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Research Article

Fuzzy Model on Online Treatment Analysis for COVID-19

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Abstract. The purpose of this paper is to introduce a fuzzy model for patients suffering from COVID-19. We have defined primary, normal and abnormal infections based on the symptoms of the patients. Fuzzy numbers and Reverse order fuzzy numbers have been used to identify the stages of each patient. These models may help the doctors for online treatment of patients.

Keywords. Fuzzy number, Reverse order fuzzy number, COVID-19, Infections

Mathematics Subject Classification (2020). 03E72; 93C42; 94D05

1. Introduction

COVID-19 is a severe infectious virus that is not only dangerous for human beings but also animals. Medical experts say that people cannot escape from the infection of the virus. As per the data given by the World Health Organization [10], 85% of the people are having primary infections from the virus whereas 10% are having normal infections from the virus. It is evident that only 5% of the people are attacked severely by COVID-19. Such infections are abnormal infections.

The symptoms for primary infections include *Fever*, *Dry dough*, and *Tiredness*, and are categorized as the *Most Common Symptoms* (*MCS*). People having primary infections will be cured by taking health care at the primary level. The *Less Common Symptoms* (*LCS*) for normal infections include Aches and Pains, Sore Throat, Diarrhea, Conjunctivitis, Headache, Loss of

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Taste or Smell, Rash on the skin, or discoloration of fingers or toes. The normally infected people will be cured under medical treatment and by improving their immunity power. The *Serious Symptoms* (SS) for abnormal infections include difficulty in breathing or shortness of breath, chest pain or pressure, and loss of speech or movement. The abnormal infected people may be facing critical conditions that may lead to loss of life.

Fuzzy health concepts were deliberated and we have introduced the restricted fuzzy number for the time interval based on the duration for the COVID-19 patients. The purpose of this paper is to introduce fuzzy models for the primarily infected, normally infected and abnormally infected people before and after the recovery from COVID-19.

We have divided this paper into three sections. The first section illustrated primarily infected people and their online medical treatment. The second section exemplified a normal infection disease people and their online medical treatment and the final section elucidated as abnormal infected people and their online medical treatment.

1.1 Fuzzy Health Concepts

Most of the medical concepts are fuzzy. Since the symptoms of COVID-19 infections are vague, the symptoms of *MCS*, *LCS*, and *SS* can be considered as fuzzy concepts or fuzzy variables. The next table gives details of the above fuzzy concepts.

S. No.	Fuzzy Health Concept	Fuzzy variable	Range
1	*Fever*	F	Normal to high
2	*Dry cough*	DC	Low to high
3	*Tiredness*	T	Lazy to dull
4	*Aches and pains*	AP	Light pain to heavy pain
5	*Sore throat*	ST	Light to heavy
6	*Diarrhea*	D	Thin solid to dilute
7	*Conjunctivitis*	C	Light pain to heavy pain
8	*Headache*	H	Light pain to heavy pain
9	*Loss of taste*	LT	Bad to good
10	*Loss of smell*	LS	Bad to good
11	*Discoloration of fingers*	DF	Bright to fade
12	*Discoloration of toes*	DT	Bright to fade
13	*Difficult breathing*	DB	Normal to abnormal
14	*Shortness of breath*	SB	Low frequency to high frequency
15	*Chest pain*	CP	Light pain to heavy pain
16	*Pressure*	P	Low to high
17	*Loss of speech*	LS	Normal to abnormal
18	*Loss of movement*	LM	Normal to abnormal

Depending upon the medical history of the patients, specific real numbers may be given to the fuzzy health concepts. For example $(Fever, Dry\ cough, Tiredness) = (a, b, c)$. Since *Fever*, *Dry cough*, and *Tiredness* have different ranges, we assume that $a < b < c$. This motivates us to create a fuzzy number model for a Primary infectious disease before treatment and another one

for the same after treatment. It is interesting to see that the former is a fuzzy number and the latter is the corresponding reverse order fuzzy number. The health conditions of the patient can be computed by giving the appropriate 't' value. As people are affected by the virus's medical glitches during some interval of time, we need the notions of fuzzy numbers and the reverse order fuzzy numbers that are restricted to the time interval. Thus the notions of restricted fuzzy numbers and the restricted reverse order fuzzy numbers have been introduced in this paper.

2. Preliminaries

Definition 2.1 ([11])**.** A fuzzy set A of X is defined as $A = \{x, \mu_A(x)/x \in X\}$, where x is an element in the universe of discourse X and $\mu_A : X \to [0,1]$ is a mapping called the degree of membership function and $\mu_A(x)$ is the membership value of $x \in X$.

