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Research Article

Common Fixed Point Theorems on Compatibility and Continuity in Soft Metric Spaces

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Abstract. In this paper, basic notions of soft sets are introduced and some important properties of soft metric spaces are established. It is shown that soft metric extensions of several important fixed point theorems for metric spaces can be directly deduce from comparable existing results. Some examples are given to validate and illustrate the approach. Obtained results modify, improve, sharpen, enrich and generalize various known results.

Keywords. Soft metric space, Soft element, Soft set, Soft mappings, Soft continuous mapping, Soft contractive mapping, Fixed point theorem

Mathematics Subject Classification (2020). 47H10; 54H25

1. Introduction

In the year 1999, Molodtsov [18] initiated a novel concept of soft set theory as a new mathematical tool for dealing with uncertainties. A soft set is a collection of approximate descriptions of an object. Soft systems provide a very general framework with the involvement of parameters. Since soft set theory has a rich potential, applications of soft set theory in other disciplines and real life problems are progressing rapidly.

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Maji *et al*. [15, 16] worked on soft set theory and presented an application of soft sets in decision making problems. Chen [5] introduced a new definition of soft set parametrization reduction and a comparison of it with attribute reduction in rough set theory, Ali *et al*. [1] gave some new operations in soft set theory. Shabir and Naz [22] presented soft topological spaces and investigated some properties of soft topological spaces. Later, many researches about soft topological spaces were studied in [10, 13, 17, 22]. In these studies, the concept of soft point is expressed by different approaches. In the study we use the concept of soft point which was given in [4].

It is known that there are many generalizations of metric spaces: Menger spaces, fuzzy metric spaces, generalized metric spaces, abstract (cone) metric spaces or K-metric and K-normed spaces etc. Das and Samanta [7, 8] introduced a different notion of soft metric space by using a different concept of soft point and investigated some important properties of these spaces. A number of authors have defined contractive type mapping on a complete metric space which are generalizations of the well-known Banach contraction, and which have the property that each such mapping has a unique fixed point [12, 20]. The fixed point can always be found by using Picard iteration, beginning with some initial choice. Some new results on fixed point and applications can be viewed in [2–4, 19, 23].

In the present study, we give some well-known results in soft set theory as preliminaries. Firstly, we examine some important properties of soft metric spaces defined in [7]. Secondly, we investigate properties of soft continuous mappings on soft metric spaces. Finally, we introduced soft contractive mappings on soft metric spaces and prove some common fixed point theorems of soft metric spaces.

Definition 1.1 ([15])**.** Let $\tilde{X}$ be an initial universe set and E be a set of parameters. A pair (F,E) is called a soft set over $\tilde{X}$, if and only if F is a mapping from E into the set of all subsets of the set $\tilde{X}$, i.e., $F : E \to P(\tilde{X})$, where $P(\tilde{X})$ is the power set of $\tilde{X}$.

Definition 1.2 ([16])**.** A soft set (F,E) over $\tilde{X}$ is said to be an absolute soft set denoted by $\tilde{X}$ if for all $e \in E$, $F(e) = \tilde{X}$.

Definition 1.3 ([6])**.** Let R be the set of real numbers and $B(R)$ be the collection of all non-empty bounded subsets of R and E be taken as a set of parameters. Then a mapping $F : E \to B(R)$ is called a soft real set. If a real soft set is a singleton soft set, it will be called a soft real number and denoted by $\tilde{r}$, $\tilde{t}$, $\tilde{s}$ etc. $\tilde{0}$ and $\tilde{1}$ are the soft real numbers where $\tilde{0}(e) = 0$, $\tilde{1}(e) = 1$ for all $e \in E$, respectively.

Definition 1.4 (Soft element)**.** Let $\tilde{X}$ be a non-empty set and E be a non-empty parameter set. Then a function $\varepsilon : E \to \tilde{X}$ is said to be a soft element of $\tilde{X}$. A soft element ε of $\tilde{X}$ is said to belong to a soft set A of $\tilde{X}$, denoted by $\varepsilon \in A$, if $\varepsilon(e) \in A(e)$, $e \in E$. Thus a soft set A of $\tilde{X}$ with respect to the index set E can be expressed as $A(e) = \{\varepsilon(e), \varepsilon \in A\}$, $e \in E$.

Note. It is to be noted that every singleton soft set (a soft set $(F;A)$ for which $F(e)$ is a singleton set, for all $\lambda \in A$) can be identified with a soft element by simply identifying the singleton set with the element that it contains, for all $\lambda \in A$.

Definition 1.5 ([6])**.** Let $\tilde{r}$, $\tilde{s}$ be two soft real numbers. Then the following statements hold:

(i) $\tilde{r} \tilde{\leq} \tilde{s}$ if $\tilde{r}(e) \tilde{\leq} \tilde{s}(e)$ for all $e \in E$;

(ii) $\tilde{r} \tilde{\geq} \tilde{s}$ if $\tilde{r}(e) \tilde{\geq} \tilde{s}(e)$ for all $e \in E$;

(iii) $\tilde{r} \tilde{<} \tilde{s}$ if $\tilde{r}(e) \tilde{<} \tilde{s}(e)$ for all $e \in E$;

(iv) $\tilde{r} \tilde{>} \tilde{s}$ if $\tilde{r}(e) \tilde{>} \tilde{s}(e)$ for all $e \in E$.

Let $SE(\tilde{X})$ be the collection of all soft points of $\tilde{X}$ and $R(E)^*$ denote the set of all non-negative soft real numbers.

Definition 1.6 ([8])**.** A mapping $\tilde{d} : SE(\tilde{X}) \times SE(\tilde{X}) \to R(E)^*$, is said to be a soft metric on the soft set $\tilde{X}$ if d satisfies the following conditions:

(1) $\tilde{0} \tilde{\leq} \tilde{d}(\tilde{x}, \tilde{y})$ for all $\tilde{x}, \tilde{y} \in \tilde{X}$;

(2) $\tilde{d}(\tilde{x}, \tilde{y}) = 0$ if and only if $\tilde{x} = \tilde{y}$;

(3) $\tilde{d}(\tilde{x}, \tilde{y}) = \tilde{d}(\tilde{y}, \tilde{x})$ for all $\tilde{x}, \tilde{y} \in \tilde{X}$;

(4) $\tilde{d}(\tilde{x}, \tilde{y}) \tilde{\leq} \tilde{d}(\tilde{x}, \tilde{z}) + \tilde{d}(\tilde{z}, \tilde{y})$ for all $\tilde{x}, \tilde{y}, \tilde{z} \in \tilde{X}$.

The soft set $\tilde{X}$ with a soft metric d on $\tilde{X}$ is called a soft metric space and denoted by $(\tilde{X}, \tilde{d}, E)$.

Theorem 1.7 ([8], Decomposition Theorem)**.** *If a soft metric $\tilde{d}$ satisfies the condition:*

(5) *for $(\xi, \eta) \in \tilde{x} \times \tilde{X}$ and $\lambda \in A$, $\{\tilde{d}(\tilde{x}, \tilde{y})(\lambda) : \tilde{x}(\lambda) = \xi,\ \tilde{y}(\lambda) = \lambda\}$ is a singleton set, and if for $\lambda \in A$, $\tilde{d}_\lambda : \tilde{X} \times \tilde{X} \to R^+$ is defined by $\tilde{d}_\lambda(\tilde{x}(\lambda), \tilde{y}(\lambda)) = \tilde{d}(\tilde{x}, \tilde{y})(\lambda)$, $\tilde{x}, \tilde{y} \in \tilde{X}$ then $\tilde{d}_\lambda$ is a metric on $\tilde{X}$.*

Definition 1.8 ([8])**.** Let $(\tilde{x}, \tilde{d}, E)$ be a soft metric space $\tilde{\varepsilon}$ be a non-negative soft real number. $B(\tilde{x}, \tilde{\varepsilon}) = \{\tilde{y} \in \tilde{x}, \tilde{d}(\tilde{x}, \tilde{y}) \tilde{<} \tilde{\varepsilon} \subseteq SE(\tilde{x})\}$ is called the soft open ball with center at $\tilde{X}$ and radius $\tilde{\varepsilon}$ and $B[\tilde{x}, \tilde{\varepsilon}] = \{\tilde{y} \in \tilde{x}, \tilde{d}(\tilde{x}, \tilde{y}) \tilde{\leq} \tilde{\varepsilon} \subseteq SE(\tilde{x})\}$ is called the soft closed ball with center at $\tilde{X}$ and radius $\tilde{\varepsilon}$.

Definition 1.9 ([8])**.** Let $\{\tilde{X}_n\}$ be a sequence of soft elements in a soft metric space $(\tilde{X}, \tilde{d}, E)$. The sequence $\{\tilde{x}_n\}$ is said to be convergent in $(\tilde{X}, \tilde{d}, E)$ if there is a soft element $\tilde{y} \in \tilde{X}$ such that $\tilde{d}(\tilde{x}_n, \tilde{y}) \to 0$ as $n \to \infty$. This means for every $\tilde{\varepsilon} \tilde{\geq} \tilde{0}$, chosen arbitrarily, $\exists$ a natural number $N = N(\tilde{\varepsilon})$ such that $\tilde{0} \tilde{\leq} \tilde{d}(\tilde{x}, \tilde{y}) \tilde{<} \tilde{\varepsilon}$, whenever $n > N$.

Theorem 1.10 ([8])**.** *Limit of a sequence in a soft metric space, if exist, is unique.*

Definition 1.11 ([8])**.** A sequence $\{\tilde{x}_n\}$ of soft elements in $(\tilde{X}, \tilde{d}, E)$ is considered as a Cauchy sequence in $\tilde{X}$ if corresponding to every $\tilde{\varepsilon} > \tilde{0}$, $\exists\ m \in N$ such that $\tilde{d}(\tilde{x}_i, \tilde{x}_j) \tilde{\leq} \tilde{\varepsilon}$, for all $i, j \geq m$.

Definition 1.12 ([8])**.** A soft metric space $(\tilde{X}, \tilde{d}, E)$ is called complete if every Cauchy sequence in $\tilde{X}$ converges to some point of $\tilde{X}$. The soft metric space $(\tilde{X}, \tilde{d}, E)$ is called incomplete if it is not complete.

Definition 1.13 ([8]). Let $(\tilde{X},\tilde{d},E)$ be a soft metric space. We can consider $\tilde{X}$ as the collection of all soft elements of $\tilde{X}$ with respect to a parameter set A. Let $f:(\tilde{X},\tilde{d},E)\to(\tilde{X},\tilde{d},E)$ be a mapping. If there exists a soft element $\tilde{x}_0 \in \tilde{X}$ such that $f(\tilde{x}_0)=\tilde{x}_0$, then $\tilde{x}_0$ is called a fixed element of f.

Definition 1.14 ([8]). Let $(\tilde{X},\tilde{d},E)$ be a soft metric space. We can consider $\tilde{X}$ as the collection of all soft elements of $\tilde{X}$ with respect to a parameter set A. A mapping $f:(\tilde{X},\tilde{d},E)\to(\tilde{X},\tilde{d},E)$ is said to be a contraction mapping in $(\tilde{X},\tilde{d},E)$, if there is positive soft real number $\tilde{t}$ with $\tilde{0}<\tilde{t}<\tilde{1}$ such that $\tilde{d}(f(\tilde{x}),f(\tilde{y}))\leq \tilde{t}\tilde{d}(\tilde{x},\tilde{y})$, $\tilde{x},\tilde{y}\in\tilde{X}$.

Definition 1.15 ([8]). Let $(\tilde{X},\tilde{d},E)$ be a complete soft metric space. Let $f:(\tilde{X},\tilde{d},E)\to(\tilde{X},\tilde{d},E)$ be a contraction mapping. Then f has a unique fixed element.

Definition 1.16. Let Ψ be the family of functions $\psi: R(E)^*\to R(E)^*$ satisfying the following conditions:

(i) ψ is non-decreasing,

(ii) $\sum\psi_n(\tilde{t})<\infty$ for all $\tilde{t}>\tilde{0}$, where ψ_n is the nth iterative of ψ.

Remark. For every function $\psi: R(E)^*\to R(E)^*$ the following holds: if ψ is non-decreasing, then for each $\tilde{t}>\tilde{0}$, $\lim_{n\to\infty}\psi_n(\tilde{t})=\tilde{0}\Rightarrow\psi(\tilde{t})<\tilde{t}\Rightarrow\psi(\tilde{0})=\tilde{0}$.
Therefore if $\psi\in\Psi$ then for each $\tilde{t}>\tilde{0}$, $\psi(\tilde{t})<\tilde{t}\Rightarrow\psi(\tilde{0})=\tilde{0}$. The notations $F(f,T)$ and $C(f,T)$ stand for the set of all common fixed point and and the set of all coincidence points of f and T, respectively.

