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Research Article

On $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed Sets

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Abstract. We introduce a new class of sets namely $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed sets, $(1,2)^{\star}$-$\Lambda_{\mathcal{G}}$-set, $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-set and $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Locally closed sets are study in bitopological spaces. We prove that this classes lies between $(1,2)^{\star}$-α-closed sets and $(1,2)^{\star}$-αg-closed sets. Furthermore, we discuss some essential properties of $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed sets in present of this paper.

Keywords. $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed sets, $(1,2)^{\star}$-$\mathcal{G}$-$ker(A)$, $(1,2)^{\star}$-$\Lambda_{\mathcal{G}}$-set, $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-set and $(1,2)^{\star}$-$\breve{g}_{\alpha}$-LC

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1. Introduction

The perceptions of bitopological spaces is to introduced and studied by J.C. Kelly [4]. Recently, more generalizations of closed sets and it is properties were introduced and investigated by various researchers for some example ([7]) and so on. We introduce and study a new classes of sets namely $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed sets, $(1,2)^{\star}$-$\Lambda_{\mathcal{G}}$-set, $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-set and $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Locally closed sets in bitopological spaces. We prove that this classes lies between $(1,2)^{\star}$-α-closed sets and $(1,2)^{\star}$-αg-closed sets. Also, we discuss some essential properties of $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed sets in present of this paper.

2. Preliminaries

Throughout this paper, (X,τ_1,τ_2) and (Y,σ_1,σ_2) (or simply X and Y) represents the non-empty bitopological spaces on which no separation axiom are assumed, unless otherwise mentioned.

For a subset A of X, $\tau_{1,2}$-$cl(A)$ and $\tau_{1,2}$-$int(A)$ represents the closure of A and interior of A, respectively.

Definition 2.1. A subset A of a bitopological space (X,τ_1,τ_2) or X is called

(i) a $(1,2)^\star$-semi open set if $A \subseteq \tau_{1,2}\text{-}cl(\tau_{1,2}\text{-}int(A))$.

(ii) a $(1,2)^\star$-pre open set if $A \subseteq \tau_{1,2}\text{-}int(\tau_{1,2}\text{-}cl(A))$.

(iii) a $(1,2)^\star$-α-open set [3] if $A \subseteq \tau_{1,2}\text{-}int(\tau_{1,2}\text{-}cl(\tau_{1,2}\text{-}int(A)))$.

(iv) a $(1,2)^\star$-β-open (or) a $(1,2)^\star$-semi-pre open set if $A \subseteq \tau_{1,2}\text{-}cl(\tau_{1,2}\text{-}int(\tau_{1,2}\text{-}cl(A)))$.

The complements of the above mentioned sets are called their respective closed sets.

Definition 2.2. A subset A of a bitopological space (X,τ_1,τ_2) or X is said to be

(i) a $(1,2)^\star$-generalized closed set (briefly, $(1,2)^\star$-g-closed) [5] if $\tau_{1,2}$-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open.

(ii) a $(1,2)^\star$-semi generalized closed set (briefly, $(1,2)^\star$-sg-closed) [2] if $(1,2)^\star$-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^\star$-semi-open.

(iii) a $(1,2)^\star$-generalized semi-closed (briefly, $(1,2)^\star$-gs-closed) set [2] if $(1,2)^\star$-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open.

(iv) an $(1,2)^\star$-α-generalized closed (briefly, $(1,2)^\star$-αg-closed) set [2] if $(1,2)^\star$-$\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open.

(v) a $(1,2)^\star$-generalized semi-preclosed (briefly, $(1,2)^\star$-gsp-closed) set [2] if $(1,2)^\star$-$\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open.

(vi) a $(1,2)^\star$-$\hat{\hat{g}}$-closed set [2] if $\tau_{1,2}$-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^\star$-sg-open.

(vii) a $(1,2)^\star$-$\hat{\hat{g}}_1$-closed set [7] if $\tau_{1,2}$-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^\star$-$\hat{\hat{g}}$-open.