Definition 2.2 ([7])**.** A fuzzy real number is a fuzzy set A on the real axis, satisfying the following conditions:

(i) A must be normal.

(ii) $A^{\geq\alpha}$ is a closed interval for all $\alpha \in (0,1]$.

(iii) The support A is bounded.

Definition 2.3 ([5])**.** The fuzzy set $A^{\leq\alpha} = \{x : A(x) \leq \alpha\}$ is called the dual α-cut of A, $A^{<\alpha} = \{x : A(x) < \alpha\}$ is known as the dual strong α-cut of $A_\sim$ and $A_\sim = \alpha = \{x : A(x) = \alpha\}$ is the α-level cut of A.

Definition 2.4 ([5])**.** If A is a reverse order fuzzy number if the following conditions hold:

(i) A is down-normal.

(ii) $A^{>\alpha\alpha}$ is a union of two disjoint unbounded open intervals for each $\alpha \in [0,1)$.

(iii) The level set $A^{=1}$ is unbounded and $A^{<1}$ is bounded.

3. Restricted Fuzzy Numbers

Let A denote the fuzzy number and J be the time interval that may be $[a,b]$, (a,b), $[a,b)$, $(a,b]$. Is a function from the set of real numbers to $[0,1]$. If $\tilde{B} = \tilde{A} \mid J$, the restriction of A to J then B is known as J-fuzzy number. That is, the membership grade of $\tilde{B}$ at x is $(A)(x)$ for each $x \in \tilde{J}$. In the same way, J-reverse order fuzzy number can be defined.

4. Fuzzy Models for Primary Infections

In this section, primary infections for the patients were identified, and online treatment has been given based on their medical history. Depending upon the order in which the patient is affected by Fever, Dry Cough, Tiredness, the parameters and membership grades can be chosen for the fuzzy concepts. Let (Fever, Dry Cough, Tiredness) $= (a,b,c)$ where $0 < a < b < c < \infty$. Let α, β, γ be the membership grades for the fuzzy variables F, DC, and T with $0 \leq \alpha < \beta < \gamma \leq 1$. Let A denote the fuzzy number with membership grades α, β, γ respectively at a, b, c. Clearly, $(A)'$ is the corresponding reverse order fuzzy number. If J is an interval containing the parameters

a, b, c then the J-fuzzy number $\underset{\sim}{B} = \underset{\sim}{A} \mid r$ represents the premedical history of the patient, and its fuzzy complement $(\underset{\sim}{B})'$ represents the post medical history of the patient. The algebraic models for Primary infections are given below.

$$(\underset{\sim}{B})(x) = \begin{cases} \alpha + \frac{\beta-\alpha}{b-a}(x-a), & a \le x \le b \\ \beta + \frac{\gamma-\beta}{c-b}(x-b), & b \le x \le c \end{cases}$$

$$(\underset{\sim}{B})'(x) = \begin{cases} 1-\alpha - \frac{\beta-\alpha}{b-a}(x-a), & a \le x \le b \\ 1-\beta - \frac{\gamma-\beta}{c-b}(x-b), & b \le x \le c \end{cases}$$

This model is illustrated below.

Illustration 4.1. Normally COVID virus is identified after 4 days since its infection and it affects health for up to 14 days. So we take $a = 4$, $b = 10$, and $c = 14$ depending upon the health conditions of the patient. The medical expert can give the membership grades $\alpha = 0.2$, $\beta = 0.6$, $\gamma = 0.8$ on the fuzzy health concepts F, D, C and T. The fuzzy models for this type of patient are given below:

$$(\underset{\sim}{B})(x) = \begin{cases} \frac{0.4}{6}(x-1), & 4 \le x \le 10 \\ \frac{0.2}{4}(x+2), & 10 \le x \le 14 \end{cases}$$

Figure 1 describes the health conditions of a patient suffering from COVID-19 at the primary level.

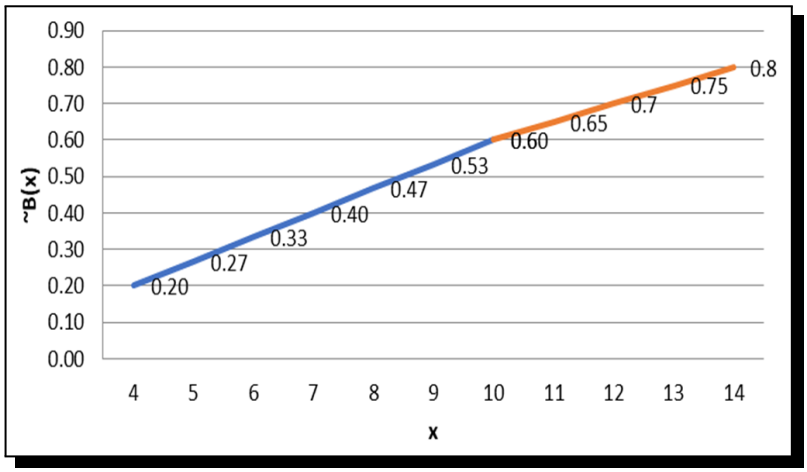

Figure 1

The next model describes the patient after treatment.