Definition 1.17 ([21]). Let $T:\tilde{X}\to\tilde{X}$ and $\alpha:\tilde{X}\times\tilde{X}\to[0,\infty)$ we say that T is α-admissible if $\tilde{x},\tilde{y}\in\tilde{X}$; $\alpha(\tilde{x},\tilde{y})\gtrsim\tilde{1}\Rightarrow\alpha(T\tilde{x},T\tilde{y})\gtrsim\tilde{1}$.

Definition 1.18 ([21]). Let $T:\tilde{X}\to\tilde{X}$ and $\alpha,\beta:\tilde{X}\times\tilde{X}\to R(E)^*$ we say that T is soft (α,β)-admissible if $\tilde{x},\tilde{y}\in\tilde{X}$, $\alpha(\tilde{x},\tilde{y})\gtrsim\tilde{1}$, $\beta(\tilde{x},\tilde{y})\gtrsim\tilde{1}\Rightarrow\alpha(T\tilde{x},T\tilde{y})\gtrsim\tilde{1}$, $\beta(T\tilde{x},T\tilde{y})\gtrsim\tilde{1}$.

Definition 1.19 ([21]). Let $f,g,S,T:\tilde{X}\to\tilde{X}$ be four self mappings of a nonempty set $\tilde{X}$, and let $\alpha: S(\tilde{X})\cup T(\tilde{X})\times S(\tilde{X})\cup T(\tilde{X})\to[0,\infty)$ be mappings, then the pair (f,g) is called an α-admissible with respect to S and T (in short $\alpha(S,T)$-admissible) if for all $\tilde{x},\tilde{y}\in\tilde{X}$, $\alpha(S\tilde{x},T\tilde{y})\gtrsim 1$ or $\alpha(T\tilde{x},S\tilde{y})\gtrsim 1$ implies $\alpha(f\tilde{x},g\tilde{y})\gtrsim 1$ and $\alpha(g\tilde{x},f\tilde{y})\gtrsim 1$.

Definition 1.20 ([21]). Let $f,g,S,T:\tilde{X}\to\tilde{X}$ be four self mappings of a nonempty set $\tilde{X}$ and let $\alpha,\beta: f(\tilde{X})\cup g(\tilde{X})\times f(\tilde{X})\cup g(\tilde{X})\to[0,\infty)$ be mappings, then the pair (S,T) is called a soft (α,β)-admissible with respect to f and g (in short (S,T) is soft $(\alpha,\beta)_{(f,g)}$-admissible) if for all $\tilde{x},\tilde{y}\in\tilde{X}$, $\alpha(f\tilde{x},g\tilde{y})\gtrsim\tilde{1}$, $\beta(f\tilde{x},g\tilde{y})\gtrsim\tilde{1}$ or $\alpha(g\tilde{X},f\tilde{y})\gtrsim\tilde{1}$, $\beta(g\tilde{x},f\tilde{y})\gtrsim\tilde{1}$ implies $\alpha(S\tilde{x},T\tilde{y})\gtrsim\tilde{1}$, $\beta(S\tilde{x},T\tilde{y})\gtrsim\tilde{1}$ or $\alpha(T\tilde{x},S\tilde{y})\gtrsim\tilde{1}$, $\beta(T\tilde{x},S\tilde{y})\gtrsim\tilde{1}$.

Definition 1.21. Let $(\tilde{X},\tilde{d},E)$ be a soft metric space and $f,g,S,T:\tilde{X}\to\tilde{X}$ be mappings and (S,T) is soft $(\alpha,\beta)_{(f,g)}$-admissible pair, we say that (S,T) is soft $(\alpha,\beta)_{(f,g)}$-contraction if

$$(f\tilde{x},g\tilde{y})\beta(f\tilde{x},g\tilde{y})\tilde{d}(S\tilde{x},T\tilde{y})\leq\psi(M(\tilde{x},\tilde{y})), \tag{1.1}$$

where

$$M(\tilde{x},\tilde{y}) = \max(\tilde{d}(f\tilde{x},g\tilde{y}),\tilde{d}(f\tilde{x},S\tilde{x}),\tilde{d}(g\tilde{y},T\tilde{y}),\frac{1}{2}[\tilde{d}(f\tilde{x},T\tilde{y})+\tilde{d}(g\tilde{y},S\tilde{x})]),$$

for all $\tilde{x},\tilde{y}\in\tilde{X}$ and $\psi\in\Psi$.

Definition 1.22 ([9])**.** Let $(\tilde{X},\tilde{d},E)$ be a soft metric space and two mappings $f,g:(\tilde{X},\tilde{d},E)\to(\tilde{X},\tilde{d},E)$ are said to be soft weakly compatible if $f(g(\tilde{x}))=g(f(\tilde{x}))$ for all $\tilde{x}\in\tilde{X}$ which satisfy $f(\tilde{x})=g(\tilde{x})$.

2. Main Results

Theorem 2.1. *Let $(\tilde{X},\tilde{d},E)$ be a soft metric space and A, B, D, M, S and T be six self mappings in $\tilde{X}$ satisfying the conditions:*

(1) $S(\tilde{X})\subset BD(\tilde{X})$ *and* $T(\tilde{X})\subset AM(\tilde{X})$,

(2) *for each $x,y\in\tilde{X}$, such that $\tilde{x}\neq\tilde{y}$, $\tilde{d}(T\tilde{y},AM\tilde{x})+\tilde{d}(S\tilde{x},BD\tilde{y})+\tilde{d}(BD\tilde{y},AM\tilde{x})\neq 0$, where α, β, γ, η and ξ are non-negative real number with $\alpha+\beta+2\gamma+\eta+\xi<1$, or $\tilde{d}(S\tilde{x},T\tilde{y})=0$ if $\tilde{d}(T\tilde{y},AM\tilde{x})+\tilde{d}(S\tilde{x},BD\tilde{y})+\tilde{d}(BD\tilde{y},AM\tilde{x})=0$, such that*

$$\begin{aligned}\tilde{d}(S\tilde{x},T\tilde{y})\leq{}&\alpha\left[\frac{\tilde{d}(AM\tilde{x},S\tilde{x})+\tilde{d}(BD\tilde{y},T\tilde{y}).\tilde{d}(AM\tilde{x},S\tilde{x})}{1+\tilde{d}(S\tilde{x},T\tilde{y})}\right]\\&+\beta\max\left\{\frac{\tilde{d}(AM\tilde{x},BD\tilde{y})}{1+\tilde{d}(AM\tilde{x},BD\tilde{y})},\frac{\tilde{d}(AM\tilde{x},S\tilde{x})}{1+\tilde{d}(AM\tilde{x},S\tilde{x})},\frac{\tilde{d}(BD\tilde{y},S\tilde{x})}{1+\tilde{d}(BD\tilde{y},S\tilde{x})}\right\}\\&+\gamma\left[\frac{\tilde{d}(BD\tilde{y},T\tilde{y})+\tilde{d}(T\tilde{y},AM\tilde{x})+\tilde{d}(S\tilde{x},BD\tilde{y})}{1+\tilde{d}(BD\tilde{y},T\tilde{y})+\tilde{d}(T\tilde{y},AM\tilde{x})+\tilde{d}(S\tilde{x},BD\tilde{y})}\right]\\&+\eta\left[\frac{\tilde{d}(T\tilde{y},BD\tilde{y}).\tilde{d}(S\tilde{x},AM\tilde{x})}{1+\tilde{d}(T\tilde{y},AM\tilde{x})+\tilde{d}(S\tilde{x},BD\tilde{y})+\tilde{d}(BD\tilde{y},AM\tilde{x})}\right]\\&+\xi\left[\frac{\tilde{d}(S\tilde{x},BD\tilde{y})+\tilde{d}(BD\tilde{y},AM\tilde{x})}{1+\tilde{d}(S\tilde{x},BD\tilde{y}).\tilde{d}(T\tilde{y},BD\tilde{y}).\tilde{d}(S\tilde{x},AM\tilde{x})}\right].\end{aligned}$$

The pair (AM,S) and (BD,T) are commute. The pair (AM,S) and (BD,T) are weakly compatible. Then A, B, D, M, S and T have a unique common fixed point.

Proof. Let $\tilde{x}_0\in\tilde{X}$. Since $S(\tilde{X})\subset BD(\tilde{X})$ and $T(\tilde{X})\subset AM(\tilde{X})$, define for each $n\geq 0$, the sequence $\{\tilde{y}_n\}$ in $\tilde{X}$ by

$$\tilde{y}_{2n+1}=S\tilde{x}_{2n}=BD\tilde{x}_{2n+1} \text{ and } \tilde{y}_{2n+2}=T\tilde{x}_{2n+1}=AM\tilde{x}_{2n+2},\quad n=0,1,2,\dots,$$

$$\begin{aligned}\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+2})&=\tilde{d}(S\tilde{x}_{2n},T\tilde{x}_{2n+1})\\&\leq\alpha\left[\frac{\tilde{d}(AM\tilde{x}_{2n},S\tilde{x}_{2n})+\tilde{d}(BD\tilde{x}_{2n+1},T\tilde{x}_{2n+1}).\tilde{d}(AM\tilde{x}_{2n},S\tilde{x}_{2n})}{1+\tilde{d}(S\tilde{x}_{2n},T\tilde{x}_{2n+1})}\right]\\&\quad+\beta\max\left\{\frac{\tilde{d}(AM\tilde{x}_{2n},BD\tilde{x}_{2n+1})}{1+\tilde{d}(AM\tilde{x}_{2n},BD\tilde{x}_{2n+1})},\frac{\tilde{d}(AM\tilde{x}_{2n},S\tilde{x}_{2n})}{1+\tilde{d}(AM\tilde{x}_{2n},S\tilde{x}_{2n})},\frac{\tilde{d}(BD\tilde{x}_{2n+1},S\tilde{x}_{2n})}{1+\tilde{d}(BD\tilde{x}_{2n+1},S\tilde{x}_{2n})}\right\}\\&\quad+\gamma\left[\frac{\tilde{d}(BD\tilde{x}_{2n+1},T\tilde{x}_{2n+1})+\tilde{d}(T\tilde{x}_{2n+1},AM\tilde{x}_{2n})+\tilde{d}(S\tilde{x}_{2n},BD\tilde{x}_{2n+1})}{1+\tilde{d}(BD\tilde{x}_{2n+1},T\tilde{x}_{2n+1})+\tilde{d}(T\tilde{x}_{2n+1},AM\tilde{x}_{2n})+\tilde{d}(S\tilde{x}_{2n},BD\tilde{x}_{2n+1})}\right]\end{aligned}$$

$$
\begin{aligned}
&+\eta\left[\frac{\tilde{d}(T\tilde{x}_{2n+1},BD\tilde{x}_{2n+1}).\tilde{d}(S\tilde{x}_{2n},AM\tilde{x}_{2n})}{1+\tilde{d}(T\tilde{x}_{2n+1},AM\tilde{x}_{2n})+\tilde{d}(S\tilde{x}_{2n},BD\tilde{x}_{2n+1})+\tilde{d}(BD\tilde{x}_{2n+1},AM\tilde{x}_{2n})}\right]\\
&+\xi\left[\frac{\tilde{d}(S\tilde{x}_{2n},BD\tilde{x}_{2n+1})+\tilde{d}(BD\tilde{x}_{2n+1},AM\tilde{x}_{2n})}{1+\tilde{d}(S\tilde{x}_{2n},BD\tilde{x}_{2n+1}).\tilde{d}(T\tilde{x}_{2n+1},BD\tilde{x}_{2n+1}).\tilde{d}(S\tilde{x}_{2n},AM\tilde{x}_{2n})}\right]\\
\le{}&\alpha\left[\frac{\tilde{d}(\tilde{y}_{2n},\tilde{y}_{2n+1})+\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+2}).\tilde{d}(\tilde{y}_{2n},\tilde{y}_{2n+1})}{1+\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+2})}\right]\\
&+\beta\max\left\{\frac{\tilde{d}(\tilde{y}_{2n},\tilde{y}_{2n+1})}{1+\tilde{d}(\tilde{y}_{2n},\tilde{y}_{2n+1})},\frac{\tilde{d}(\tilde{y}_{2n},\tilde{y}_{2n+1})}{1+\tilde{d}(\tilde{y}_{2n},\tilde{y}_{2n+1})},\frac{\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+1})}{1+\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+1})}\right\}\\
&+\gamma\left[\frac{\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+2})+\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n})+\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+1})}{1+\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+2})+\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n})+\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+1})}\right]\\
&+\eta\left[\frac{\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+1}).\tilde{d}(y_{2n+1},\tilde{y}_{2n})}{1+\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n})+\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+1})+\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n})}\right]\\
&+\xi\left[\frac{\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+1})+\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n})}{1+\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+1}).\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+1}).\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n})}\right]\\
\le{}&\alpha\tilde{d}(\tilde{y}_{2n},\tilde{y}_{2n+1})+\beta\max\{\tilde{d}(\tilde{y}_{2n},\tilde{y}_{2n+1}),\tilde{d}(\tilde{y}_{2n},\tilde{y}_{2n+1}),\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+1})\}\\
&+\gamma[\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+2})+\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n})+\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+1})]\\
&+\eta\tilde{d}(\tilde{y}_{2n},\tilde{y}_{2n+1})+\xi\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n})\\
\le{}&(\alpha+\beta+\gamma+\eta+\xi)\tilde{d}(\tilde{y}_{2n},\tilde{y}_{2n+1}),
\end{aligned}
$$

that is

$$
|\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+2})|\le(\alpha+\beta+\gamma+\eta+\xi)|\tilde{d}(\tilde{y}_{2n},\tilde{y}_{2n+1})|. \tag{2.1}
$$