(viii) a $(1,2)^\star$-$\mathcal{G}$-closed set [7] if $(1,2)^\star$-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^\star$-$\hat{\hat{g}}_1$-open.

(ix) a $(1,2)^\star$-$\breve{g}$-closed set [7] if $\tau_{1,2}$-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^\star$-$\mathcal{G}$-open.

The complements of the above mentioned closed sets are called their respective open sets.

3. On $(1,2)^\star$-$\breve{g}_\alpha$-closed Sets

Definition 3.1. A subset A of a space (X,τ_1,τ_2) is said to be an $(1,2)^\star$-$\breve{g}_\alpha$-closed set if $\tau_{1,2}$-$\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^\star$-$\mathcal{G}$-open.

The complement of $(1,2)^\star$-$\breve{g}_\alpha$-closed set is called $(1,2)^\star$-$\breve{g}_\alpha$-open set.

The collection of all $(1,2)^\star$-$\breve{g}_\alpha$-closed (resp. $(1,2)^\star$-$\breve{g}_\alpha$-open) sets in (X,τ_1,τ_2) is denoted by $(1,2)^\star$-$\breve{g}_\alpha C(X)$ (resp. $(1,2)^\star$-$\breve{g}_\alpha O(X)$).

Proposition 3.2. *In a space* (X,τ_1,τ_2)*, every* $(1,2)^\star$*-*α*-closed set is* $(1,2)^\star$*-*$\breve{g}_\alpha$*-closed.*

Proof. Let A be an $(1,2)^\star$-α-closed set and U be any $(1,2)^\star$-$\mathcal{G}$-open set containing A. Since A is $(1,2)^\star$-α-closed, we have $\tau_{1,2}$-$\alpha cl(A) = A \subseteq U$. Thus A is $(1,2)^\star$-$\breve{g}_\alpha$-closed. □

Remark 3.3. The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.4. Let $X = \{a,b,c\}$ with $\tau_1 = \{\phi,\{a,b\},X\}$ and $\tau_2 = \{\phi,X\}$ then $\tau_{1,2} = \{\phi,\{a,b\},X\}$. In the space X, then $(1,2)^{\star}$-$\breve{g}_{\alpha}C(X) = \{\phi,\{c\},\{a,c\},\{b,c\},X\}$ and $(1,2)^{\star}$-$\alpha C(X) = \{\phi,\{c\},X\}$. We have the subset $\{a,c\}$ is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed set but not $(1,2)^{\star}$-α-closed.

Proposition 3.5. *In a space* (X,τ_1,τ_2)*, every* $(1,2)^{\star}$*-*$\breve{g}$*-closed set is* $(1,2)^{\star}$*-*$\breve{g}_{\alpha}$*-closed.*

Proof. Let A be a $(1,2)^{\star}$-$\breve{g}$-closed set and U be any $(1,2)^{\star}$-$\mathcal{G}$-open set containing A. Since A is $(1,2)^{\star}$-$\breve{g}$-closed, we have $U \supseteq cl(A) \supseteq \tau_{1,2}$-$\alpha cl(A)$. Hence A is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed. □

Remark 3.6. The converse of Proposition 3.5 need not be true as seen from the following Example.

Example 3.7. Let $X = \{a,b,c\}$ with $\tau_1 = \{\phi,X\}$ and $\tau_2 = \{\phi,\{b\},X\}$ then $\tau_{1,2} = \{\phi,\{b\},X\}$. In a space X, then $(1,2)^{\star}$-$\breve{g}C(X) = \{\phi,\{a,c\},X\}$ and $(1,2)^{\star}$-$\breve{g}_{\alpha}C(X) = \{\phi,\{a\},\{c\},\{a,c\},X\}$. We have the subset $\{a\}$ is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed set but not $(1,2)^{\star}$-$\breve{g}$-closed.

