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Research Article

On Some Fixed Point Results for Cyclic (α,β)-admissible Almost z-Contraction in Metric-Like Space with Simulation Function

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Abstract. In this paper, we present some fixed point results in the setting of metric-like space by defining a cyclic (α,β)-admissible almost z-contraction embedded in simulation function. Suitable examples are also established to verify the validity of the results obtained.

Keywords. Metric-like space, Fixed point, Simulation function, Cyclic (α,β)-admissible almost z-contraction

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1. Introduction

As generalizations of standard metric spaces, metric-like spaces were considered first by Hitzler and Seda [12] under the name of dislocated metric spaces. After then Amini-Harandi [3] proved some fixed point theorems in the class of metric-like space. Very recently many authors proved fixed point results in the setting of metric-like spaces (e.g, see [5–7, 15, 17]). Also, Khojasteh *et al*. [14] introduced the notion of z-contraction by defining the concept of simulation functions. They unified the some existing metric fixed point results. Then various authors studied in this direction [2, 4, 11, 16, 18].

Alizadeh *et al*. [1] introduced the concept of cyclic (α,β)-admissible mapping and proved some new fixed point theorems which generalize and extend some recent results in the literature. By using this concept, they [15, 16] proved several fixed point theorems with different type contractive conditions. Berinde [9, 10] extended the class of contractive mapping, introducing the notion of almost contractions and proved that every almost contraction mapping defined on a complete metric space has at least one fixed point. Subsequently, Babu *et al*. [8] demonstrated that almost contraction type mappings have a unique fixed point under conditions that present the notion of B-almost contraction. Also, Isik *et al*. [13] proved fixed point theorems for almost z-contraction with an application.

In this paper, we consider some simulation functions to show the existence of fixed points of cyclic (α,β)-admissible almost z-contraction in metric-like space. Furthermore, we also give some examples to illustrate the main results. We modify and generalize the results of Isik *et al*. [13], and Qawaqneh [15].

Let us recall some notations and definitions, we will need in the squeal. Throughout this paper, we assume the symbols $\mathbb{R}$ and $\mathbb{N}$ as a set of real numbers and a set of natural numbers respectively.

2. Basic Facts and Definitions

Definition 2.1 ([3])**.** Let X be a non empty set. A function $\sigma : X \times X \to \mathbb{R}^+$ is said to be a metric-like (or a dislocated metric) on X, if for any $x,y,z \in X$ the following conditions hold:

(σ_1) $\sigma(x,y) = 0 \Rightarrow x = y$;

(σ_2) $\sigma(x,y) = \sigma(y,x)$;

(σ_3) $\sigma(x,z) \le \sigma(x,y) + \sigma(y,z)$.

The pair (X,σ) is called a metric-like space. Then a metric-like on X satisfies all conditions of a metric except that $\sigma(x,x)$ may be positive for $x \in X$. Each metric-like σ on X generates a topology τ_σ on X, whose base is the family of open σ-balls, then for all $x \in X$ and $\epsilon > 0$

$$B_\sigma(X,\epsilon) = \{y \in X : \sigma(x,y) - \sigma(x,x) < \epsilon\}.$$

Now, let (X,σ) be a metric-like space. A sequence $\{x_n\}$ in the metric-like space (X,σ) converges to a point $x \in X$ if and only if

$$\lim_{n\to\infty} \sigma(x_n,x) = \sigma(x,x).$$

Let (X,σ) be metric-like space, and let $T : X \to X$ be a continuous mapping. Then

$$\lim_{n\to\infty} x_n = x \Rightarrow \lim_{n\to\infty} T(x_n) = T(x).$$

A sequence $\{x_n\}$ is Cauchy in (X,σ), if and only if $\lim_{n,m\to\infty} \sigma(x_m,x_n)$ exists and is finite. Moreover, the metric-like space (X,σ) is called complete, if and only if for every Cauchy sequence $\{x_n\}$ in X, there exists $x \in X$ such that

$$\lim_{n\to+\infty} \sigma(x_n,x) = \sigma(x,x) = \lim_{n,m\to\infty} \sigma(x_n,x_m).$$

It is clear that every metric space and partial metric space is a metric-like space but the converse is not true.

