(w)^4 + \sum_{w \in V(G_2^i)} d_G(w)^4 \\
&= \sum_{i=1}^{n_1} (d_{G_1}(u_i))^4 + \sum_{i=1}^{m_1} (2+n_2)^4 + \sum_{i=1}^{m_1} \sum_{j=1}^{n_2} (d_{G_2}(v_j)+1)^4 \\
&= \sum_{i=1}^{n_1} d_{G_1}(u_i)^4 + \sum_{i=1}^{m_1} (2+n_2)^4 + \sum_{i=1}^{m_1} \sum_{j=1}^{n_2} (d_{G_2}(v_j)+1)^4 \\
&= Y(G_1) + m_1(n_2+2)^4 + m_1 \sum_{j=1}^{n_2} (d_{G_2}(v_j)^4 + 4d_{G_2}(v_j)^3 + 6d_{G_2}(v_j)^2 + 4d_{G_2}(v_j) + 1) \\
&= Y(G_1) + m_1(n_2+2)^4 + m_1 Y(G_2) + 4m_1 F(G_2) + 6m_1 M_1(G_2) + 8m_1 m_2 + m_1 n_2
\end{aligned}$$

from where the desired result follows. □

Example 2.6. Let C_n, P_n, K_n be the cycle, the path and the complete graph, respectively, on n vertices. Then by Theorem 2.5, we obtain the Y-index of the following graphs.

(1) $Y(C_n \ominus C_m) = nm^4 + 8nm^3 + 24nm^2 + 113nm + 32n$, $n,m \ge 3$.

(2) $Y(C_n \ominus P_m) = nm^4 + 8nm^3 + 24nm^2 + 113nm - 98n$, $n \ge 3$, $m \ge 2$.

(3) $Y(C_n \ominus K_m) = nm^5 + nm^4 + 8nm^3 + 24nm^2 + 32nm + 32n$, $n \ge 3$, $m \ge 1$.

2.3 Subdivision-vertex Neighborhood Corona

Definition 2.3. Let G_1 and G_2 be two simple connected graphs with n_i number of vertices and m_i number of edges respectively, $i \in \{1,2\}$. The subdivision-vertex neighborhood corona of G_1 and G_2, denoted by $G_1 \diamondsuit G_2$, is obtained from $S(G_1)$ and n_1 copies of G_2, all vertex-disjoint, by joining the neighbors of the i-th vertex of $V(G_1)$ to every vertex in the i-th copy of G_2.

Let $V(G_1) = \{u_1, u_2, \dots, u_{n_1}\}$ and $V(G_2) = \{v_1, v_2, \dots, v_{n_2}\}$. Denote by G_2^i, the ith copy of G_2 in $G_1 \diamondsuit G_2$. Let $V(G_2^i) = \{v_{i1}, v_{i2}, \dots, v_{in_2}\}$, $1 \le i \le n_1$. Let $W(G_1) = \{w_1, w_2, \dots, w_{m_1}\}$ be the set of new vertices inserted on the edges of G_1.

The degree of a vertex $w \in G_1 \diamondsuit G_2$ is given in the following lemma.

Lemma 2.7 ([12]). *Let G_1 and G_2 be two vertex disjoint graphs. Then the degree of $w \in V(G_1 \diamondsuit G_2)$ is*

$$d_{G_1 \diamondsuit G_2}(w) = \begin{cases} d_{G_1}(w), & \text{if } w \in V(G_1), \\ 2 + 2n_2, & \text{if } w \in W(G_1), \\ d_{G_2}(v_j) + d_{G_1}(u_i), & \text{if } w = v_{ij} \in V(G_2^i) \text{ for some } i,j. \end{cases}$$

Theorem 2.8. *The Y-index of $G_1 \diamondsuit G_2$ is given by*

$$Y(G_1\diamondsuit G_2) = Y(G_1) + 16m_1(n_2+1)^4 + n_1Y(G_2) + 8m_1F(G_2) + 6M_1(G_2)M_1(G_1) + 8m_2F(G_1) + n_2Y(G_1).$$