Proposition 3.8. *In a space* (X,τ_1,τ_2)*, every* $(1,2)^{\star}$*-*$\breve{g}_{\alpha}$*-closed set is* $(1,2)^{\star}$*-*αg*-closed.*

Proof. Let A be an $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed set and U be any $\tau_{1,2}$-open set containing A. Since any $\tau_{1,2}$-open set is $(1,2)^{\star}$-$\mathcal{G}$-open, then $\tau_{1,2}$-$\alpha cl(A) \subseteq U$. Thus A is $(1,2)^{\star}$-αg-closed. □

Remark 3.9. The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.10. Let $X = \{a,b,c\}$ with $\tau_1 = \{\phi,\{c\},X\}$ and $\tau_2 = \{\phi,X\}$ then $\tau_{1,2} = \{\phi,\{c\},X\}$. Then $(1,2)^{\star}$-$\breve{g}_{\alpha}C(X) = \{\phi,\{a\},\{b\},\{a,b\},X\}$ and $(1,2)^{\star}$-$\alpha gC(X) = \{\phi,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\},X\}$. In a space X, we have the subset $\{a,c\}$ is $(1,2)^{\star}$-αg set but not $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed.

Proposition 3.11. *In a space* (X,τ_1,τ_2)*, every* $(1,2)^{\star}$*-*$\breve{g}_{\alpha}$*-closed set is* $(1,2)^{\star}$*-*gs*-closed ((*$(1,2)^{\star}$*-*sg*-closed).*

Proof. Let A be an $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed set and U be any $\tau_{1,2}$-open set ($(1,2)^{\star}$-semi-open set) containing A. Since any $\tau_{1,2}$-open ($(1,2)^{\star}$-semi-open) set is $(1,2)^{\star}$-$\mathcal{G}$-open, then $\tau_{1,2}$-$scl(A) \subseteq \tau_{1,2}$-$\alpha cl(A) \subseteq U$. Thus A is $(1,2)^{\star}$-gs-closed ($(1,2)^{\star}$-sg-closed). □

Remark 3.12. The converse of Proposition 3.11 need not be true as seen from the following example.

Example 3.13. Let $X = \{a,b,c\}$ with $\tau_1 = \{\phi,\{a\},\{a,b\},X\}$ and $\tau_1 = \{\phi,\{b\},\{a,b\},X\}$ then $\tau_1 = \{\phi,\{a\},\{b\},\{a,b\},X\}$. We have $(1,2)^{\star}$-$\breve{g}_{\alpha}C(X) = \{\phi,\{c\},\{a,c\},\{b,c\},X\}$ and $(1,2)^{\star}$-$sgC(X) = (1,2)^{\star}$-$gsC(X) = \{\phi,\{a\},\{b\},\{c\},\{a,c\},\{b,c\},X\}$. In the space X, then the subset $\{a\}$ is both $(1,2)^{\star}$-sg-closed set and $(1,2)^{\star}$-gs-closed set but not $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed.

Proposition 3.14. *In a space* (X,τ_1,τ_2)*, every* $(1,2)^{\star}$*-*$\breve{g}_{\alpha}$*-closed set is* $(1,2)^{\star}$*-*gsp*-closed.*

Proof. Let A be an $(1,2)^\star$-$\breve{g}_\alpha$-closed set and U be any $\tau_{1,2}$-open set containing A. Since any $\tau_{1,2}$-open set is $(1,2)^\star$-$\mathcal{G}$-open, then $\tau_{1,2}$-$spcl(A) \subseteq \tau_{1,2}$-$cl(A) \subseteq U$. Hence A is $(1,2)^\star$-gsp-closed. □

Remark 3.15. The converse of Proposition 3.14 need not be true as seen from the following example.

Example 3.16. In Example 3.7, we have $(1,2)^\star$-$gspC(X) = \{\phi,\{a\},\{c\},\{a,b\},\{a,c\},\{b,c\},X\}$. In the space X, then the subset $\{a,b\}$ is $(1,2)^\star$-gsp-closed set but not $(1,2)^\star$-$\breve{g}_\alpha$-closed.