Example 2.2. Let $X = \{0,1\}$ and $\sigma(x,y) = \begin{cases} 2, & \text{if } x = y = 0; \\ 1, & \text{otherwise.} \end{cases}$

Then (X,σ) is a metric-like space. It is neither a partial metric space ($\sigma(0,0) \nleq \sigma(0,1)$) nor a metric space ($\sigma(0,0) = 2 \neq 0$).

Remark 2.3. A subset A of a metric-like space (X,σ) is bounded if there is a point $b \in X$ and a positive constant k such that $\sigma(a,b) \leq k$ for all $a \in A$.

Remark 2.4 ([3]). Let $X = \{0,1\}$ such that $\sigma(x,y) = 1$ for each $x,y \in X$ and let $x_n = 1$ for each $n \in N$. Then it is easy to see that $x_n \to 0$ and $x_n \to 1$ and so in metric-like space, the limit of a convergence sequence is not necessarily unique.

The following Lemma is useful to prove our results.

Lemma 2.5 ([3,9]). *Let (X,σ) be a metric-like space. Let $\{x_n\}$ be a sequence in X such that $x_n \to x$, where $x \in X$ and $\sigma(x,y) = 0$. Then for all $y \in X$ we have $\lim_{n\to\infty} \sigma(x_n,y) = \sigma(x,y)$.*

Definition 2.6 ([14]). A function $\zeta : [0,\infty)\times[0,\infty) \to \mathbb{R}$ is called a simulation function if ζ satisfies the following conditions:

(ζ_1) $\zeta(0,0) = 0$.

(ζ_2) $\zeta(t,s) < s - t$, for all $t,s > 0$.

(ζ_3) If $\{t_n\}$ and $\{s_n\}$ are sequences in $(0,\infty)$ such that $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n = l \in (0,\infty)$, then $\lim_{n\to\infty} \sup \zeta(t_n,s_n) < 0$.

The following unique fixed point theorem is established by Khojasteh *et al.* in [14].

Theorem 2.7. *Let (X,d) be a metric space and $T : X \to X$ be a z-contraction with respect to a simulation function ζ, that is*

$$\zeta(d(Tx,Ty),d(x,y)) \geq 0$$

for all $x,y \in X$. Then T has a unique fixed point.

It is worth mentioning that the Banach contraction is an example of z-contractions by defining $\zeta : [0,\infty)\times[0,\infty) \to \mathbb{R}$ via $\zeta(t,s) = \lambda s - t$, for all $s,t \in [0,\infty)$, where $\lambda \in [0,1)$.

Argoubi *et al.* [4] modified Definition 2.6 as follows.

Definition 2.8 ([4]). A simulation function is a function $\zeta : [0,\infty)\times[0,\infty) \to \mathbb{R}$ that satisfies the following conditions:

(i) $\zeta(t,s) < s - t$, for all $t,s > 0$;

(ii) if $\{t_n\}$ and $\{s_n\}$ are sequences in $(0,\infty)$ such that $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n = l \in (0,\infty)$, then $\lim_{n\to\infty} \sup \zeta(t_n, s_n) < 0$.

It is clear that any simulation function in the sense of Khojasteh *et al*. [14, Definition 2.6] is also a simulation function in the sense of Argoubi *et al*. [4, Definition 2.8]. The converse is not true.

Example 2.9 ([4])**.** Define a function $\zeta : [0,\infty)\times[0,\infty) \to \mathbb{R}$ by

$$\zeta(t,s) = \begin{cases} 1, & \text{if } (s,t) = (0,0), \\ \lambda s - t, & \text{otherwise,} \end{cases}$$

where $\lambda \in (0,1)$. Then ζ is a simulation function in the sense of Argoubi *et al*. [4].