Proof. From definition of $G_1\diamondsuit G_2$, we have

$$\begin{aligned}
Y(G_1\diamondsuit G_2) &= \sum_{w\in V(G_1\diamondsuit G_2)} d_G(w)^4 \\
&= \sum_{w\in V(G_1)} d_G(w)^4 + \sum_{w\in W(G_1)} d_G(w)^4 + \sum_{w\in V(G_2^i)} d_G(w)^4 \\
&= \sum_{i=1}^{n_1}(d_{G_1}(u_i))^4 + \sum_{i=1}^{m_1}(2+2n_2)^4 + \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}(d_{G_2}(v_j)+d_{G_1}(u_i))^4 \\
&= Y(G_1) + m_1(2n_2+2)^4 + \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}[d_{G_2}(v_j)^4 + 4d_{G_2}(v_j)^3 d_{G_1}(u_i) \\
&\quad + 6d_{G_2}(v_j)^2 d_{G_1}(u_i)^2 + 4d_{G_2}(v_j)d_{G_1}(u_i)^3 + d_{G_1}(u_i)^4] \\
&= Y(G_1) + 16m_1(n_2+1)^4 + n_1Y(G_2) + 8m_1F(G_2) + 6M_1(G_2)M_1(G_1) \\
&\quad + 8m_2F(G_1) + n_2Y(G_1)
\end{aligned}$$

from where the desired result follows. □

Example 2.9. Let C_n, P_n, K_n be the cycle, the path and the complete graph, respectively, on n vertices. Then by Theorem 2.8, we obtain the Y-index of the following graphs.

(1) $Y(C_n\diamondsuit C_m) = 16nm^4 + 64nm^3 + 96nm^2 + 320nm + 32n$, $n,m \geq 3$.

(2) $Y(C_n\diamondsuit P_m) = 16nm^4 + 64nm^3 + 96nm^2 + 320nm - 318n$, $n \geq 3$, $m \geq 2$.

(3) $Y(C_n\diamondsuit K_m) = nm^5 + 20nm^4 + 70nm^3 + 100nm^2 + 65nm + 32n$,$n \geq 3$, $m \geq 1$.

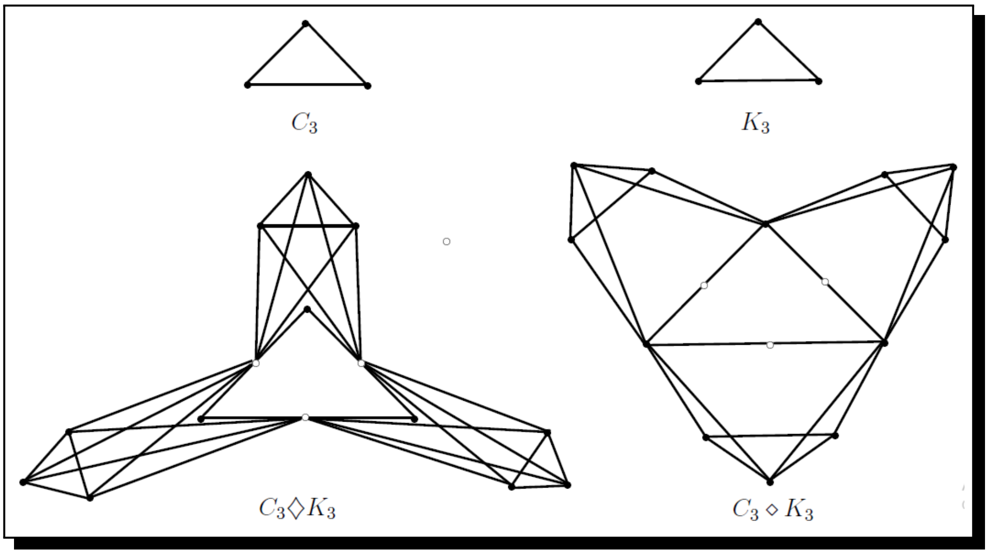

Figure 2. Subdivision-vertex and subdivision-edge neighborhood corona products of C_3 and K_3

2.4 Subdivision-edge Neighborhood Corona

Definition 2.4. For two vertex disjoint graphs G_1 and G_2, the subdivision-edge neighborhood corona of G_1 and G_2, denoted by $G_1 \diamond G_2$, is obtained from $S(G_1)$ and m_1 copies of G_2, all vertex-disjoint, by joining the neighbors of the i-th new vertex of $S(G_1)$ to every vertex in the i-th copy of G_2.