Similarly,

$$
\begin{aligned}
\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+4})&=\tilde{d}(S\tilde{x}_{2n+2},T\tilde{x}_{2n+3})\\
&\le\alpha+\eta\left[\frac{\tilde{d}(T\tilde{x}_{2n+3},BD\tilde{x}_{2n+3}).\tilde{d}(S\tilde{x}_{2n+2},AM\tilde{x}_{2n+2})}{1+\tilde{d}(T\tilde{x}_{2n+3},AM\tilde{x}_{2n+2})+\tilde{d}(S\tilde{x}_{2n+2},BD\tilde{x}_{2n+3})+\tilde{d}(BD\tilde{x}_{2n+3},AM\tilde{x}_{2n+2})}\right]\\
&\quad+\xi\left[\frac{\tilde{d}(S\tilde{x}_{2n+2},BD\tilde{x}_{2n+3})+\tilde{d}(BD\tilde{x}_{2n+3},AM\tilde{x}_{2n+2})}{1+\tilde{d}(S\tilde{x}_{2n+2},BD\tilde{x}_{2n+3}).\tilde{d}(T\tilde{x}_{2n+3},BD\tilde{x}_{2n+3}).\tilde{d}(S\tilde{x}_{2n+2},AM\tilde{x}_{2n+2})}\right]\\
&\le\alpha\left[\frac{\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+3})+\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+4}).\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+3})}{1+\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+4})}\right]\\
&\quad+\beta\max\left\{\frac{\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+3})}{1+\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+3})},\frac{\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+3})}{1+\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+3})},\frac{\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+3})}{1+\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+3})}\right\}\\
&\quad+\gamma\left[\frac{\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+4})+\tilde{d}(\tilde{y}_{2n+4},\tilde{y}_{2n+2})+\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+3})}{1+\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+4})+\tilde{d}(\tilde{y}_{2n+4},\tilde{y}_{2n+2})+\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+3})}\right]\\
&\quad+\eta\left[\frac{\tilde{d}(\tilde{y}_{2n+4},\tilde{y}_{2n+3}).\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+2})}{1+\tilde{d}(\tilde{y}_{2n+4},\tilde{y}_{2n+2})+\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+3})+\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+2})}\right]\\
&\quad+\xi\left[\frac{\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+3})+\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+2})}{1+\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+3}).\tilde{d}(\tilde{y}_{2n+4},\tilde{y}_{2n+3}).\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+2})}\right]\\
&\le\alpha\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+3})+\beta\max\left\{\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+3}),\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+3}),\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+3})\right\}
\end{aligned}
$$

$$+\gamma[\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+4})+\tilde{d}(\tilde{y}_{2n+4},\tilde{y}_{2n+2})+\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+3})]+\eta\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+3})$$
$$+\xi\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+2})$$
$$\leq(\alpha+\beta+\gamma+\eta+\xi)\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+3}),$$

that is

$$|\tilde{d}(\tilde{y}_{2n+3},\tilde{y}_{2n+4})|\leq(\alpha+\beta+\gamma+\eta+\xi)|\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+3})|. \tag{2.2}$$

Therefore from (2.1) and (2.2) $\tilde{d}(\tilde{y}_n,\tilde{y}_{n+1})\leq(\alpha+\beta+\gamma+\eta+\xi)\tilde{d}(\tilde{y}_{n-1},\tilde{y}_n)$. If $\delta=\alpha+\beta+\gamma+\eta+\xi<1$. Then it is concluded that $\tilde{d}(\tilde{y}_n,\tilde{y}_{n+1})\leq\delta\tilde{d}(\tilde{y}_{n-1},\tilde{y}_n)$

$$\tilde{d}(\tilde{y}_n,\tilde{y}_{n+1})\leq\delta^2\tilde{d}(\tilde{y}_{n-2},\tilde{y}_{n-1})\leq\delta^3\tilde{d}(\tilde{y}_{n-3},\tilde{y}_{n-2})\leq\cdots\leq\delta^n\tilde{d}(\tilde{y}_0,\tilde{y}_1).$$

Now, for all $m>n$, we have

$$\tilde{d}(\tilde{y}_m,\tilde{y}_n)\leq\tilde{d}(\tilde{y}_n,\tilde{y}_{n+1})+\tilde{d}(\tilde{y}_{n+1},\tilde{y}_{n+2})+\ldots+\tilde{d}(\tilde{y}_{m-1},\tilde{y}_m)$$
$$\leq\delta^n\tilde{d}(\tilde{y}_0,\tilde{y}_1)+\delta^{n+1}\tilde{d}(\tilde{y}_0,\tilde{y}_1)+\cdots+\delta^{m-1}\tilde{d}(\tilde{y}_0,\tilde{y}_1)$$
$$|\tilde{d}(\tilde{y}_m,\tilde{y}_n)|\leq\frac{\delta^n}{1-\delta}|\tilde{d}(\tilde{y}_0,\tilde{y}_1)|$$

Hence $|\tilde{d}(\tilde{y}_m,\tilde{y}_n)|\leq\frac{\delta^n}{1-\delta}|\tilde{d}(\tilde{y}_0,\tilde{y}_1)|\to0$ as $m,n\to\infty$, that is $\lim_{n\to\infty}|\tilde{d}(\tilde{y}_m,\tilde{y}_n)|=0$.
Hence $\{\tilde{y}_n\}$ is Cauchy sequence. Since $\tilde{X}$ is completed, so $\{\tilde{y}_n\}$ is converges to some point z, that is $\lim_{n\to\infty}\tilde{y}_n=\lim_{n\to\infty}S\tilde{x}_{2n}=\lim_{n\to\infty}BD\tilde{x}_{2n+1}=\lim_{n\to\infty}T\tilde{x}_{2n+1}=\lim_{n\to\infty}AM\tilde{x}_{2n+2}=z$. There exist some $u\in\tilde{X}$ such that $\tilde{y}_n\to u$ as $n\to\infty$.
$S\tilde{u}=AM\tilde{u}=BD\tilde{u}=T\tilde{u}=\tilde{z}$. Since the pair (AM,S) and (BD,T) are weakly compatible. Then they commute at their coincidence point. Hence $S\tilde{z}=S(AM\tilde{u})=AM(S\tilde{u})=AM\tilde{z}$ and $BD\tilde{z}=BD(T\tilde{u})=T(BD\tilde{u})=T\tilde{z}$.
Now, we shall show that $T\tilde{z}=S\tilde{z}$. Putting $\tilde{x}=\tilde{z}$ and $\tilde{y}=\tilde{x}_{2n+1}$ we have

$$\tilde{d}(S\tilde{z},T\tilde{x}_{2n+1})\leq\alpha\left[\frac{\tilde{d}(AM\tilde{z},S\tilde{z})+\tilde{d}(BD\tilde{x}_{2n+1},T\tilde{x}_{2n+1}).\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(Sz,T\tilde{x}_{2n+1})}\right]$$
$$+\beta\max\left\{\frac{\tilde{d}(AM\tilde{z},BD\tilde{x}_{2n+1})}{1+\tilde{d}(AM\tilde{z},BD\tilde{x}_{2n+1})},\frac{\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(AM\tilde{z},S\tilde{z})},\frac{\tilde{d}(BD\tilde{x}_{2n+1},S\tilde{z})}{1+\tilde{d}(BD\tilde{x}_{2n+1},S\tilde{z})}\right\}$$
$$+\gamma\left[\frac{\tilde{d}(BD\tilde{x}_{2n+1},T\tilde{x}_{2n+1})+\tilde{d}(T\tilde{x}_{2n+1},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{x}_{2n+1})}{1+\tilde{d}(BD\tilde{x}_{2n+1},T\tilde{x}_{2n+1})+\tilde{d}(T\tilde{x}_{2n+1},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{x}_{2n+1})}\right]$$
$$+\eta\left[\frac{\tilde{d}(T\tilde{x}_{2n+1},BD\tilde{x}_{2n+1}).\tilde{d}(S\tilde{z},AM\tilde{z})}{1+\tilde{d}(T\tilde{x}_{2n+1},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{x}_{2n+1})+\tilde{d}(BD\tilde{x}_{2n+1},AM\tilde{z})}\right]$$
$$+\xi\left[\frac{\tilde{d}(S\tilde{z},BD\tilde{x}_{2n+1})+\tilde{d}(BD\tilde{x}_{2n+1},AM\tilde{z})}{1+\tilde{d}(S\tilde{z},BD\tilde{x}_{2n+1}).\tilde{d}(T\tilde{x}_{2n+1},BD\tilde{x}_{2n+1}).\tilde{d}(S\tilde{z},AM\tilde{z})}\right],$$

$$\tilde{d}(S\tilde{z},\tilde{y}_{2n+2})\leq\alpha\left[\frac{\tilde{d}(S\tilde{z},S\tilde{z})+\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+2}).\tilde{d}(S\tilde{z},S\tilde{z})}{1+\tilde{d}(S\tilde{z},\tilde{y}_{2n+2})}\right]$$
$$+\beta\max\left\{\frac{\tilde{d}(S\tilde{z},\tilde{y}_{2n+2})}{1+\tilde{d}(Sz,\tilde{y}_{2n+2})},\frac{\tilde{d}(S\tilde{z},S\tilde{z})}{1+\tilde{d}(S\tilde{z},S\tilde{z})},\frac{\tilde{d}(\tilde{y}_{2n+1},S\tilde{z})}{1+\tilde{d}(\tilde{y}_{2n+1},S\tilde{z})}\right\}$$
$$+\gamma\left[\frac{\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+2})+\tilde{d}(\tilde{y}_{2n+2},S\tilde{z})+\tilde{d}(S\tilde{z},\tilde{y}_{2n+1})}{1+\tilde{d}(\tilde{y}_{2n+1},\tilde{y}_{2n+2})+\tilde{d}(\tilde{y}_{2n+2},S\tilde{z})+\tilde{d}(S\tilde{z},\tilde{y}_{2n+1})}\right]$$

$$+\eta\left[\frac{\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+1}).\tilde{d}(S\tilde{z},S\tilde{z})}{1+\tilde{d}(\tilde{y}_{2n+2},S\tilde{z})+\tilde{d}(S\tilde{z},\tilde{y}_{2n+1})+\tilde{d}(\tilde{y}_{2n+1},S\tilde{z})}\right]$$
$$+\xi\left[\frac{\tilde{d}(S\tilde{z},\tilde{y}_{2n+1})+\tilde{d}(\tilde{y}_{2n+1},S\tilde{z})}{1+\tilde{d}(S\tilde{z},\tilde{y}_{2n+1}).\tilde{d}(\tilde{y}_{2n+2},\tilde{y}_{2n+1}).\tilde{d}(S\tilde{z},S\tilde{z})}\right].$$