In the sense of Definition 2.6, some other examples of simulation functions are given below:

(i) $\zeta(t,s) = cs - t$, for all $t,s \in [0,\infty)$, where $c \in [0,1)$,

(ii) $\zeta(t,s) = s - \phi(s) - t$, for all $t,s \in [0,\infty)$,

where $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ is a lower semi-continuous function such that $\phi(t) = 0$ if and only if $t = 0$.

Definition 2.10 ([1])**.** Let $f : X \to X$ be a mapping and $\alpha, \beta : X \to \mathbb{R}^+$ be two functions. We say that f is a cyclic (α,β)-admissible mapping if:

(1) $\alpha(x) \ge 1$ for some $x \in X \Rightarrow \beta(f(x)) \ge 1$.

(2) $\beta(x) \ge 1$ for some $x \in X \Rightarrow \alpha(f(x)) \ge 1$.

Definition 2.11 ([9,10])**.** Let (X,d) be a metric space. A self mapping T on X is called an almost contraction if there are constants $\lambda \in (0,1)$ and $\theta \ge 0$ such that

$$d(Tx,Ty) \le \lambda d(x,y) + \theta d(y,Tx), \quad \text{for all } x,y \in X.$$

Definition 2.12 ([8])**.** Let (X,d) be a metric space. A self mapping T on X is called an B-almost contraction if there are constants $\lambda \in (0,1)$ and $\theta \ge 0$ such that

$$d(Tx,Ty) \le \lambda d(x,y) + \theta N(x,y), \quad \text{for all } x,y \in X,$$

where $N(x,y) = min\{d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\}$.

3. Main Results

In this section, we present the class of cyclic (α,β)-admissible almost z-contraction mapping and prove some fixed point theorems on complete metric-like space with simulation function.

Theorem 3.1. *Let (X,σ) be a complete metric-like space and $T : X \to X$ be a cyclic (α,β)-admissible almost z-contraction mapping with respect to a ζ simulation function if there exist $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ with $\psi(t) < t$ such that*

$$\zeta(\psi(\sigma(Tx,Ty)), \psi(\sigma(x,y) + \theta N(x,y))) \ge 0, \tag{3.1}$$

for all $x, y \in X$ satisfying $\alpha(x)\beta(y) \geq 1$ where

$$N(x,y) = \min\{\sigma(x,Tx), \sigma(y,Ty), \sigma(x,Ty), \sigma(y,Tx)\} \text{ and } \theta \geq 0.$$

Assume that,

(1) *there exists $x_0 \in X$ such that $\alpha(x_0) \geq 1$ and $\beta(x_0) \geq 1$,*

(2) *T is continuous, or*

(3) *if $\{x_n\} \subseteq X$ such that $x_n \to x$ and $\beta(x_n) \geq 1$ for all n, then $\beta(x) \geq 1$.*

Then T has a fixed point $u \in X$ such that $\sigma(u,u) = 0$. Moreover, if $\alpha(x) \geq 1$ and $\beta(y) \geq 1$, for all $x, y \in Fix(T)$, then T has a unique fixed point.

Proof. Since T is a cyclic (α,β)-admissible mapping and $\alpha(x_0) \geq 1$ then $\beta(x_1) = \beta(Tx_0) \geq 1$ which implies that $\alpha(Tx_1) = \alpha(x_2) \geq 1$. By continuing this method, we have $\alpha(x_{2n}) \geq 1$ and $\beta(x_{2n-1}) \geq 1$ for all $n \in \mathbb{N}$. Again, since T is cyclic (α,β)-admissible mapping and $\beta(x_0) \geq 1$, we have $\beta(x_{2n}) \geq 1$ and $\alpha(x_{2n-1}) \geq 1$, then we deduce

$$\alpha(x_n) \geq 1 \quad \text{and} \quad \beta(x_n) \geq 1, \tag{3.2}$$

for all $n \in \mathbb{N}_0$. Equivalently, $\alpha(x_{n-1})\beta(x_n) \geq 1$. Applying (3.1), we obtain

$$\begin{aligned}&\zeta(\psi(\sigma(Tx_{n-1},Tx_n)), \psi(\sigma(x_{n-1},x_n) + \theta N(x_{n-1},x_n)))\\ &\quad = \zeta(\psi(\sigma(x_n,x_{n+1})), \psi(\sigma(x_{n-1},x_n) + \theta N(x_{n-1},x_n))) \geq 0.\end{aligned} \tag{3.3}$$