Remark 3.17. The following Examples show that $(1,2)^\star$-$\breve{g}_\alpha$-closedness is independent of $(1,2)^\star$-semi-closedness and $(1,2)^\star$-g-closedness.

Example 3.18. In Example 3.13, we have $(1,2)^\star$-$sC(X) = \{\phi,\{a\},\{b\},\{c\},\{a,c\},\{b,c\},X\}$. In the space, then the subset $\{b\}$ is $(1,2)^\star$-semi-closed set but not $(1,2)^\star$-$\breve{g}_\alpha$-closed.

Example 3.19. In Example 3.4, we have $(1,2)^\star$-$sC(X) = \{\phi,\{c\},X\}$. In the space, then the subset $\{b,c\}$ is $(1,2)^\star$-$\breve{g}_\alpha$-closed set but not $(1,2)^\star$-semi-closed.

Example 3.20. Let $X = \{a,b,c\}$ with $\tau_1 = \{\phi,\{a\},X\}$ and $\tau_2 = \{\phi,\{a,b\},X\}$ then $\tau_{1,2} = \{\phi,\{a\},\{a,b\},X\}$. We have $(1,2)^\star$-$\breve{g}_\alpha C(X) = \{\phi,\{b\},\{c\},\{b,c\},X\}$ and $(1,2)^\star$-$gC(X) = \{\phi,\{c\},\{a,c\},\{b,c\},X\}$. In the space, then

(i) the subset $\{b\}$ is $(1,2)^\star$-$\breve{g}_\alpha$-closed set but not $(1,2)^\star$-g-closed.

(ii) the subset $\{a,c\}$ is $(1,2)^\star$-g-closed set but not $(1,2)^\star$-$\breve{g}_\alpha$-closed.

Remark 3.21. From the above discussions are obtain in the following diagram.

$$\begin{array}{ccccc}
(1,2)^\star\text{-}g\text{-closed} & \Longleftarrow & (1,2)^\star\text{-}\breve{g}\text{-closed} & \Longleftarrow & \tau_{1,2}\text{-closed} \\
\Downarrow & & \Downarrow & & \Downarrow \\
(1,2)^\star\text{-}\alpha g\text{-closed} & \Longleftarrow & (1,2)^\star\text{-}\breve{g}_\alpha\text{-closed} & \Longleftarrow & (1,2)^\star\text{-}\alpha\text{-closed} \\
\Downarrow & & \Downarrow & & \Downarrow \\
(1,2)^\star\text{-}gsp\text{-closed} & \Longleftarrow & (1,2)^\star\text{-}sg\text{-closed} & \Longleftarrow & (1,2)^\star\text{-semi-closed}
\end{array}$$

4. Properties of $(1,2)^\star$-$\breve{g}_\alpha$-closed Sets

Definition 4.1. The intersection of all $(1,2)^\star$-$\mathcal{G}$-open subsets in (X,τ_1,τ_2) containing A is said to be a $(1,2)^\star$-$\mathcal{G}$-kernel of A and denoted by $(1,2)^\star$-$\mathcal{G}$-$ker(A)$.

Lemma 4.2. *A subset A of (X,τ_1,τ_2) is $(1,2)^\star$-$\breve{g}_\alpha$-closed* $\Longleftrightarrow$ $\tau_{1,2}$-$\alpha cl(A) \subseteq (1,2)^\star$-$\mathcal{G}$-$ker(A)$.

Proof. Suppose that A is $(1,2)^\star$-$\breve{g}_\alpha$-closed. Then $(1,2)^\star$-$\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^\star$-$\mathcal{G}$-open. Let $x \notin \tau_{1,2}$-$\alpha cl(A)$. If $x \notin (1,2)^\star$-$\mathcal{G}$-$ker(A)$, then there is $(1,2)^\star$-$\mathcal{G}$-open set U containing A such that $x \notin U$. Since U is $(1,2)^\star$-$\mathcal{G}$-open set containing A, we have $x \notin \tau_{1,2}$-$\alpha cl(A)$ and this is a contradiction.