Let $V(G_1) = \{u_1, u_2, \dots, u_{n_1}\}$, $E(G_1) = \{e_1, e_2, \dots, e_{m_1}\}$ and $V(G_2) = \{v_1, v_2, \dots, v_{n_2}\}$. Let $V(G_2^i) = \{v_{i1}, v_{i2}, \dots, v_{in_2}\}$, $1 \le i \le n_1$. Let $W(G_1) = \{w_1, w_2, \dots, w_{m_1}\}$ be the set of new vertices inserted on the edges of G_1.

The degree of a vertex $w \in G_1 \diamond G_2$ is given in the following lemma.

Lemma 2.10 ([12]). *Let G_1 and G_2 be two vertex disjoint graphs. Then the degree of $w \in V(G_1 \diamond G_2)$ is*

$$d_{G_1\diamond G_2}(w) = \begin{cases} (n_2+1)d_{G_1}(w), & \text{if } w \in V(G_1), \\ 2, & \text{if } w \in W(G_1), \\ d_{G_2}(v_j)+2, & \text{if } w = v_{ij} \in V(G_2^i) \text{ for some } i,j. \end{cases}$$

Theorem 2.11. *The Y-index of $G_1 \diamond G_2$ is given by*

$$Y(G_1 \diamond G_2) = (n_2+1)^4 Y(G_1) + 16m_1 + n_1 Y(G_2) + 8n_1 F(G_2) + 24n_1 M_1(G_2) + 64n_1 m_2 + 16n_1 n_2.$$

Proof. From definition of $G_1 \diamond G_2$, we have

$$\begin{aligned} Y(G_1 \diamond G_2) &= \sum_{w \in V(G_1 \diamond G_2)} d_G(w)^4 \\ &= \sum_{w \in V(G_1)} d_G(w)^4 + \sum_{w \in W(G_1)} d_G(w)^4 + \sum_{w \in V(G_2^i)} d_G(w)^4 \\ &= \sum_{i=1}^{n_1} (n_2+1)^4 d_{G_1}(v_i)^4 + \sum_{i=1}^{m_1} 2^4 + \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} (d_{G_2}(v_j)+2)^4 \\ &= (n_2+1)^4 Y(G_1) + 16m_1 + \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} (d_{G_2}(v_j)^4 + 8d_{G_2}(v_j)^3 + 24d_{G_2}(v_j)^2 + 32d_{G_2}(v_j) + 16) \\ &= (n_2+1)^4 Y(G_1) + 16m_1 + n_1(Y(G_2) + 8F(G_2) + 24M_1(G_2) + 64m_2 + 16n_2) \end{aligned}$$

from where the desired result follows. □

Example 2.12. Let C_n, P_n, K_n be the cycle, the path and the complete graph, respectively, on n vertices. Then by Theorem 2.11, we obtain the Y-index of the following graphs.

(1) $Y(C_n \diamond C_m) = 16nm^4 + 64nm^3 + 96nm^2 + 320nm + 32n$, $n,m \ge 3$.

(2) $Y(C_n \diamond P_m) = 16nm^4 + 64nm^3 + 96nm^2 + 320nm - 318n$, $n \ge 3$, $m \ge 2$.

(3) $Y(C_n \diamond K_m) = nm^5 + 20nm^4 + 70nm^3 + 100nm^2 + 65nm + 32n$, $n \ge 3$, $m \ge 1$.

2.5 The Vertex-edge Corona

Definition 2.5. The vertex-edge corona of two graphs G_1 and G_2 is denoted by $G_1 \circledast G_2$, is the graph obtained by taking one copy of G_1, n_1 copies of G_2 and also m_1 copies of G_2, then joining the i-th vertex of G_1 to every vertex in the i-th vertex copy of G_2 and also joining the end vertices of j-th edge of G_1 to every vertex in the j-th edge copy of G_2, where $1 \le i \le n_1$ and $1 \le j \le m_1$.