Since,

$$\begin{aligned}N(x_{n-1},x_n) &= \min\{\sigma(x_{n-1},Tx_{n-1}), \sigma(x_n,Tx_n), \sigma(x_{n-1},Tx_n), \sigma(x_n,Tx_{n-1})\}\\ &= \min\{\sigma(x_{n-1},x_n), \sigma(x_n,x_{n+1}), \sigma(x_{n-1},x_{n+1}), \sigma(x_n,x_n)\}\\ &= 0.\end{aligned}$$

From (3.3), we have

$$\zeta(\psi(\sigma(x_n,x_{n+1})), \psi(\sigma(x_{n-1,x_n}))) \geq 0. \tag{3.4}$$

If $\sigma(x_n,x_{n+1}) = 0$ for some n, then $x_n = x_{n+1} = Tx_n$, that is x_n is a fixed point of T and so the proof is finished. Therefore, we suppose that $x_n \neq x_{n+1}$ for all $n \geq 0$. Now, we shall show that $\sigma(x_n,x_{n+1}) \leq \sigma(x_{n-1},x_n)$. Now from (3.4) and by (ζ_2), we have

$$\begin{aligned}0 &\leq \zeta(\psi(\sigma(x_n,x_{n+1})), \psi(\sigma(x_{n-1},x_n)))\\ &< \psi(\sigma(x_{n-1},x_n)) - \psi(\sigma(x_n,x_{n+1})),\end{aligned}$$

by using the properties of ψ, we get

$$\sigma(x_{n+1},x_n) < \sigma(x_{n-1},x_n).$$

Hence, we obtain

$$\sigma(x_n,x_{n+1}) \leq \sigma(x_{n-1},x_n), \tag{3.5}$$

for all $n \geq 1$ which implies that $\{\sigma(x_n, x_{n+1})\}$ is non increasing sequence, so there exists $r \geq 0$ such that

$$\lim_{n\to\infty} \sigma(x_n, x_{n+1}) = r.$$

Suppose that $r > 0$. By the properties of ψ, (3.4), (3.5) and the condition (ζ_3)

$$0 \leq \lim_{n\to\infty} \sup \zeta(\psi(\sigma(x_n, x_{n+1})), \psi(\sigma(x_{n-1}, x_n))) < 0,$$

which is a contradiction. Therefore $r = 0$. This implies that

$$\lim_{n\to\infty} \sigma(x_n, x_{n+1}) = 0. \tag{3.6}$$

Again, we show that $\{x_n\}$ is Cauchy sequence in (X, σ), i.e.,

$$\lim_{n,m\to\infty} \sigma(x_n, x_m) = 0. \tag{3.7}$$

Suppose on the contrary that is $\{x_n\}$ is not a Cauchy sequence. Then there exist $\epsilon > 0$ for which we can assume subsequences $\{x_{m_{(k)}}\}$ and $\{x_{n_{(k)}}\}$ of $\{x_n\}$ with $m(k) > n(k) > k$ such that for every k,

$$\sigma(x_{n_{(k)}}, x_{m_{(k)}}) \geq \epsilon. \tag{3.8}$$

This means that

$$\sigma(x_{n_{(k)}}, x_{m_{(k)}-1}) < \epsilon. \tag{3.9}$$

By the triangular inequality and using (3.8) and (3.9), we get

$$\begin{aligned}\epsilon \leq \sigma(x_{n_{(k)}}, x_{m_{(k)}}) &\leq \sigma(x_{n_{(k)}}, x_{m_{(k)-1}}) + \sigma(x_{m_{(k)}-1}, x_{m_{(k)}}) \\ &< \epsilon + \sigma(x_{m_{(k)}-1}, x_{m_{(k)}}).\end{aligned}$$