Conversely, let $\tau_{1,2}$-$\alpha cl(A) \subseteq (1,2)^\star$-$\mathcal{G}$-$ker(A)$. If U is any $(1,2)^\star$-$\mathcal{G}$-open set containing A, then $\tau_{1,2}$-$\alpha cl(A) \subseteq (1,2)^\star$-$\mathcal{G}$-$ker(A) \subseteq U$. Therefore, A is $(1,2)^\star$-$\breve{g}_\alpha$-closed. □

Proposition 4.3. *If A and B are $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed sets in (X,τ_1,τ_2), then $A\cup B$ is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed in (X,τ_1,τ_2).*

Proof. If $A\cup B\subseteq U$ and U is $(1,2)^{\star}$-$\mathcal{G}$-open, then $A\subseteq U$ and $B\subseteq U$. Since A and B are $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed, $U\supseteq \tau_{1,2}$-$\alpha cl(A)$ and $U\supseteq \tau_{1,2}$-$\alpha cl(B)$ and hence $U\supseteq \tau_{1,2}$-$\alpha cl(A)\cup\tau_{1,2}$-$\alpha cl(B)=\tau_{1,2}$-$\alpha cl(A\cup B)$. Thus $A\cup B$ is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed in (X,τ_1,τ_2). □

Proposition 4.4. *If a set A is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed in (X,τ_1,τ_2) and $A\subseteq B\subseteq\tau_{1,2}$-$\alpha cl(A)$, then B is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed in (X,τ_1,τ_2).*

Proof. Let U be $(1,2)^{\star}$-$\mathcal{G}$-open set in (X,τ_1,τ_2) such that $B\subseteq U$. Then $A\subseteq U$. Since A is an $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed set, $\tau_{1,2}$-$\alpha cl(A)\subseteq U$. Also $\tau_{1,2}$-$\alpha cl(B)=\tau_{1,2}$-$\alpha cl(A)\subseteq U$. Hence B is also an $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed in (X,τ_1,τ_2). □

Proposition 4.5. *If A is $(1,2)^{\star}$-$\mathcal{G}$-open and $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed in (X,τ_1,τ_2), then A is $(1,2)^{\star}$-α-closed in (X,τ_1,τ_2).*

Proof. Since A is $(1,2)^{\star}$-$\mathcal{G}$-open and $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed, $\tau_{1,2}$-$\alpha cl(A)\subseteq A$ and hence A is $(1,2)^{\star}$-α-closed in (X,τ_1,τ_2). □

Proposition 4.6. *For each $x\in X$, either $\{x\}$ is $(1,2)^{\star}$-$\mathcal{G}$-closed or $\{x\}^c$ is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed in (X,τ_1,τ_2).*

Proof. Suppose that $\{x\}$ is not $(1,2)^{\star}$-$\mathcal{G}$-closed in (X,τ_1,τ_2). Then $\{x\}^c$ is not $(1,2)^{\star}$-$\mathcal{G}$-open and the only $(1,2)^{\star}$-$\mathcal{G}$-open set containing $\{x\}^c$ is the space X itself. Therefore $\tau_{1,2}$-$\alpha cl(\{x\}^c)\subseteq X$ and so $\{x\}^c$ is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed in (X,τ_1,τ_2). □

Definition 4.7. A subset A of a space (X,τ_1,τ_2) is said to be $(1,2)^{\star}$-$\Lambda_{\mathcal{G}}$-set if $A=(1,2)^{\star}$-$\mathcal{G}$-$ker(A)$.