Letting $n\to\infty$, we get

$$\begin{aligned}\tilde{d}(S\tilde{z},\tilde{z})\le{}&\alpha\left[\frac{\tilde{d}(S\tilde{z},S\tilde{z})+\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(S\tilde{z},S\tilde{z})}{1+\tilde{d}(S\tilde{z},\tilde{z})}\right]+\beta\max\left\{\frac{\tilde{d}(S\tilde{z},\tilde{z})}{1+\tilde{d}(S\tilde{z},\tilde{z})},\frac{\tilde{d}(S\tilde{z},S\tilde{z})}{1+\tilde{d}(S\tilde{z},S\tilde{z})},\frac{\tilde{d}(\tilde{z},S\tilde{z})}{1+\tilde{d}(\tilde{z},S\tilde{z})}\right\}\\&+\gamma\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},S\tilde{z})+\tilde{d}(S\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},S\tilde{z})+\tilde{d}(S\tilde{z},\tilde{z})}\right]+\eta\left[\frac{\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(S\tilde{z},S\tilde{z})}{1+\tilde{d}(\tilde{z},S\tilde{z})+\tilde{d}(S\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},S\tilde{z})}\right]\\&+\xi\left[\frac{\tilde{d}(S\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},S\tilde{z})}{1+\tilde{d}(S\tilde{z},\tilde{z}).\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(S\tilde{z},S\tilde{z}))}\right]\\\le{}&\beta\tilde{d}(S\tilde{z},\tilde{z})+2\gamma\tilde{d}(S\tilde{z},\tilde{z})+2\xi\tilde{d}(S\tilde{z},\tilde{z}).\end{aligned}$$

Then $\tilde{d}(S\tilde{z},\tilde{z})\le(\beta+2\gamma+2\xi)\tilde{d}(S\tilde{z},\tilde{z})$, that is $|\tilde{d}(S\tilde{z},\tilde{z})|\le(\beta+2\gamma+2\xi)|\tilde{d}(S\tilde{z},\tilde{z})|$, which is contradiction $\beta+2\gamma+2\xi<1$. Therefore $S\tilde{z}=\tilde{z}$, which implies $AM\tilde{z}=\tilde{z}$. Now we prove that $T\tilde{z}=\tilde{z}$, putting $\tilde{X}=\tilde{y}=\tilde{z}$, we get

$$\begin{aligned}\tilde{d}(S\tilde{z},T\tilde{z})\le{}&\alpha\left[\frac{\tilde{d}(AM\tilde{z},S\tilde{z})+\tilde{d}(BD\tilde{z},T\tilde{z}).\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(S\tilde{z},T\tilde{z})}\right]\\&+\beta\max\left\{\frac{\tilde{d}(AM\tilde{z},BD\tilde{z})}{1+\tilde{d}(AM\tilde{z},BD\tilde{z})},\frac{\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(AM\tilde{z},S\tilde{z})},\frac{\tilde{d}(BD\tilde{z},S\tilde{z})}{1+\tilde{d}(BD\tilde{z},S\tilde{z})}\right\}\\&+\gamma\left[\frac{\tilde{d}(BD\tilde{z},T\tilde{z})+\tilde{d}(T\tilde{z},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{z})}{1+\tilde{d}(BDz,T\tilde{z})+\tilde{d}(T\tilde{z},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{z})}\right]\\&+\eta\left[\frac{\tilde{d}(T\tilde{z},BD\tilde{z}).\tilde{d}(S\tilde{z},AM\tilde{z})}{1+\tilde{d}(T\tilde{z},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{z})+\tilde{d}(BD\tilde{z},AM\tilde{z})}\right]\\&+\xi\left[\frac{\tilde{d}(S\tilde{z},BD\tilde{z})+\tilde{d}(BD\tilde{z},AM\tilde{z})}{1+\tilde{d}(S\tilde{z},BD\tilde{z}).\tilde{d}(T\tilde{z},BD\tilde{z}).\tilde{d}(S\tilde{z},AM\tilde{z})}\right],\end{aligned}$$

$$\begin{aligned}\tilde{d}(\tilde{z},T\tilde{z})\le{}&\alpha\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(T\tilde{z},T\tilde{z}).\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},T\tilde{z})}\right]+\beta\max\left\{\frac{\tilde{d}(\tilde{z},T\tilde{z})}{1+\tilde{d}(\tilde{z},T\tilde{z})},\frac{\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})},\frac{\tilde{d}(T\tilde{z},\tilde{z})}{1+\tilde{d}(T\tilde{z},\tilde{z})}\right\}\\&+\gamma\left[\frac{\tilde{d}(T\tilde{z},T\tilde{z})+\tilde{d}(T\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},T\tilde{z})}{1+\tilde{d}(T\tilde{z},T\tilde{z})+\tilde{d}(T\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},T\tilde{z})}\right]+\eta\left[\frac{\tilde{d}(T\tilde{z},T\tilde{z}).\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(T\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},T\tilde{z})+\tilde{d}(T\tilde{z},\tilde{z})}\right]\\&+\xi\left[\frac{\tilde{d}(\tilde{z},T\tilde{z})+\tilde{d}(T\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},T\tilde{z}).\tilde{d}(T\tilde{z},T\tilde{z}).\tilde{d}(\tilde{z},\tilde{z})}\right]\\\le{}&\beta\tilde{d}(\tilde{z},T\tilde{z})+2\gamma\tilde{d}(\tilde{z},T\tilde{z})+2\xi\tilde{d}(\tilde{z},T\tilde{z}).Then\tilde{d}(\tilde{z},T\tilde{z})\le(\beta+2\gamma+2\xi)\tilde{d}(T\tilde{z},\tilde{z}),\end{aligned}$$

that is $|\tilde{d}(\tilde{z},T\tilde{z})|\le(\beta+2\gamma+2\xi)|\tilde{d}(T\tilde{z},\tilde{z})|$, which is contradiction $\beta+2\gamma+2\xi<1$. Therefore $T\tilde{z}=\tilde{z}$, since $BD\tilde{z}=T\tilde{z}$, which implies $BD\tilde{z}=\tilde{z}$. Now, we prove that $M\tilde{z}=\tilde{z}$, putting $\tilde{X}=M\tilde{z}$ and $\tilde{y}=\tilde{z}$, we get

$$\begin{aligned}\tilde{d}(\tilde{s}(M\tilde{z}),T\tilde{z})\le{}&\alpha\left[\frac{\tilde{d}(AM(M\tilde{z}),\tilde{S}(M\tilde{z}))+\tilde{d}(BD\tilde{z},T\tilde{z}).\tilde{d}(AM(M\tilde{z}),\tilde{S}(M\tilde{z}))}{1+\tilde{d}(\tilde{S}(M\tilde{z}),T\tilde{z})}\right]\\&+\beta\max\left\{\frac{\tilde{d}(AM(M\tilde{z}),BD\tilde{z})}{1+\tilde{d}(AM(M\tilde{z}),BD\tilde{z})},\frac{\tilde{d}(AM(M\tilde{z}),\tilde{S}(M\tilde{z}))}{1+\tilde{d}(AM(M\tilde{z}),\tilde{S}(M\tilde{z}))},\frac{\tilde{d}(BD\tilde{z},\tilde{S}(M\tilde{z}))}{1+\tilde{d}(BD\tilde{z},\tilde{S}(M\tilde{z}))}\right\}\end{aligned}$$

$$+\gamma\left[\frac{\tilde{d}(BD\tilde{z},T\tilde{z})+\tilde{d}(T\tilde{z},AM(M\tilde{z}))+\tilde{d}(\tilde{S}(M\tilde{z}),BD\tilde{z})}{1+\tilde{d}(BD\tilde{z},T\tilde{z})+\tilde{d}(T\tilde{z},AM(M\tilde{z}))+\tilde{d}(\tilde{S}(M\tilde{z}),BD\tilde{z})}\right]$$
$$+\eta\left[\frac{\tilde{d}(T\tilde{z},BD\tilde{z}).\tilde{d}(\tilde{S}(M\tilde{z}),AM(M\tilde{z}))}{1+\tilde{d}(T\tilde{z},AM(M\tilde{z}))+\tilde{d}(\tilde{S}(M\tilde{z}),BD\tilde{z})+\tilde{d}(BD\tilde{z},AM(M\tilde{z}))}\right]$$
$$+\xi\left[\frac{\tilde{d}(\tilde{S}(M\tilde{z}),BD\tilde{z})+\tilde{d}(BD\tilde{z},AM(M\tilde{z}))}{1+\tilde{d}(\tilde{S}(M\tilde{z}),BD\tilde{z}).\tilde{d}(T\tilde{z},BD\tilde{z}).\tilde{d}(\tilde{S}(M\tilde{z}),AM(M\tilde{z}))}\right]$$

$$\tilde{d}(M\tilde{z},\tilde{z})\le\alpha\left[\frac{\tilde{d}(M\tilde{z},M\tilde{z})+\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(M\tilde{z},M\tilde{z}))}{1+\tilde{d}(M\tilde{z},\tilde{z})}\right]$$
$$+\beta\max\left\{\frac{\tilde{d}(M\tilde{z},\tilde{z})}{1+\tilde{d}(M\tilde{z},\tilde{z})},\frac{\tilde{d}(M\tilde{z},M\tilde{z})}{1+\tilde{d}(M\tilde{z},M\tilde{z})},\frac{\tilde{d}(z,M\tilde{z})}{1+\tilde{d}(\tilde{z},M\tilde{z})}\right\}$$
$$+\gamma\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},M\tilde{z})+\tilde{d}(M\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},M\tilde{z})+\tilde{d}(M\tilde{z},\tilde{z})}\right]+\eta\left[\frac{\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(M\tilde{z},M\tilde{z})}{1+\tilde{d}(\tilde{z},M\tilde{z})+\tilde{d}(M\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},M\tilde{z})}\right]$$
$$+\xi\left[\frac{\tilde{d}(M\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},M\tilde{z})}{1+\tilde{d}(M\tilde{z},\tilde{z}).\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(M\tilde{z},M\tilde{z})}\right]$$
$$\le\beta\tilde{d}(M\tilde{z},\tilde{z})+2\gamma\tilde{d}(M\tilde{z},\tilde{z})+2\xi\tilde{d}(M\tilde{z},\tilde{z}).$$

Then $\tilde{d}(\tilde{z},T\tilde{z})\le(\beta+2\gamma+2\xi)\tilde{d}(M\tilde{z},\tilde{z})$. That is $|\tilde{d}(M\tilde{z},\tilde{z})|\le(\beta+2\gamma+2\xi)|\tilde{d}(M\tilde{z},\tilde{z})|$, which is contradiction $\beta+2\gamma+2\xi<1$. Therefore $M\tilde{z}=\tilde{z}$, since $AM\tilde{z}=\tilde{z}$, which implies $A\tilde{z}=\tilde{z}$. Now, we prove that $D\tilde{z}=\tilde{z}$, putting $\tilde{X}=\tilde{z}$ and $\tilde{y}=D\tilde{z}$, we get

$$\tilde{d}(S\tilde{z},T(D\tilde{z}))\le\alpha\left[\frac{\tilde{d}(AM\tilde{z},S\tilde{z})+\tilde{d}(BD(D\tilde{z}),T(D\tilde{z}).\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(S\tilde{z},T(D\tilde{z}))}\right]$$
$$+\beta\max\left\{\frac{\tilde{d}(AM\tilde{z},BD(D\tilde{z}))}{1+\tilde{d}(AM\tilde{z},BD(D\tilde{z}))},\frac{\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(AM\tilde{z},S\tilde{z})},\frac{\tilde{d}(BD(D\tilde{z}),S\tilde{z})}{1+\tilde{d}(BD(D\tilde{z}),S\tilde{z})}\right\}$$
$$+\gamma\left[\frac{\tilde{d}(BD(d\tilde{z}),T(d\tilde{z}))+\tilde{d}(T(d\tilde{z}),AM\tilde{z})+\tilde{d}(S\tilde{z},BD(D\tilde{z}))}{1+\tilde{d}(BD(d\tilde{z}),T(d\tilde{z}))+\tilde{d}(T(d\tilde{z}),AM\tilde{z})+\tilde{d}(S\tilde{z},BD(D\tilde{z}))}\right]$$
$$+\eta\left[\frac{\tilde{d}(T(D\tilde{z}),BD(D\tilde{z})).\tilde{d}(S\tilde{z},AM(D\tilde{z}))}{1+\tilde{d}(T(d\tilde{z}),AM\tilde{z})+\tilde{d}(S\tilde{z},BD(d\tilde{z}))+\tilde{d}(BD(D\tilde{z}),AM\tilde{z})}\right]$$
$$+\xi\left[\frac{\tilde{d}(S\tilde{z},BD(d\tilde{z}))+\tilde{d}(BD(D\tilde{z}),AM\tilde{z})}{1+\tilde{d}(S\tilde{z},BD(D\tilde{z})).\tilde{d}(T(D\tilde{z}),BD(D\tilde{z})).\tilde{d}(S\tilde{z},AM\tilde{z})}\right]$$