Letting $n \to \infty$ in the above inequalities and by using (3.7) and (3.8), we have

$$\lim_{n,m\to\infty} \sigma(x_{n_{(k)}}, x_{m_{(k)}}) = \epsilon. \tag{3.10}$$

Since

$$\sigma(x_{n_{(k)}}, x_{m_{(k)}}) \leq \sigma(x_{m_{(k)}}, x_{n_{(k)}+1}) + \sigma(x_{n_{(k)}+1}, x_{n_{(k)}})$$

and

$$\sigma(x_{n_{(k)}+1}, x_{m_{(k)}+1}) \leq \sigma(x_{m_{(k)}}, x_{n_{(k)}+1}) + \sigma(x_{n_{(k)}+1}, x_{m_{(k)}}),$$

then by letting the limit as $k \to \infty$ in above inequalities and using (3.6) and (3.10), we deduce that

$$\lim_{n,m\to\infty} \sigma(x_{n_{(k)}+1}, x_{m_{(k)}}) = \epsilon. \tag{3.11}$$

Similarly, one can easily show that

$$\lim_{n,m\to\infty} \sigma(x_{n_{(k)}+1}, x_{m_{(k)}+1}) = \lim_{n,m\to\infty} \sigma(x_{n_{(k)}}, x_{m_{(k)}+1}) = \epsilon. \tag{3.12}$$

Again since T is a cyclic (α,β)-admissible almost z-contraction mapping and

$$\alpha(x_{n_{(k)}})\beta(x_{m_{(k)}}) \geq 1,$$

then

$$N(x_{n_{(k)}}, x_{m_{(k)}}) = \min\{\sigma(x_{n_{(k)}}, x_{n_{(k)}+1}), \sigma(x_{m_{(k)}}, x_{m_{(k)}+1}), \sigma(x_{n_{(k)}}, x_{m_{(k)}+1}), \sigma(x_{m_{(k)}}, x_{n_{(k)}+1})\}$$

taking $n\to\infty$ and using (3.6), (3.10) and (3.11), we obtain

$$\lim_{n,m\to\infty} N(x_{n(k)},x_{m(k)})=\epsilon. \tag{3.13}$$

If $x_n=x_m$ for some $n<m$, then $x_n=Tx_n=Tx_m=x_{m+1}$ and since $\{\sigma(x_n,x_{n+1})\}$ is non increasing sequence then

$$0<\sigma(x_n,x_{n+1})=\sigma(x_m,x_{m+1})<\sigma(x_{m-1},x_m)<\ldots<\sigma(x_n,x_{n+1})$$

which is a contradiction. Then $x_n\neq x_m$ for all $n<m$. From condition (ζ_2), we have

$$0\le \lim_{k\to\infty}\sup\zeta(\psi(\sigma(x_{n(k)+1},x_{m(k)+1})),\psi(\sigma(x_{n(k)},x_{m(k)}))+\theta N(x_{n(k)},x_{m(k)}))<0,$$

which is a contradiction, due to our assumption, so $\{x_n\}$ is a Cauchy sequence. Since (X,σ) is complete, there exists $u\in X$ such that

$$\lim_{n\to\infty}\sigma(x_n,u)=\sigma(u,u)=\lim_{n,m\to\infty}\sigma(x_n,x_m)=0. \tag{3.14}$$

Now, if T is continuous, we obtain from (3.14)

$$\lim_{n\to\infty}\sigma(x_{n+1},Tu)=\lim_{n\to\infty}\sigma(Tx_{n+1},Tu)=\sigma(Tu,Tu)=0. \tag{3.15}$$

Using Lemma 2.5 and (3.15), we have

$$\lim_{n\to\infty}\sigma(x_n,Tu)=\sigma(u,Tu). \tag{3.16}$$

Combining (3.15) and (3.16), we deduce that $\sigma(Tu,u)=\sigma(Tu,Tu)$. That is $Tu=u$. Assume that condition (3) is hold, that is $\alpha(x_n)\beta(u)\ge 1$. From (3.1), we get

$$\begin{aligned}0&\le\zeta(\psi(\sigma(x_{n+1},Tu)),\psi(\sigma(x_n,u)+\theta N(x_n,u)))\\&=\zeta(\psi(\sigma(Tx_n,Tu)),\psi(\sigma(x_n,u)+\theta N(x_n,u))),\end{aligned} \tag{3.17}$$

where

$$\begin{aligned}N(x_n,u)&=\min\{\sigma(x_n,Tx_n),\sigma(u,Tu),\sigma(x_n,Tu),\sigma(u,Tx_n)\}\\&=\min\{\sigma(x_n,x_{n+1}),\sigma(u,u),\sigma(x_n,u),\sigma(u,x_{n+1})\}=0.\end{aligned} \tag{3.18}$$