Let the vertex set of the j-th edge copy of G_2 is denoted by $V_{j_e}(G_2) = \{u_{j1}, u_{j2}, \dots, u_{jn_2}\}$ and the vertex set of the i-th vertex copy of G_2 is denoted by $V_{i_v}(G_2) = \{w_{i1}, w_{i2}, \dots, w_{in_2}\}$. Also, let us denote the edge set of the j-th edge and i-th vertex copy of G_2 by $E_{j_e}(G_2)$ and $E_{i_v}(G_2)$ respectively. From definition we have the vertex-edge corona $G_1 \otimes G_2$ has $m_1 + m_1(m_2 + 2n_2) + n_1(n_2 + m_2)$ edge and $n_1 + n_2(n_1 + m_1)$ vertices.

The degree of the vertices of $G_1 \otimes G_2$ is given in the following lemma.

Lemma 2.13 ([13]). *Let G_1 and G_2 be two vertex disjoint graphs. Then the degree of $G_1 \otimes G_2$ is*

$$d_{G_1 \otimes G_2} = \begin{cases} (n_2+1)d_{G_1}(v_i) + n_2, & \forall\ v_i \in V(G_1), \\ d_{G_2}(u_j) + 2, & \forall\ u_{ij} \in V_{i_e}(G_2), \\ d_{G_2}(w_j) + 1, & \forall\ w_{ij} \in V_{i_e}(G_2). \end{cases}$$

Theorem 2.14. *The Y-index of $G_1 \otimes G_2$ is given by*

$$\begin{aligned} Y(G_1 \otimes G_2) = {} & (n_2+1)^4 Y(G_1) + 4n_2(n_2+1)^3 F(G_1) + 6n_2^2(n_2+1)^2 M_1(G_1) + 8n_2^3(n_2+1)m_1 \\ & + n_1 n_2^4 + m_1 Y(G_2) + m_1 8F(G_2) + 24 m_1 M_1(G_2) + 64 m_1 m_2 + 16 m_1 n_2 \\ & + n_1 Y(G_2) + 4n_1 F(G_2) + 6n_1 M_1(G_2) + 8n_1 m_2 + n_1 n_2. \end{aligned}$$