$$(\underset{\sim}{B})'(x) = \begin{cases} \frac{6.4-0.4x}{6}, & 4 \le x \le 10 \\ \frac{3.6-0.2x}{4}, & 10 \le x \le 14 \end{cases}$$

After the treatment, the critical trend level of the patient slowly decreases from high to normal as seen in Figure 2.

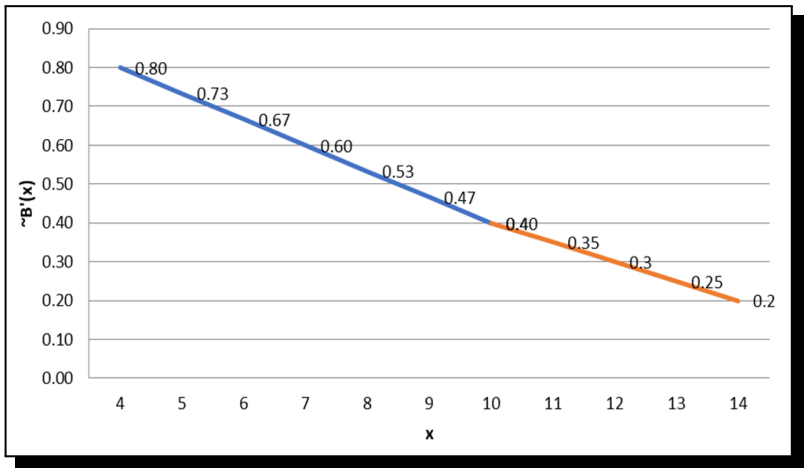

Figure 2

Remark. The fuzzy models depend upon the order of the fuzzy concepts F, DC and T. Therefore, we can have six different models for the virus infection. The expert has to choose the right model for the investigations.

5. Fuzzy Models for Normal Infections

In this section, normal infections for the patients were exemplified, and online treatment has been given based on their medical history. The Symptoms of Aches and Pains, Sore Throat, Diarrhea for normal infections have different ranges whereas Conjunctivitis & Headache, and Loss of Taste & Smell have the same ranges. Therefore, the symptoms having the same range may be combined for constructing the fuzzy model. Thus the parameters and membership grades can be taken as

$$(AP, ST, D, C\&H, LT\&LS) = (a, b, c, d, e),$$

where $a < b < c < d < e$ and their membership grades $= (\alpha, \beta, \gamma, \delta, \varepsilon)$ where $0 \le \alpha < \beta < \gamma < \delta < \varepsilon \le 1$. Let J be an interval containing the parameters a, b, c, d, e. The right membership grades can be chosen by the doctors who involved in the treatment of the patient.

$$(C)(x) = \begin{cases} \alpha + \frac{\beta-\alpha}{b-a}(x-a), & a \le x \le b \\ \beta + \frac{\gamma-\beta}{c-b}(x-b), & b \le x \le c \\ \gamma + \frac{\delta-\gamma}{d-c}(x-c), & c \le x \le d \\ \delta + \frac{\delta-\delta}{\theta-d}(x-d), & d \le x \le e \end{cases}$$

$$\underset{\sim}{(C)}'(x) = \begin{cases} 1-\alpha - \frac{\beta-\alpha}{b-a}(x-a), & a \le x \le b \\ 1-\beta - \frac{\gamma-\beta}{c-b}(x-b), & b \le x \le c \\ 1-\gamma - \frac{\delta-\gamma}{d-c}(x-c), & c \le x \le d \\ 1-\delta - \frac{\delta-\delta}{\theta-d}(x-d), & d \le x \le e \end{cases}$$

This model has been demonstrated in the next illustration.