From (3.17), (3.18) and (ζ_2), we have

$$\begin{aligned}0&\le\zeta(\psi(\sigma(Tx_n,Tu)),\psi(\sigma(x_n,u)))\\&\le\psi(\sigma(x_n,u))-\psi(\sigma(Tx_n,Tu))<0.\end{aligned}$$

Since ψ is strictly increasing, we have $\sigma(u,Tu)<(u,Tu)$, which is not possible and hence $\sigma(u,Tu)=0$, that is $Tu=u$ and so u is a fixed point of T. Now, we shall show that the uniqueness of fixed point of u. Let v be another fixed point of T. Since $\alpha(u)\beta(v)\ge 1$, it follows from (3.1) that

$$\begin{aligned}0&\le\zeta(\psi(\sigma(Tu,Tv)),\psi(\sigma(u,v)+\theta N(u,v)))\\&=\zeta(\psi(\sigma(u,v)),\psi(\sigma(u,v)+\theta N(u,v))),\end{aligned} \tag{3.19}$$

where

$$\begin{aligned}N(u,v)&=\min\{\sigma(u,Tu),\sigma(v,Tv),\sigma(u,Tv),\sigma(v,Tu)\}\\&=\min\{\sigma(u,u),\sigma(v,v),\sigma(u,v),\sigma(v,u)\}=0.\end{aligned} \tag{3.20}$$

From (3.19) and (3.20), we get

$$0 \le \zeta(\psi(\sigma(u,v)),\psi(\sigma(u,v)))$$
$$< \psi(\sigma(u,v)) - \psi(\sigma(u,v)).$$

Since ψ is strictly increasing, we have $\sigma(u,v) < \sigma(u,v)$, which is a contradiction. Hence $u = v$ that is T has a unique fixed point. □

Corollary 3.2. *Let (X,σ) be a complete metric-like space and $T : X \to X$ be a cyclic (α,β)-admissible z-contraction mapping with respect to ζ simulation function if there exist $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ with $\psi(t) < t$ such that*

$$\zeta(\psi(\sigma(Tx,Ty)),\psi(\sigma(x,y))) \ge 0, \tag{3.21}$$

for all $x,y \in X$ satisfying $\alpha(x)\beta(y) \ge 1$. Assume that

(1) *there exists $x_0 \in X$ such that $\alpha(x_0) \ge 1$ and $\beta(x_0) \ge 1$,*

(2) *T is continuous, or*

(3) *if $\{x_n\} \subseteq X$ such that $x_n \to x$ and $\beta(x_n) \ge 1$ for all n, then $\beta(x) \ge 1$.*

Then T has a unique fixed point.

Proof. The rest of proof follows from Theorem 3.1 by considering cyclic (α,β)-admissible z-contraction mapping that is $N(x,y) = 0$. □

Corollary 3.3. *Let (X,σ) be a complete metric-like space and $T : X \to X$ be a cyclic (α,β)-admissible z-contraction mapping with respect to ζ simulation function if there exist $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ with $\psi(t) < t$ such that*

$$\zeta(\psi(\alpha(x)\beta(y)\sigma(Tx,Ty)),\psi(\sigma(x,y))) \ge 0, \tag{3.22}$$

for all $x,y \in X$ satisfying $\alpha(x)\beta(y) \ge 1$. Assume that

(1) *there exists $x_0 \in X$ such that $\alpha(x_0) \ge 1$ and $\beta(x_0) \ge 1$,*

(2) *T is continuous, or*

(3) *if $\{x_n\} \subseteq X$ such that $x_n \to x$ and $\beta(x_n) \ge 1$ for all n, then $\beta(x) \ge 1$.*

Then T has a unique fixed point.