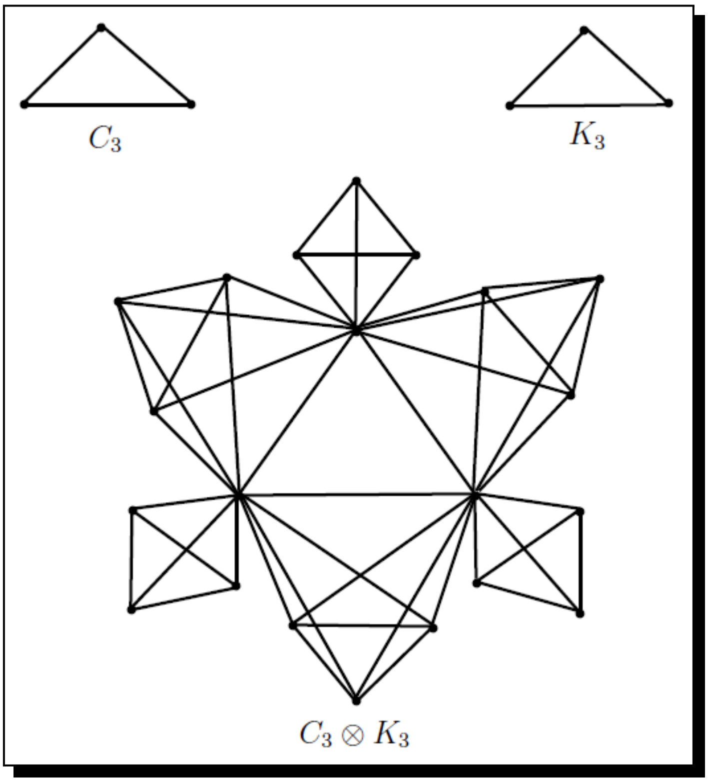

Figure 3. Vertex-edge corona product of C_3 and K_3

Proof. From definition of $G_1 \otimes G_2$, we have

$$Y(G_1 \otimes G_2) = \sum_{v_i \in V(G_1)} d_G(v_i)^4 + \sum_{e_i \in E(G_1)} \sum_{u_{ij} \in V(G_2)} d_G(U_{ij})^4 + \sum_{v_i \in V(G_1)} \sum_{w_{ij} \in V(G_2)} d_G(w_{ij})^4 .$$

Now to calculate the contribution of A_1, we have

$$\begin{aligned} A_1 &= \sum_{v_i \in V(G_1)} d_G(v_i)^4 \\ &= \sum_{vi \in V(G_1)} ((n_2+1)d_{G_1}(v_i)+n_2)^4 \\ &= \sum_{v_i \in V(G_1)} ((n_2+1)^4 d_{G_1}(v_i)^4 + 4n_2(n_2+1)^3 d_{G_1}(v_i)^3 + 6n_2^2(n_2+1)^2 d_{G_1}(v_i)^2 \\ &\quad + 4n_2^3(n_2+1)d_{G_1}(v_i) + n_2^4) \\ &= (n_2+1)^4 Y(G_1) + 4n_2(n_2+1)^3 F(G_1) + 6n_2^2(n_2+1)^2 M_1(G_1) + 8n_2^3(n_2+1)m_1 + n_1 n_2^4, \\ A_2 &= \sum_{e_i \in E(G_1)} \sum_{u_{ij} \in V(G_2)} d_G(u_{ij})^4 \\ &= \sum_{e_i \in E(G_1)} \sum_{u_{ij} \in V(G_2)} (d_{G_2}(u_{ij})+2)^4 \\ &= \sum_{e_i \in E(G_1)} \sum_{u_j \in V_e(G_2)} (d_{G_2}(u_j)^4 + 8d_{G_2}(u_j)^3 + 24d_{G_2}(u_j)^2 + 32d_{G_2}(u_j) + 16) \\ &= m_1(Y(G_2) + 8F(G_2) + 24M_1(G_2) + 64m_2 + 16n_2). \end{aligned}$$

Similarly, we get the contribution of A_3 as follows,

$$\begin{aligned} A_3 &= \sum_{v_i \in V(G_1)} \sum_{w_{ij} \in V(G_2)} d_G(w_{ij})^4 \\ &= \sum_{v_i \in V(G_1)} \sum_{w_{ij} \in V_{iv}(G_2)} (d_{G_2}(w_{ij})+1)^4 \\ &= \sum_{v_i \in V(G_1)} \sum_{w_{ij} \in V_{iv}(G_2)} (d_{G_2}(w_{ij})^4 + 4d_{G_2}(w_{ij})^3 + 6d_{G_2}(w_{ij})^2 + 4d_{G_2}(w_{ij}) + 1) \\ &= n_1(Y(G_2) + 4F(G_2) + 6M_1(G_2) + 8m_2 + n_2). \end{aligned}$$

Adding A_1, A_2 and A_3, we get the desired result. □

Example 2.15. Let C_n, P_n, K_n be the cycle, the path and the complete graph, respectively, on n vertices. Then by Theorem (2.14), we obtain the Y-index of the following graphs.

(1) $Y(C_n \otimes C_m) = 81nm^4 + 216nm^3 + 216nm^2 + 433nm + 16n$, $n, m \geq 3$.

(2) $Y(C_n \otimes K_m) = 2nm^5 + 85nm^4 + 222nm^3 + 220nm^2 + 97nm + 16n$, $n \geq 3$, $m \geq 1$.

3. Conclusion

We calculated the Y-index of many types of corona product of two graphs such as subdivision-vertex corona, subdivision-edge corona, subdivision-vertex neighborhood corona, subdivision-edge neighborhood corona and vertex-edge corona. As an application we have given some explicit expressions for corona products of some graphs. For further study, other topological indices of these corona product of graphs can be computed.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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