Definition 4.8. A subset A of a space (X,τ_1,τ_2) is called $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-closed if $A=S\cap T$ where S is a $(1,2)^{\star}$-$\Lambda_{\mathcal{G}}$-set and T is $(1,2)^{\star}$-α-closed.

The complement of $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-closed set is called $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-open set.

The collection of all $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-closed (resp. $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-open) sets in (X,τ_1,τ_2) is denoted by $(1,2)^{\star}$-$\lambda_{\mathcal{G}}C(X)$ (resp. $(1,2)^{\star}$-$\lambda_{\mathcal{G}}O(X)$).

Lemma 4.9. *For a subset A of a topological space (X,τ_1,τ_2), the following conditions are equivalent.*

(i) *A is $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-closed.*

(ii) *$A=S\cap\tau_{1,2}$-$\alpha cl(A)$ where S is a $(1,2)^{\star}$-$\Lambda_{\mathcal{G}}$-set.*

(iii) *$A=(1,2)^{\star}$-$\mathcal{G}$-$ker(A)\cap\tau_{1,2}$-$\alpha cl(A)$.*

Lemma 4.10. *In a space* (X,τ_1,τ_2),

(i) *every* $(1,2)^{\star}$-α-*closed set is* $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-*closed.*

(ii) *every* $(1,2)^{\star}$-$\Lambda_{\mathcal{G}}$-*set is* $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-*closed.*

Remark 4.11. The converses of Lemma 4.10 need not be true as seen from the following examples.

Example 4.12. Let $X = \{a,b,c,d,e\}$ with $\tau_1 = \{\phi,\{a\},X\}$ and $\tau_2 = \{\phi,X\}$ then $\tau_{1,2} = \{\phi,\{a\},X\}$, we have

(i) $(1,2)^{\star}$-$\alpha C(X) = \{\phi,\{b\},\{c\},\{b,c\},X\}$ and $(1,2)^{\star}$-$\lambda_{\mathcal{G}}C(X) = \wp(X)$. In the space X, then the subset $\{a\}$ is $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-closed set but not $(1,2)^{\star}$-α-closed.

(ii) $(1,2)^{\star}$-$\Lambda_{\mathcal{G}}$-sets are $\{\phi,\{a\},\{a,b\},\{a,c\},X\}$ and $(1,2)^{\star}$-$\lambda_{\mathcal{G}}C(X) = \wp(X)$. In the space X, then the subset $\{b\}$ is $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-closed set but not $(1,2)^{\star}$-$\Lambda_{\mathcal{G}}$-set.

Theorem 4.13. *For a subset A of a topological space (X,τ_1,τ_2), the following conditions are equivalent.*

(i) *A is $(1,2)^{\star}$-α-closed.*

(ii) *A is $(1,2)^{\star}$-$\breve{g}_{\alpha}$ and $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$.*

Proof. (i)⇒(ii). Obvious.

(ii)⇒(i). Since A is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed, so by Lemma 4.2, $\tau_{1,2}$-$\alpha cl(A) \subseteq (1,2)^{\star}$-$\mathcal{G}$-$ker(A)$. Since A is $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-closed, so by Lemma 4.9, $A = (1,2)^{\star}$-$\mathcal{G}$-$ker(A) \cap \tau_{1,2}$-$cl(A) = \tau_{1,2}$-$cl(A)$. Hence A is $(1,2)^{\star}$-α-closed. □

Remark 4.14. The following examples show that concepts of $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed sets and $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-closed sets are independent of each other.

Example 4.15. In Example 4.12, we have $(1,2)^{\star}$-$\breve{g}_{\alpha}C(X) = \{\phi,\{b\},\{c\},\{b,c\},X\}$ and $(1,2)^{\star}$-$\lambda_{\mathcal{G}}C(X) = \wp(X)$. In the space X, then the subset $\{a\}$ is $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-closed set but not $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed.