$$\tilde{d}(\tilde{z},D\tilde{z})\le\alpha\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(D\tilde{z},D\tilde{z}).\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},D\tilde{z})}\right]+\beta\max\left\{\frac{\tilde{d}(\tilde{z},D\tilde{z})}{1+\tilde{d}(\tilde{z},D\tilde{z})},\frac{\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})},\frac{\tilde{d}(D\tilde{z},\tilde{z})}{1+\tilde{d}(D\tilde{z},\tilde{z})}\right\}$$
$$+\gamma\left[\frac{\tilde{d}(d\tilde{z},d\tilde{z})+\tilde{d}(d\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},D\tilde{z})}{1+\tilde{d}(d\tilde{z},d\tilde{z})+\tilde{d}(d\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},D\tilde{z})}\right]+\eta\left[\frac{\tilde{d}(D\tilde{z},D\tilde{z}).\tilde{d}(\tilde{z},D\tilde{z})}{1+\tilde{d}(d\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},d\tilde{z})+\tilde{d}(D\tilde{z},\tilde{z})}\right]$$
$$+\xi\left[\frac{\tilde{d}(\tilde{z},d\tilde{z})+\tilde{d}(D\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},D\tilde{z}).\tilde{d}(D\tilde{z},D\tilde{z}).\tilde{d}(\tilde{z},\tilde{z})}\right]$$
$$\le(\beta+2\gamma+2\xi)\tilde{d}(\tilde{z},D\tilde{z}),$$

that is $|\tilde{d}(\tilde{z},D\tilde{z})|\le(\beta+2\gamma+2\xi)|\tilde{d}(\tilde{z},D\tilde{z})|$, which is contradiction $\beta+2\gamma+2\xi<1$. Therefore $D\tilde{z}=\tilde{z}$, since $BD\tilde{z}=\tilde{z}$, which implies $B\tilde{z}=\tilde{z}$.
Therefore, $\tilde{z}$ is a unique common fixed point of A, B, D, M, S and T.

Uniqueness: Let $\tilde{u}$ be an another common fixed point of A, B, D, M, S and T. Then, we have

$$\begin{aligned}
\tilde{d}(S\tilde{z},T\tilde{u}) \le \alpha &\left[\frac{\tilde{d}(AM\tilde{z},S\tilde{z})+\tilde{d}(BD\tilde{u},T\tilde{u}).\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(S\tilde{z},T\tilde{u})}\right] \\
&+\beta\max\left\{\frac{\tilde{d}(AM\tilde{z},BD\tilde{u})}{1+\tilde{d}(AM\tilde{z},BD\tilde{u})},\frac{\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(AM\tilde{z},S\tilde{z})},\frac{\tilde{d}(BD\tilde{u},S\tilde{z})}{1+\tilde{d}(BD\tilde{u},S\tilde{z})}\right\} \\
&+\gamma\left[\frac{\tilde{d}(BD\tilde{u},T\tilde{u})+\tilde{d}(T\tilde{u},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{u})}{1+\tilde{d}(BD\tilde{u},T\tilde{u})+\tilde{d}(T\tilde{u},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{u})}\right] \\
&+\eta\left[\frac{\tilde{d}(T\tilde{u},BD\tilde{u}).\tilde{d}(S\tilde{z},AM\tilde{z})}{1+\tilde{d}(T\tilde{u},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{u})+\tilde{d}(BD\tilde{u},AM\tilde{z})}\right] \\
&+\xi\left[\frac{\tilde{d}(S\tilde{z},BD\tilde{u})+\tilde{d}(BD\tilde{u},AM\tilde{z})}{1+\tilde{d}(S\tilde{z},BD\tilde{u}).\tilde{d}(T\tilde{u},BD\tilde{u}).\tilde{d}(S\tilde{z},AM\tilde{z})}\right] \\
\tilde{d}(\tilde{z},\tilde{u}) \le \alpha &\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{u},\tilde{u}).\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{u})}\right]+\beta\max\left\{\frac{\tilde{d}(\tilde{z},\tilde{u})}{1+\tilde{d}(\tilde{z},\tilde{u})},\frac{\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})},\frac{\tilde{d}(\tilde{u},\tilde{z})}{1+\tilde{d}(\tilde{u},\tilde{z})}\right\} \\
&+\gamma\left[\frac{\tilde{d}(\tilde{u},\tilde{u})+\tilde{d}(\tilde{u},\tilde{z})+\tilde{d}(\tilde{z},\tilde{u})}{1+\tilde{d}(\tilde{u},\tilde{u})+\tilde{d}(\tilde{u},\tilde{z})+\tilde{d}(\tilde{z},\tilde{u})}\right]+\eta\left[\frac{\tilde{d}(\tilde{u},\tilde{u}).\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{u},\tilde{z})+\tilde{d}(\tilde{z},\tilde{u})+\tilde{d}(\tilde{u},\tilde{z})}\right] \\
&+\xi\left[\frac{\tilde{d}(\tilde{z},\tilde{u})+\tilde{d}(\tilde{u},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{u}).\tilde{d}(\tilde{u},\tilde{u}).\tilde{d}(\tilde{z},\tilde{z})}\right] \\
&\le (\beta+2\gamma+2\xi)\tilde{d}(\tilde{z},\tilde{u}),
\end{aligned}$$

that is $|\tilde{d}(\tilde{z},\tilde{u})| \le (\beta+2\gamma+2\xi)|\tilde{d}(\tilde{z},\tilde{u})|$, which is contradiction $\beta+2\gamma+2\xi<1$. Therefore $\tilde{z}=\tilde{u}$, therefore $\tilde{z}$ is a unique common fixed point of A, B, D, M, S and T.

Case II: We consider the case: $\tilde{d}(T\tilde{x}_{2n+1},AM\tilde{x}_{2n})+\tilde{d}(S\tilde{x}_{2n},BD\tilde{x}_{2n+1})+\tilde{d}(BD\tilde{x}_{2n+1},AM\tilde{x}_{2n})=0$ (for any n) implies that $\tilde{d}(S\tilde{x}_{2n},T\tilde{x}_{2n+1})$. So that $\tilde{y}_{2n}=S\tilde{x}_{2n}=\tilde{y}_{2n+1}=BD\tilde{x}_{2n+1}=T\tilde{x}_{2n+1}=AM\tilde{x}_{2n+2}=\tilde{y}_{2n+2}$. Thus we have $\tilde{y}_{2n+1}=S\tilde{x}_{2n}=AM\tilde{x}_{2n}=\tilde{y}_{2n}$, there exists n_1 and m_1, such that $n_1=Sm_1=AMm_1=m_1$. Similarly, $\tilde{y}_{2n+2}=T\tilde{x}_{2n+1}=BD\tilde{x}_{2n+1}=\tilde{y}_{2n+1}$, there exists n_2 and m_2 such that $n_2=Tm_2=BD_{m2}=m_2$. As $\tilde{d}(Tm_2,AMm_1)+\tilde{d}(Sm_1,BDm_2)+\tilde{d}(BDm_2,AMm_1)=0$ implies that, $\tilde{d}(Sm_1,Tm_2)=0$, so that $n_1=Sm_1=AMm_1=Tm_2=BDm_2=n_2$, which is turn yields that $n_1=Sn_1=AMm_1=AMn_1$, similarly, one can also have $n_2=Tn_2=BDn_2$. As $n_1=n_2$, implies $n_1=Sn_1=Tn_1=BDn_1$, therefore $n_1=Sn_1=An_1=Mn_1=Tn_1=Bn_1=Dn_1$. Hence, $n_1=n_2$, is common fixed point.

Uniqueness: Let v_1 is an another common fixed point of A, B, D, M, S and T. Then, we have $v_1=Sv_1=Av_1=Mv_1=Tv_1=Bv_1=Dv_1$. Therefore, $d(Tv_1,AMv_1)+d(Sn_1,BDv_1)+d(BDv_1,AMn_1)=0$, so that $d(n_1,v_1)=0$. Hence this implies that $n_1=v_1$. Hence n_1 is a unique common fixed point of A, B, D, M, S and T. □

Corollary 2.2. *Let $(\tilde{X},\tilde{d})$ be a complex valued soft metric space and D, M, S and T be four self mappings in $\tilde{X}$ satisfying the conditions:*

(i) *$S(\tilde{X})\subset D(\tilde{X})$ and $T(\tilde{X})\subset M(\tilde{X})$,*

(ii) *for each $\tilde{x},\tilde{y}\in\tilde{X}$, such that $\tilde{x}\ne\tilde{y}$, $\tilde{d}(T\tilde{y},M\tilde{x})+\tilde{d}(S\tilde{x},D\tilde{y})+\tilde{d}(D\tilde{y},M\tilde{x})\ne 0$, where α,β,γ,η and ξ are non-negative real number with $\alpha+\beta+2\gamma+\eta+\xi<1$, or $\tilde{d}(S\tilde{x},T\tilde{y})=0$ if*

$\tilde{d}(T\tilde{y},M\tilde{x})+\tilde{d}(S\tilde{x},D\tilde{y})+\tilde{d}(D\tilde{y},M\tilde{x})=0$, *such that*

$$\begin{aligned}\tilde{d}(S\tilde{x},T\tilde{y})\le\alpha&\left[\frac{\tilde{d}(M\tilde{x},S\tilde{x})+\tilde{d}(D\tilde{y},T\tilde{y}).\tilde{d}(M\tilde{x},S\tilde{x})}{1+\tilde{d}(S\tilde{x},T\tilde{y})}\right]\\&+\beta\max\left\{\frac{\tilde{d}(M\tilde{x},D\tilde{y})}{1+\tilde{d}(M\tilde{x},D\tilde{y})},\frac{\tilde{d}(M\tilde{x},S\tilde{x})}{1+\tilde{d}(M\tilde{x},S\tilde{x})},\frac{\tilde{d}(D\tilde{y},S\tilde{x})}{1+\tilde{d}(D\tilde{y},S\tilde{x})}\right\}\\&+\gamma\left[\frac{\tilde{d}(D\tilde{y},T\tilde{y})+\tilde{d}(T\tilde{y},M\tilde{x})+\tilde{d}(S\tilde{x},D\tilde{y})}{1+\tilde{d}(D\tilde{y},T\tilde{y})+\tilde{d}(T\tilde{y},M\tilde{x})+\tilde{d}(S\tilde{x},D\tilde{y})}\right]\\&+\eta\left[\frac{\tilde{d}(T\tilde{y},D\tilde{y}).\tilde{d}(S\tilde{x},M\tilde{x})}{1+\tilde{d}(T\tilde{y},M\tilde{x})+\tilde{d}(S\tilde{x},D\tilde{y})+\tilde{d}(D\tilde{y},M\tilde{x})}\right]\\&+\xi\left[\frac{\tilde{d}(S\tilde{x},D\tilde{y})+\tilde{d}(D\tilde{y},M\tilde{x})}{1+\tilde{d}(S\tilde{x},D\tilde{y}).\tilde{d}(T\tilde{y},D\tilde{y}).\tilde{d}(S\tilde{x},M\tilde{x})}\right],\end{aligned}$$

(iii) *the pair* (M,S) *and* (D,T) *are weakly compatible.*

Then D, M, S and T have a unique common fixed point.