Proof. The rest of proof follows from Theorem 3.1 by considering cyclic (α,β)-admissible z-contraction mapping that is $N(x,y) = 0$ and $\alpha(x)\beta(y) \ge 1$. □

Example 3.4. Let $X = [0,\infty)$ endowed with the metric-like $\sigma(x,y) = x^2 + y^2$. Consider the mapping $T : X \to X$ given by

$$T(x) = \begin{cases} \frac{x^2}{2}, & \text{if } x \in [0,1], \\ x+1, & \text{otherwise.} \end{cases}$$

Note that (X,σ) is complete metric-like space. Define mappings $\alpha,\beta: X \to \mathbb{R}^+$ by

$$\alpha(x) = \begin{cases} 1, & \text{if } x \in [0,1], \\ 0, & \text{otherwise,} \end{cases}$$

$$\beta(x) = \begin{cases} 1, & \text{if } x \in [0,1], \\ 0, & \text{otherwise.} \end{cases}$$

Let $\zeta(t,s) = \frac{s}{1+s} - t$ for all $s,t \ge 0$ and $\psi(t) = t$. Note that T is a cyclic (α,β)-admissible. In fact, let $x,y \in X$ such that $\alpha(x) \ge 1$ and $\beta(x) \ge 1$. By definition of α and β this implies that $x,y \in [0,1]$.

Thus $\beta(T(x)) \ge 1$, $\alpha(T(x)) \ge 1$. Now, if $\{x_n\} \subset X$ such that $\beta(x_n) \ge 1$ and $x_n \to x$ as $n \to \infty$. Therefore, $x_n \in [0,1]$ hence $x \in [0,1]$, i.e., $\beta(x) \ge 1$.

Let $\alpha(x)\beta(y) \ge 1$. Then $x,y \in [0,1]$ and so we have

$$\psi(\sigma(Tx,Ty)), \psi(\sigma(x,y) + \theta N(x,y)) = \sigma(Tx,Ty), (\sigma(x,y) + \theta N(x,y)). \tag{3.23}$$

Hence $\theta \ge 0$ and

$$\begin{aligned} N(x,y) &= \min\{\sigma(x,Tx), \sigma(y,Ty), \sigma(x,Ty), \sigma(y,Tx)\} \\ &= \min\left\{\left(x^2 + \frac{x^4}{4}\right), \left(y^2 + \frac{y^4}{4}\right), \left(x^2 + \frac{y^4}{4}\right), \left(y^2 + \frac{x^4}{4}\right)\right\}. \end{aligned}$$

Since $x,y \in [0,1]$

$$N(x,y) = 0. \tag{3.24}$$

From (3.23) and (3.24), we have

$$\psi(\sigma(Tx,Ty)), \psi(\sigma(x,y) + \theta N(x,y)) = \sigma(Tx,Ty), \sigma(x,y).$$

It follows that

$$\begin{aligned} \zeta(\psi(\sigma(Tx,Ty)), \psi(\sigma(x,y) + \theta N(x,y))) &= \zeta(\sigma(Tx,Ty), \sigma(x,y)) \\ &= \frac{\sigma(x,y)}{1+\sigma(x,y)} - \sigma(Tx,Ty) \\ &= \frac{x^2+y^2}{1+x^2+y^2} - \left(\frac{x^4}{4} + \frac{y^4}{4}\right) \\ &= \frac{x^2+y^2}{1+x^2+y^2} - \frac{x^4+y^4}{4} \\ &\ge 0. \end{aligned}$$

So, the hypothesis of Corollary 3.2 hold and therefore, T has a unique fixed point $x = 0$.

4. Conclusion

In this paper, we have presented some fixed point results for cyclic (α,β)-admissible almost z-contraction mapping in metric like space via simulation function. Our results are generalization of many existing results in the literature. Finally, we show one example to support the obtained results.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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