Example 4.16. In Example 3.4, we have $(1,2)^{\star}$-$\breve{g}_{\alpha}C(X) = \{\phi,\{c\},\{a,c\},\{b,c\},X\}$ and $(1,2)^{\star}$-$\lambda_{\mathcal{G}}C(X) = \{\phi,\{a\},\{b\},\{c\},\{a,b\},X\}$. In the space X, then the subset $\{a\}$ is $(1,2)^{\star}$-$\lambda_{\mathcal{G}}$-closed set but not $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed.

5. $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Locally Closed Sets and It's Property

Definition 5.1. Let (X,τ_1,τ_2) be a bitopological space. A subset A of X is called $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Locally closed sets (briefly $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Lc) if $A = S \cap T$ where S is $(1,2)^{\star}$-$\mathcal{G}$-open and T is $(1,2)^{\star}$-α-closed in (X,τ_1,τ_2).

Example 5.2. In Example 3.4, we have the subset $\{a\}$ is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Lc-set in X.

Proposition 5.3. *In a space (X,τ_1,τ_2), every $(1,2)^{\star}$-$\mathcal{G}$-open set is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Lc-set.*

Remark 5.4. The converse of Proposition 5.3 need not be true seen from the following Example.

Example 5.5. In Example 3.4, we have $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Lc-sets are $\{\phi,\{a\},\{b\},\{c\},\{a,b\},X\}$ and $(1,2)^{\star}$-$\mathcal{G}$-open sets are $\{\phi,\{a\},\{b\},\{a,b\},X\}$. In the space X, then the subset $\{c\}$ is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Lc-set but not $(1,2)^{\star}$-$\mathcal{G}$-open.

Proposition 5.6. *In a space (X,τ_1,τ_2), every $(1,2)^{\star}$-α-closed set is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Lc-set.*

Remark 5.7. The converse of Proposition 5.6 need not be true seen from the following Example.

Example 5.8. In Example 3.4, we have $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Lc-sets are $\{\phi,\{a\},\{b\},\{c\},\{a,b\},X\}$ and $(1,2)^{\star}$-$\alpha C(X)=\{\phi,\{c\},X\}$. In the space X, then the subset $\{a\}$ is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Lc-set but not $(1,2)^{\star}$-α-closed.

Theorem 5.9. *Let (X,τ_1,τ_2) be a bitopological space and A a subset of X. Then, A is $(1,2)^{\star}$-α-closed $\Longleftrightarrow$ $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed and $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Lc-set.*

Proof. Let A be an $(1,2)^{\star}$-α-closed. By Propositions 3.2 and 5.6, A is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed and $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Lc-set.

Conversely, let $A=S\cap T$. Then S is $(1,2)^{\star}$-$\mathcal{G}$-open and T is $(1,2)^{\star}$-α-closed. Since A is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed, $\tau_{1,2}$-$cl(A)\subseteq S$. Also $\tau_{1,2}$-$cl(A)\subseteq\tau_{1,2}$-$cl(T)=T$. We have $\tau_{1,2}$-$cl(A)\subseteq S\cap T=A$. Hence A is $(1,2)$-α-closed. □

Remark 5.10. The following Example shows that the concepts of $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed sets and $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Lc-sets are independent of each other.

Example 5.11. In Example 3.4, we have $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Lc-sets are $\{\phi,\{a\},\{b\},\{c\},\{a,b\},X\}$ and $(1,2)^{\star}$-$\breve{g}_{\alpha}C(X)=\{\phi,\{c\},\{a,c\},\{b,c\},X\}$. In the space X, then

(i) the subset $\{a\}$ is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Lc-set but not $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed.

(ii) the subset $\{a,c\}$ is $(1,2)^{\star}$-$\breve{g}_{\alpha}$-closed set but not $(1,2)^{\star}$-$\breve{g}_{\alpha}$-Lc-set.

6. Conclusion

This paper extend an temptation to the budding mathematicians to make use of these above concept in several area for better understanding and can be applied in other fields of science and technology which always craves for new applications to solve troubles that baffle experts.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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