Corollary 2.3. (M,S) *and* (D,T) *are four commuting self mappings defined on a complete complex valued metric space* $(\tilde{X},\tilde{d})$ *satisfying the condition:*

$$\begin{aligned}\tilde{d}(S\tilde{x},T\tilde{y})\le\alpha&\left[\frac{\tilde{d}(M^m\tilde{x},S^m\tilde{x})+\tilde{d}(d^n\tilde{y},T^n\tilde{y}).\tilde{d}(M^m\tilde{x},S^m\tilde{x})}{1+\tilde{d}(S^m\tilde{x},T^n\tilde{y})}\right]\\&+\beta\max\left\{\frac{\tilde{d}(M^m\tilde{x},d^n\tilde{y})}{1+\tilde{d}(M^m\tilde{x},d^n\tilde{y})},\frac{\tilde{d}(M^m\tilde{x},S^m\tilde{x})}{\tilde{d}(M^m\tilde{x},S^m\tilde{x})},\frac{\tilde{d}(d^n\tilde{y},S^m\tilde{x})}{\tilde{d}(d^n\tilde{y},S^m\tilde{x})}\right\}\\&+\gamma\left[\frac{\tilde{d}(d^n\tilde{y},T^n\tilde{y})+\tilde{d}(T^n\tilde{y},M^m\tilde{x})+\tilde{d}(S^m\tilde{x},d^n\tilde{y})}{1+\tilde{d}(d^n\tilde{y},T^n\tilde{y})+\tilde{d}(T^n\tilde{y},M^m\tilde{x})+\tilde{d}(S^m\tilde{x},d^n\tilde{y})}\right]\\&+\eta\left[\frac{\tilde{d}(T^n\tilde{y},d^n\tilde{y}).\tilde{d}(S^m\tilde{x},M^m\tilde{x})}{1+\tilde{d}(T^n\tilde{y},M^m\tilde{x})+\tilde{d}(S^m\tilde{x},d^n\tilde{y})+\tilde{d}(d^n\tilde{y},M^m\tilde{x})}\right]\\&+\xi\left[\frac{\tilde{d}(S^m\tilde{x},d^n\tilde{y})+\tilde{d}(d^n\tilde{y},M^m\tilde{x})}{1+\tilde{d}(S^m\tilde{x},d^n\tilde{y}).\tilde{d}(T^n\tilde{y},d^n\tilde{y}).\tilde{d}(S^m\tilde{x},M^m\tilde{x})}\right].\end{aligned}$$

For each $\tilde{X},\tilde{y}\in\tilde{X}$, *such that* $\tilde{X}\neq\tilde{y}$, *where* α,β,γ,η *and* ξ *are non-negative real number with* $\alpha+\beta+2\gamma+\eta+\xi<1$, *or if* $\tilde{d}(S^m\tilde{X},T^n\tilde{y})=0$ *if* $\tilde{d}(T^n\tilde{y},M^m\tilde{X})+\tilde{d}(S^m\tilde{X},D^n\tilde{y})+\tilde{d}(D^n\tilde{y},M^m\tilde{X})=0$. *Then D, M, S and T have a unique common fixed point.*

Theorem 2.4. *Let A, B, D, M, S and T be self mappings of a complete complex valued soft metric space* $(\tilde{X},\tilde{d})$ *satisfying conditions:*

(i) $S(\tilde{X})\subset BD(\tilde{X})$ *and* $T(\tilde{X})\subset AM(\tilde{X})$,

(ii) *for each* $\tilde{x},\tilde{y}\in\tilde{X}$, *where* α,β,γ,η *and* ξ *are non-negative real number with* $\alpha+\beta+2\gamma+\eta+\xi<1$, *such that*

$$\begin{aligned}\tilde{d}(S\tilde{x},T\tilde{y})\le\alpha&\left[\frac{\tilde{d}(AM\tilde{x},S\tilde{x})+\tilde{d}(BD\tilde{y},T\tilde{y}).\tilde{d}(AM\tilde{x},S\tilde{x})}{1+\tilde{d}(S\tilde{x},T\tilde{y})}\right]\\&+\beta\max\left\{\frac{\tilde{d}(AM\tilde{x},BD\tilde{y})}{1+\tilde{d}(AM\tilde{x},BD\tilde{y})},\frac{\tilde{d}(AM\tilde{x},S\tilde{x})}{1+\tilde{d}(AM\tilde{x},S\tilde{x})},\frac{\tilde{d}(BD\tilde{y},S\tilde{x})}{1+\tilde{d}(BD\tilde{y},S\tilde{x})}\right\}\end{aligned}$$

$$+\gamma\left[\frac{\tilde{d}(BD\tilde{y},T\tilde{y})+\tilde{d}(T\tilde{y},AM\tilde{x})+\tilde{d}(S\tilde{x},BD\tilde{y})}{1+\tilde{d}(BD\tilde{y},T\tilde{y})+\tilde{d}(T\tilde{y},AM\tilde{x})+\tilde{d}(S\tilde{x},BD\tilde{y})}\right]$$
$$+\eta\left[\frac{\tilde{d}(T\tilde{y},BD\tilde{y}).\tilde{d}(S\tilde{x},AM\tilde{x})}{1+\tilde{d}(T\tilde{y},AM\tilde{x})\tilde{d}(S\tilde{x},BD\tilde{y})+\tilde{d}(BD\tilde{y},AM\tilde{x})}\right]$$
$$+\xi\left[\frac{\tilde{d}\tilde{x}+\tilde{d}(BD\tilde{y},AM\tilde{x})}{1+\tilde{d}(S\tilde{x},BD\tilde{y}).\tilde{d}(T\tilde{y},BD\tilde{y}).\tilde{d}(S\tilde{x},AM\tilde{x})}\right],$$

(iii) *(AM,S) are compatible, and AM or S is continuous and (BD,T) are weakly compatible,*

(iv) *(BD,T) are compatible, and BD or T is continuous and (AM,S) are weakly compatible. Then A, B, D, M, S and T have a unique common fixed point.*

Proof. By above theorem $\{\tilde{y}_n\}$ is a Cauchy sequence. Since $\tilde{X}$ is completed, so $\{\tilde{y}_n\}$ is converges to some point $\tilde{z}$. Thus subsequence $\{S\tilde{x}_{2n}\}$, $\{BD\tilde{x}_{2n+1}\}$, $\{T\tilde{x}_{2n+1}\}$ and $\{AM\tilde{x}_{2n+2}\}$ also converges to $\tilde{z}$, that is

$$\lim_{n\to\infty}\tilde{y}_n=\lim_{n\to\infty}S\tilde{x}_{2n}=\lim_{n\to\infty}BD\tilde{x}_{2n+1}=\lim_{n\to\infty}AM\tilde{x}_{2n+2}=\lim_{n\to\infty}T\tilde{x}_{2n+1}. \tag{2.3}$$

Assume that $\tilde{s}$ is continuous. Since (AM,S) are compatible, we have

$$\lim_{n\to\infty}AM(S\tilde{x}_{2n+2})=\lim_{n\to\infty}\tilde{S}(AM\tilde{x}_{2n+2})=S\tilde{z}. \tag{2.4}$$

Putting $\tilde{X}=\tilde{X}_{2n+2}$, $\tilde{y}=\tilde{X}_{2n+1}$, then we have

$$\tilde{d}(AM(S\tilde{x}_{2n}),T\tilde{x}_{2n+1})$$
$$\le\alpha\left[\frac{\tilde{d}(AM\tilde{x}_{2n+2},S\tilde{x}_{2n+2})+\tilde{d}(BD\tilde{x}_{2n+1},T\tilde{x}_{2n+1}).\tilde{d}(AM\tilde{x}_{2n+2},S\tilde{x}_{2n+2})}{1+\tilde{d}(S\tilde{x}_{2n},T\tilde{x}_{2n+1})}\right]$$
$$+\beta\max\left\{\frac{\tilde{d}(AM\tilde{x}_{2n+2},BD\tilde{x}_{2n+1})}{1+\tilde{d}(AM\tilde{x}_{2n+2},BD\tilde{x}_{2n+1})},\frac{\tilde{d}(AM\tilde{x}_{2n+2},S\tilde{x}_{2n+2})}{1+\tilde{d}(AM\tilde{x}_{2n+2},S\tilde{x}_{2n+2})},\frac{\tilde{d}(BD\tilde{x}_{2n+1},S\tilde{x}_{2n+2})}{1+\tilde{d}(BD\tilde{x}_{2n+1},S\tilde{x}_{2n+2})}\right\}$$
$$+\gamma\left[\frac{\tilde{d}(BD\tilde{x}_{2n+1},T\tilde{x}_{2n+1})+\tilde{d}(T\tilde{x}_{2n+1},AM\tilde{x}_{2n+2})+\tilde{d}(S\tilde{x}_{2n+2},BD\tilde{x}_{2n+1})}{1+\tilde{d}(BD\tilde{x}_{2n+1},T\tilde{x}_{2n+1})+\tilde{d}(T\tilde{x}_{2n+1},AM\tilde{x}_{2n+2})+\tilde{d}(S\tilde{x}_{2n+2},BD\tilde{x}_{2n+1})}\right]$$
$$+\eta\left[\frac{\tilde{d}(T\tilde{x}_{2n+1},BD\tilde{x}_{2n+1}).\tilde{d}(S\tilde{x}_{2n+2},AM\tilde{x}_{2n+2})}{1+\tilde{d}(T\tilde{x}_{2n+1},AM\tilde{x}_{2n+2})+\tilde{d}(S\tilde{x}_{2n+2},BD\tilde{x}_{2n+1})+\tilde{d}(BD\tilde{x}_{2n+1},AM\tilde{x}_{2n+2})}\right]$$
$$+\xi\left[\frac{\tilde{d}(S\tilde{x}_{2n+2},BD\tilde{x}_{2n+1})+\tilde{d}(BD\tilde{x}_{2n+1},AM\tilde{x}_{2n+2})}{1+\tilde{d}(S\tilde{x}_{2n+2},BD\tilde{x}_{2n+1}).\tilde{d}(T\tilde{x}_{2n+1},BD\tilde{x}_{2n+1}).\tilde{d}(S\tilde{x}_{2n+2},AM\tilde{x}_{2n+2})}\right].$$

Letting $n\to\infty$, in the above inequality and using (2.3) and (2.4), we get

$$\tilde{d}(S\tilde{z},\tilde{z})\le\alpha\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})}\right]+\beta\max\left\{\frac{\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})},\frac{\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})},\frac{\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})}\right\}$$
$$+\gamma\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},\tilde{z})}\right]$$
$$+\eta\left[\frac{\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},\tilde{z})}\right]$$
$$+\xi\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(\tilde{z},\tilde{z})}\right]$$
$$\le 0,$$

that is $|\tilde{d}(S\tilde{z},\tilde{z})| \le 0$, hence $S\tilde{z}=\tilde{z}$. Now putting $\tilde{x}=\tilde{z}$ and $\tilde{y}=\tilde{x}_{2n+1}$, we have

$$\begin{aligned}\tilde{d}(S\tilde{z},T\tilde{x}_{2n+1}) \le \alpha &\left[\frac{\tilde{d}(AM\tilde{z},S\tilde{z})+\tilde{d}(BD\tilde{x}_{2n+1},T\tilde{x}_{2n+1}).\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(S\tilde{z},T\tilde{x}_{2n+1})}\right]\\ &+\beta\max\left\{\frac{\tilde{d}(AM\tilde{z},BD\tilde{x}_{2n+1})}{1+\tilde{d}(AM\tilde{z},BD\tilde{x}_{2n+1})},\frac{\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(AM\tilde{z},S\tilde{z})},\frac{\tilde{d}(BD\tilde{x}_{2n+1},S\tilde{z})}{1+\tilde{d}(BD\tilde{x}_{2n+1},S\tilde{z})}\right\}\\ &+\gamma\left[\frac{\tilde{d}(BD\tilde{x}_{2n+1},T\tilde{x}_{2n+1})+\tilde{d}(T\tilde{x}_{2n+1},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{x}_{2n+1})}{1+\tilde{d}(BD\tilde{x}_{2n+1},T\tilde{x}_{2n+1})+\tilde{d}(T\tilde{x}_{2n+1},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{x}_{2n+1})}\right]\\ &+\eta\left[\frac{\tilde{d}(T\tilde{x}_{2n+1},BD\tilde{x}_{2n+1}).\tilde{d}(S\tilde{z},AM\tilde{z})}{1+\tilde{d}(T\tilde{x}_{2n+1},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{x}_{2n+1})+\tilde{d}(BD\tilde{x}_{2n+1},AM\tilde{z})}\right]\\ &+\xi\left[\frac{\tilde{d}(S\tilde{z},BD\tilde{x}_{2n+1})+\tilde{d}(BD\tilde{x}_{2n+1},AM\tilde{z})}{1+\tilde{d}(S\tilde{z},BD\tilde{x}_{2n+1}).\tilde{d}(T\tilde{x}_{2n+1},BD\tilde{x}_{2n+1}).\tilde{d}(S\tilde{z},AM\tilde{z})}\right].\end{aligned}$$

Letting $n\to\infty$, we get

$$\begin{aligned}\tilde{d}(\tilde{z},\tilde{z}) \le \alpha &\left[\frac{\tilde{d}(AM\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(AM\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})}\right]+\beta\max\left\{\frac{\tilde{d}(AM\tilde{z},\tilde{z})}{1+\tilde{d}(AM\tilde{z},\tilde{z})},\frac{\tilde{d}(AM\tilde{z},\tilde{z})}{1+\tilde{d}(AM\tilde{z},\tilde{z})},\frac{\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})}\right\}\\ &+\gamma\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},AM\tilde{z})+\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},AM\tilde{z})+\tilde{d}(\tilde{z},\tilde{z})}\right]+\eta\left[\frac{\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(\tilde{z},AM\tilde{z})}{1+\tilde{d}(\tilde{z},AM\tilde{z})+\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},AM\tilde{z})}\right]\\ &+\xi\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},AM\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(\tilde{z},AM\tilde{z})}\right]\\ &\le \alpha\tilde{d}(AM\tilde{z},\tilde{z})+\beta\tilde{d}(AM\tilde{z},\tilde{z})+\gamma\tilde{d}(AM\tilde{z},\tilde{z})\\ &\le(\alpha+\beta+\gamma)\tilde{d}(AM\tilde{z},\tilde{z}),\end{aligned}$$

that is $0\le(\alpha+\beta+\gamma)\tilde{d}(AM\tilde{z},\tilde{z})$. Then $|\tilde{d}(AM\tilde{z},\tilde{z})|\ge 0$, hence $AM\tilde{z}=\tilde{z}$.

Since $S(\tilde{X})\subset BD(\tilde{X})$, there exist a point $\tilde{w}\in\tilde{X}$ such that $S\tilde{z}=BD\tilde{w}$. Suppose that $BD\tilde{w}\ne T\tilde{w}$. Now to prove $BD\tilde{w}=T\tilde{w}$ and given that $S\tilde{z}=\tilde{z}=BD\tilde{w}$. Putting $\tilde{X}=\tilde{z}$ and $\tilde{y}=\tilde{w}$, obtain

$$\begin{aligned}\tilde{d}(S\tilde{z},T\tilde{w}) \le \alpha &\left[\frac{\tilde{d}(AM\tilde{z},S\tilde{z})+\tilde{d}(BD\tilde{z},T\tilde{w}).\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(S\tilde{z},T\tilde{w})}\right]\\ &+\beta\max\left\{\frac{\tilde{d}(AM\tilde{z},BD\tilde{w})}{1+\tilde{d}(AM\tilde{z},BD\tilde{w})},\frac{\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(AM\tilde{z},S\tilde{z})},\frac{\tilde{d}(BD\tilde{w},S\tilde{w})}{1+\tilde{d}(BD\tilde{w},S\tilde{w})}\right\}\\ &+\gamma\left[\frac{\tilde{d}(BD\tilde{w},T\tilde{w})+\tilde{d}(T\tilde{w},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{w})}{1+\tilde{d}(BD\tilde{w},T\tilde{w})+\tilde{d}(T\tilde{w},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{w})}\right]\\ &+\eta\left[\frac{\tilde{d}(T\tilde{w},BD\tilde{w}).\tilde{d}(S\tilde{z},AM\tilde{z})}{1+\tilde{d}(T\tilde{w},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{w})+\tilde{d}(BD\tilde{w},AM\tilde{z})}\right]\\ &+\xi\left[\frac{\tilde{d}(S\tilde{z},BD\tilde{w})+\tilde{d}(BD\tilde{w},AM\tilde{z})}{1+\tilde{d}(S\tilde{z},BD\tilde{w}).\tilde{d}(T\tilde{w},BD\tilde{w}).\tilde{d}(S\tilde{z},AM\tilde{z})}\right],\end{aligned}$$

$$\begin{aligned}\tilde{d}(\tilde{z},T\tilde{w}) \le \alpha &\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},T\tilde{w}).\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},T\tilde{w})}\right]+\beta\max\left\{\frac{\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})},\frac{\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})},\frac{\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})}\right\}\\ &+\gamma\left[\frac{\tilde{d}(\tilde{z},T\tilde{w})+\tilde{d}(T\tilde{w},\tilde{z})+\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},T\tilde{w})+\tilde{d}(T\tilde{w},\tilde{z})+\tilde{d}(\tilde{z},\tilde{z})}\right]+\eta\left[\frac{\tilde{d}(T\tilde{w},\tilde{z}).\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(T\tilde{w},\tilde{z})+\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},\tilde{z})}\right]\end{aligned}$$

$$+\xi\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(T\tilde{w},\tilde{z}).\tilde{d}(\tilde{z},\tilde{z})}\right]$$
$$\le 2\gamma\tilde{d}(\tilde{z},T\tilde{w}),$$

that is $|\tilde{d}(\tilde{z},T\tilde{w})| \le 2\gamma|\tilde{d}(\tilde{z},T\tilde{w})|$, which is contradiction to $2\gamma < 1$. Therefore $T\tilde{w} = \tilde{z}$. Hence $BD\tilde{w} = \tilde{z} = T\tilde{w}$. Thus $BD\tilde{w} = T\tilde{w}$. Since BD and T are weakly compatible then $BD\tilde{z} = BD(T\tilde{w}) = T(BD\tilde{w}) = T\tilde{w}$. Thus $\tilde{z}$ is a coincidence point of BD and T. Now to prove $T\tilde{z} = \tilde{z}$, putting $\tilde{X} = \tilde{z}$ and $\tilde{y} = \tilde{z}$.

$$\tilde{d}(S\tilde{z},T\tilde{z}) \le \alpha\left[\frac{\tilde{d}(AM\tilde{z},S\tilde{z})+\tilde{d}(BD\tilde{z},T\tilde{z}).\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(S\tilde{z},T\tilde{z})}\right]$$
$$+\beta\max\left\{\frac{\tilde{d}(AM\tilde{z},BD\tilde{z})}{1+\tilde{d}(AM\tilde{z},BD\tilde{z})},\frac{\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(AM\tilde{z},S\tilde{z})},\frac{\tilde{d}(BD\tilde{z},S\tilde{z})}{1+\tilde{d}(BD\tilde{z},S\tilde{z})}\right\}$$
$$+\gamma\left[\frac{\tilde{d}(BD\tilde{z},T\tilde{z})+\tilde{d}(T\tilde{z},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{z})}{1+\tilde{d}(BD\tilde{z},T\tilde{z})+\tilde{d}(T\tilde{z},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{z})}\right]$$
$$+\eta\left[\frac{\tilde{d}(T\tilde{z},BD\tilde{z}).\tilde{d}(S\tilde{z},AM\tilde{z})}{1+\tilde{d}(T\tilde{z},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{z})+\tilde{d}(BD\tilde{z},AM\tilde{z})}\right]$$
$$+\xi\left[\frac{\tilde{d}(S\tilde{z},BD\tilde{z})+\tilde{d}(BD\tilde{z},AM\tilde{z})}{1+\tilde{d}(S\tilde{z},BD\tilde{z}).\tilde{d}(T\tilde{z},BD\tilde{z}).\tilde{d}(S\tilde{z},AM\tilde{z})}\right],$$

$$\tilde{d}(\tilde{z},T\tilde{z}) \le \alpha\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(T\tilde{z},T\tilde{z}).\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},T\tilde{z})}\right]+\beta\max\left\{\frac{\tilde{d}(\tilde{z},T\tilde{z})}{1+\tilde{d}(\tilde{z},T\tilde{z})},\frac{\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})},\frac{\tilde{d}(T\tilde{z},\tilde{z})}{1+\tilde{d}(T\tilde{z},\tilde{z})}\right\}$$
$$+\gamma\left[\frac{\tilde{d}(T\tilde{z},T\tilde{z})+\tilde{d}(T\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},T\tilde{z})}{1+\tilde{d}(T\tilde{z},T\tilde{z})+\tilde{d}(T\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},T\tilde{z})}\right]+\eta\left[\frac{\tilde{d}(T\tilde{z},T\tilde{z}).\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(T\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},T\tilde{z})+\tilde{d}(T\tilde{z},\tilde{z})}\right]$$
$$+\xi\left[\frac{\tilde{d}(\tilde{z},T\tilde{z})+\tilde{d}(T\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},T\tilde{z}).\tilde{d}(T\tilde{z},T\tilde{z}).\tilde{d}(\tilde{z},\tilde{z})}\right]$$
$$\le \beta\tilde{d}(\tilde{z},T\tilde{z})+2\gamma\tilde{d}(\tilde{z},T\tilde{z})+2\xi\tilde{d}(\tilde{z},T\tilde{z}).$$

Then $\tilde{d}(\tilde{z},T\tilde{z}) \le (\beta+2\gamma+2\xi)\tilde{d}(T\tilde{z},\tilde{z})$. That is $|\tilde{d}(\tilde{z},T\tilde{z})| \le (\beta+2\gamma+2\xi)|\tilde{d}(T\tilde{z},\tilde{z})|$, which is contradiction $\beta+2\gamma+2\xi<1$. Therefore $T\tilde{z}=\tilde{z}$, since $BD\tilde{z}=T\tilde{z}$, which implies $BD\tilde{z}=\tilde{z}$. Now we prove that $M\tilde{z}=\tilde{z}$, putting $\tilde{X}=M\tilde{z}$ and $\tilde{y}=\tilde{z}$, we get

$$\tilde{d}(S(M\tilde{z}),T\tilde{z}) \le \alpha\left[\frac{\tilde{d}(AM(M\tilde{z}),S(M\tilde{z}))+\tilde{d}(BD\tilde{z},T\tilde{z}).\tilde{d}(AM(M\tilde{z}),S(M\tilde{z}))}{1+\tilde{d}(S(M\tilde{z}),T\tilde{z})}\right]$$
$$+\beta\max\left\{\frac{\tilde{d}(AM(M\tilde{z}),BD\tilde{z})}{1+\tilde{d}(AM(M\tilde{z}),BD\tilde{z})},\frac{\tilde{d}(AM(M\tilde{z}),S(M\tilde{z}))}{1+\tilde{d}(AM(M\tilde{z}),S(M\tilde{z}))},\frac{\tilde{d}(BD\tilde{z},S(M\tilde{z}))}{1+\tilde{d}(BD\tilde{z},S(M\tilde{z}))}\right\}$$
$$+\gamma\left[\frac{\tilde{d}(BD\tilde{z},T\tilde{z})+\tilde{d}(T\tilde{z},AM(M\tilde{z}))+\tilde{d}(S(M\tilde{z}),BD\tilde{z})}{1+\tilde{d}(BD\tilde{z},T\tilde{z})+\tilde{d}(T\tilde{z},AM(M\tilde{z}))+\tilde{d}(S(M\tilde{z}),BD\tilde{z})}\right]$$
$$+\eta\left[\frac{\tilde{d}(T\tilde{z},BD\tilde{z}).\tilde{d}(S(M\tilde{z}),AM(M\tilde{z}))}{1+\tilde{d}(T\tilde{z},AM(M\tilde{z}))+\tilde{d}(S(M\tilde{z}),BD\tilde{z})+\tilde{d}(BD\tilde{z},AM(M\tilde{z}))}\right]$$
$$+\xi\left[\frac{\tilde{d}(S(M\tilde{z}),BD\tilde{z})+\tilde{d}(BD\tilde{z},AM(M\tilde{z}))}{1+\tilde{d}(S(M\tilde{z}),BD\tilde{z}).\tilde{d}(T\tilde{z},BD\tilde{z}).\tilde{d}(S(M\tilde{z}),AM(M\tilde{z}))}\right],$$

$$\tilde{d}(M\tilde{z},\tilde{z}) \le \alpha\left[\frac{\tilde{d}(M\tilde{z},M\tilde{z})+\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(M\tilde{z},M\tilde{z}))}{1+\tilde{d}(M\tilde{z},\tilde{z})}\right]$$

$$+\beta\max\left\{\frac{\tilde{d}(M\tilde{z},\tilde{z})}{1+\tilde{d}(M\tilde{z},\tilde{z})},\frac{\tilde{d}(M\tilde{z},M\tilde{z})}{1+\tilde{d}(M\tilde{z},M\tilde{z})},\frac{\tilde{d}(\tilde{z},M\tilde{z})}{1+\tilde{d}(\tilde{z},M\tilde{z})}\right\}$$
$$+\gamma\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},M\tilde{z})+\tilde{d}(M\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},M\tilde{z})+\tilde{d}(M\tilde{z},\tilde{z})}\right]+\eta\left[\frac{\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(M\tilde{z},M\tilde{z})}{1+\tilde{d}(\tilde{z},M\tilde{z})+\tilde{d}(M\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},M\tilde{z})}\right]$$
$$+\xi\left[\frac{\tilde{d}(M\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},M\tilde{z})}{1+\tilde{d}(M\tilde{z},\tilde{z}).\tilde{d}(\tilde{z},\tilde{z}).\tilde{d}(M\tilde{z},M\tilde{z})}\right]$$
$$\leq\beta\tilde{d}(M\tilde{z},\tilde{z})+2\gamma\tilde{d}(M\tilde{z},\tilde{z})+2\xi\tilde{d}(M\tilde{z},\tilde{z}).$$

Then $\tilde{d}(\tilde{z},T\tilde{z})\leq(\beta+2\gamma+2\xi)\tilde{d}(M\tilde{z},\tilde{z})$. That is $|\tilde{d}(M\tilde{z},\tilde{z})|\leq(\beta+2\gamma+2\xi)|\tilde{d}(M\tilde{z},\tilde{z})|$, which is contradiction $\beta+2\gamma+2\xi<1$. Therefore $M\tilde{z}=\tilde{z}$, since $AM\tilde{z}=\tilde{z}$, which implies $A\tilde{z}=\tilde{z}$. Now we prove that $D\tilde{z}=\tilde{z}$, putting $\tilde{X}=\tilde{z}$ and $\tilde{y}=D\tilde{z}$, we get

$$\tilde{d}(S\tilde{z},T(D\tilde{z}))\leq\alpha\left[\frac{\tilde{d}(AM\tilde{z},S\tilde{z})+\tilde{d}(BD(D\tilde{z}),T(D\tilde{z})).\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(S\tilde{z},T(D\tilde{z}))}\right]$$
$$+\beta\max\left\{\frac{\tilde{d}(AM\tilde{z},BD(D\tilde{z}))}{1+\tilde{d}(AM\tilde{z},BD(D\tilde{z}))},\frac{\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(AM\tilde{z},S\tilde{z})},\frac{\tilde{d}(BD(D\tilde{z}),S\tilde{z})}{1+\tilde{d}(BD(D\tilde{z}),S\tilde{z})}\right\}$$
$$+\gamma\left[\frac{\tilde{d}(BD(d\tilde{z}),T(d\tilde{z}))+\tilde{d}(T(d\tilde{z}),AM\tilde{z})+\tilde{d}(S\tilde{z},BD(D\tilde{z}))}{1+\tilde{d}(BD(d\tilde{z}),T(d\tilde{z}))+\tilde{d}(T(d\tilde{z}),AM\tilde{z})+\tilde{d}(S\tilde{z},BD(D\tilde{z}))}\right]$$
$$+\eta\left[\frac{\tilde{d}(T(D\tilde{z}),BD(D\tilde{z})).\tilde{d}(S\tilde{z},AM(D\tilde{z}))}{1+\tilde{d}(T(d\tilde{z}),AM\tilde{z})+\tilde{d}(S\tilde{z},BD(d\tilde{z}))+\tilde{d}(BD(D\tilde{z}),AM\tilde{z})}\right]$$
$$+\xi\left[\frac{\tilde{d}(S\tilde{z},BD(d\tilde{z}))+\tilde{d}(BD(D\tilde{z}),AM\tilde{z})}{1+\tilde{d}(S\tilde{z},BD(D\tilde{z})).\tilde{d}(T(D\tilde{z}),BD(D\tilde{z})).\tilde{d}(S\tilde{z},AM\tilde{z})}\right],$$
$$\leq\alpha\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(D\tilde{z},D\tilde{z}).\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},D\tilde{z})}\right]+\beta\max\left\{\frac{\tilde{d}(\tilde{z},D\tilde{z})}{1+\tilde{d}(\tilde{z},D\tilde{z})},\frac{\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})},\frac{\tilde{d}(D\tilde{z},\tilde{z})}{1+\tilde{d}(D\tilde{z},\tilde{z})}\right\}$$
$$+\gamma\left[\frac{\tilde{d}(d\tilde{z},d\tilde{z})+\tilde{d}(d\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},D\tilde{z})}{1+\tilde{d}(d\tilde{z},d\tilde{z})+\tilde{d}(d\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},D\tilde{z})}\right]+\eta\left[\frac{\tilde{d}(D\tilde{z},D\tilde{z}).\tilde{d}(\tilde{z},D\tilde{z})}{1+\tilde{d}(d\tilde{z},\tilde{z})+\tilde{d}(\tilde{z},d\tilde{z})+\tilde{d}(D\tilde{z},\tilde{z})}\right]$$
$$+\xi\left[\frac{\tilde{d}(\tilde{z},d\tilde{z})+\tilde{d}(D\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},D\tilde{z}).\tilde{d}(D\tilde{z},D\tilde{z}).\tilde{d}(\tilde{z},\tilde{z})}\right]$$
$$\leq(\beta+2\gamma+2\xi)\tilde{d}(\tilde{z},D\tilde{z}).$$

that is $|\tilde{d}(\tilde{z},D\tilde{z})|\leq(\beta+2\gamma+2\xi)|\tilde{d}(\tilde{z},D\,\tilde{z})|$, which is contradiction $\beta+2\gamma+2\xi<1$. Therefore $D\tilde{z}=\tilde{z}$, since $BD\tilde{z}=\tilde{z}$, which implies $B\tilde{z}=\tilde{z}$. Therefore by combining all the above result, we conclude that $\tilde{z}$ is a unique common fixed point of A, B, D, M, S and T.

Uniqueness: Let $\tilde{u}$ be an another common fixed point of A, B, D, M, S and T. Then, we have

$$\tilde{d}(S\tilde{z},T\tilde{u})\leq\alpha\left[\frac{\tilde{d}(AM\tilde{z},S\tilde{z})+\tilde{d}(BD\tilde{u},T\tilde{u}).\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(S\tilde{z},T\tilde{u})}\right]$$
$$+\beta\max\left\{\frac{\tilde{d}(AM\tilde{z},BD\tilde{u})}{1+\tilde{d}(AM\tilde{z},BD\tilde{u})},\frac{\tilde{d}(AM\tilde{z},S\tilde{z})}{1+\tilde{d}(AM\tilde{z},S\tilde{z})},\frac{\tilde{d}(BD\tilde{u},S\tilde{z})}{1+\tilde{d}(BD\tilde{u},S\tilde{z})}\right\}$$
$$+\gamma\left[\frac{\tilde{d}\tilde{u}+\tilde{d}(T\tilde{u},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{u})}{1+\tilde{d}(BD\tilde{u},T\tilde{u})+\tilde{d}(T\tilde{u},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{u})}\right]$$

$$+\eta\left[\frac{\tilde{d}(T\tilde{u},BD\tilde{u}).\tilde{d}(S\tilde{z},AM\tilde{z})}{1+\tilde{d}(T\tilde{u},AM\tilde{z})+\tilde{d}(S\tilde{z},BD\tilde{u})+\tilde{d}(BD\tilde{u},AM\tilde{z})}\right]$$
$$+\xi\left[\frac{\tilde{d}(S\tilde{z},BD\tilde{u})+\tilde{d}(BD\tilde{u},AM\tilde{z})}{1+\tilde{d}(S\tilde{z},BD\tilde{u}).\tilde{d}(T\tilde{u},BD\tilde{u}).\tilde{d}(S\tilde{z},AM\tilde{z})}\right],$$
$$\leq\alpha\left[\frac{\tilde{d}(\tilde{z},\tilde{z})+\tilde{d}(\tilde{u},\tilde{u}).\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{u})}\right]+\beta\max\left\{\frac{\tilde{d}(\tilde{z},\tilde{u})}{1+\tilde{d}(\tilde{z},\tilde{u})},\frac{\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{z})},\frac{\tilde{d}(\tilde{u},\tilde{z})}{1+\tilde{d}(\tilde{u},\tilde{z})}\right\}$$
$$+\gamma\left[\frac{\tilde{d}(\tilde{u},\tilde{u})+\tilde{d}(\tilde{u},\tilde{z})+\tilde{d}(\tilde{z},\tilde{u})}{1+\tilde{d}(\tilde{u},\tilde{u})+\tilde{d}(\tilde{u},\tilde{z})+\tilde{d}(\tilde{z},\tilde{u})}\right]+\eta\left[\frac{\tilde{d}(\tilde{u},\tilde{u}).\tilde{d}(\tilde{z},\tilde{z})}{1+\tilde{d}(\tilde{u},\tilde{z})+\tilde{d}(\tilde{z},\tilde{u})+\tilde{d}(\tilde{u},\tilde{z})}\right]$$
$$+\xi\left[\frac{\tilde{d}(\tilde{z},\tilde{u})+\tilde{d}(\tilde{u},\tilde{z})}{1+\tilde{d}(\tilde{z},\tilde{u}).\tilde{d}(\tilde{u},\tilde{u}).\tilde{d}(\tilde{z},\tilde{z})}\right]$$
$$\leq(\beta+2\gamma+2\xi)\tilde{d}(\tilde{z},\tilde{u}),$$

that is $|\tilde{d}(\tilde{z},\tilde{u})|\leq(\beta+2\gamma+2\xi)|\tilde{d}(\tilde{z},\tilde{u})|$, which is contradiction $\beta+2\gamma+2\xi<1$.
Therefore $\tilde{z}=\tilde{u}$, therefore $\tilde{z}$ is a unique common fixed point of A, B, D, M, S and T. □

Corollary 2.5. *Let $(\tilde{X},\tilde{d})$ be a soft metric space and D, M, S and T be four self mappings in $\tilde{X}$ satisfying the conditions:*

(i) *$S(\tilde{X})\subset D(\tilde{X})$ and $T(\tilde{X})\subset M(\tilde{X})$,*

(ii) *for each $\tilde{x},\tilde{y}\in\tilde{X}$, such that $\tilde{x}\neq\tilde{y},\tilde{d}(T\tilde{y},M\tilde{x})+\tilde{d}(S\tilde{x},D\tilde{y})+\tilde{d}(D\tilde{y},M\tilde{x})\neq 0$, where α,β,γ,η and ξ are non-negative real number with $\alpha+\beta+2\gamma+\eta+\xi<1$, or $\tilde{d}(S\tilde{x},T\tilde{y})=0$ if $\tilde{d}(T\tilde{y},M\tilde{x})+\tilde{d}(S\tilde{x},D\tilde{y})+\tilde{d}(D\tilde{y},M\tilde{x})=0$, such that*
$$\tilde{d}(S\tilde{x},T\tilde{y})\leq\alpha\left[\frac{\tilde{d}(M\tilde{x},S\tilde{x})+\tilde{d}(D\tilde{y},T\tilde{y}).\tilde{d}(M\tilde{x},S\tilde{x})}{1+\tilde{d}(S\tilde{x},T\tilde{y})}\right]$$
$$+\beta\max\left\{\frac{\tilde{d}(M\tilde{x},D\tilde{y})}{1+\tilde{d}(M\tilde{x},D\tilde{y})},\frac{\tilde{d}(M\tilde{x},S\tilde{x})}{1+\tilde{d}(M\tilde{x},S\tilde{x})},\frac{\tilde{d}(D\tilde{y},S\tilde{x})}{1+\tilde{d}(D\tilde{y},S\tilde{x})}\right\}$$
$$+\gamma\left[\frac{\tilde{d}(D\tilde{y},T\tilde{y})+\tilde{d}(T\tilde{y},M\tilde{x})+\tilde{d}(S\tilde{x},D\tilde{y})}{1+\tilde{d}(D\tilde{y},T\tilde{y})+\tilde{d}(T\tilde{y},M\tilde{x})+\tilde{d}(S\tilde{x},D\tilde{y})}\right]$$
$$+\eta\left[\frac{\tilde{d}(T\tilde{y},D\tilde{y}).\tilde{d}(S\tilde{x},M\tilde{x})}{\tilde{d}(T,M\tilde{x})+\tilde{d}(S\tilde{x},D\tilde{y})+\tilde{d}(D\tilde{y},M\tilde{x})}\right]$$
$$+\xi\left[\frac{\tilde{d}(S\tilde{x},D\tilde{y})+\tilde{d}(D\tilde{y},M\tilde{x})}{1+\tilde{d}(S\tilde{x},D\tilde{y}).\tilde{d}(T\tilde{y},D\tilde{y}).\tilde{d}(S\tilde{x},M\tilde{x})}\right],$$

(iii) *the pair (M,S) and (D,T) are weakly compatible.*

Then D, M, S and T have a unique common fixed point.

3. Conclusion

For the imperfect knowledge or for uncertainty the theory of soft sets can be used. Obtained results of the fixed-point theorems related to soft sets can be used in decision making problems. All the results can be used in image processing also. Soft metric extensions of several important fixed point theorems for metric spaces can be directly deduce from comparable existing results.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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