Illustration 5.1. Suppose a person has been affected by COVID-19 at normal stage and is under gone a medical treatment. For this model we take $a = 4$, $b = 6$, $c = 8$, $d = 10$, $e = 14$ and $\alpha = 0.2$, $\beta = 0.4$, $\gamma = 0.6$, $\delta = 0.8$, $\varepsilon = 1$. The next model and the subsequent Figure 3, demonstrate the health conditions of the patient during the treatment.

$$(C)(x) = \begin{cases} 0.1x - 0.2, & 4 \le x \le 6 \\ 0.1x - 0.2, & 6 \le x \le 8 \\ 0.1x - 0.2, & 8 \le x \le 10 \\ \frac{1.2+0.2x}{4}, & 10 \le x \le 14 \end{cases}$$

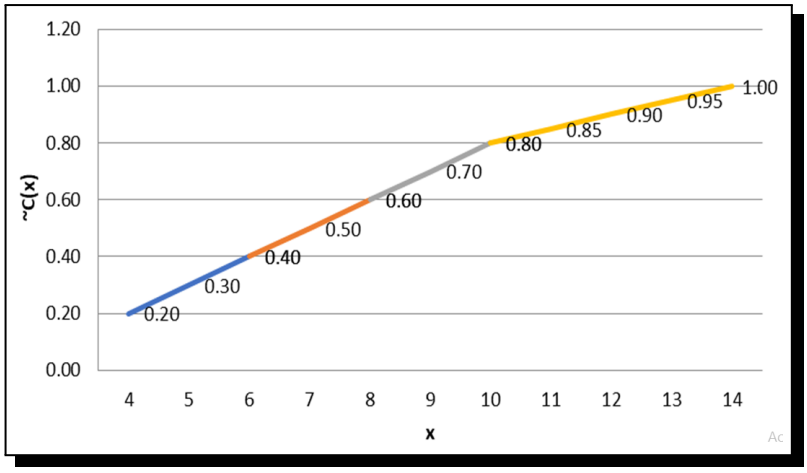

Figure 3

The next model and its graph, Figure 4 explain the decreasing trend of the risk factors of the patient from high pitch to zero pitch.

$$(C')(x) = \begin{cases} 1.2 - 0.1x, & 4 \le x \le 6 \\ 1.2 - 0.1x, & 6 \le x \le 8 \\ 1.2 - 0.1x, & 8 \le x \le 10 \\ \frac{2.8-0.2x}{4}, & 10 \le x \le 14 \end{cases}$$

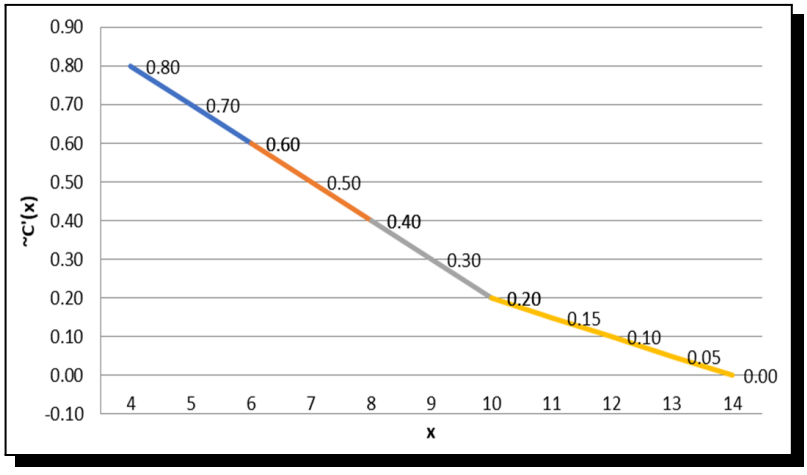

Figure 4

Remark. The fuzzy models depend upon the order of the fuzzy concepts AP, ST, D, $C\&H$, $LT\&LS$. Therefore, we can have 125 different models for the virus infection. The expert has to choose the right model for the investigations.

6. Fuzzy Models for Abnormal Infections

In this final section, abnormal infections for the patients were elucidated as abnormally infected people, and online treatment has been given based on their medical history. The Symptoms of Difficult breathing, Shortness of breath, Chest pain, Pressure for abnormal infections have distinct ranges whereas the symptoms of Loss of speech, Loss of movement have the same range. Therefore the parameters and membership grade for the models can be chosen as given below:

$$(DB, SB, CP, P, LS\&LM) = (a, b, c, d, e),$$

where $a < b < c < d < e$ and their membership grades $= (\alpha, \beta, \gamma, \delta, \varepsilon)$ where, $0 \le \alpha < \beta < \gamma < \delta < \varepsilon \le 1$. The right membership grades can be chosen by the doctors who involved in the treatment of the patient.

$$(D)(x) = \begin{cases} \alpha + \frac{\beta-\alpha}{b-a}(x-a), & a \le x \le b \\ \beta + \frac{\gamma-\beta}{c-b}(x-b), & b \le x \le c \\ \gamma + \frac{\delta-\gamma}{d-c}(x-c), & c \le x \le d \\ \delta + \frac{z-\delta}{e-d}(x-d), & d \le x \le e, \end{cases} \qquad (D)'(x) = \begin{cases} 1-\alpha - \frac{\beta-\alpha}{b-a}(x-a), & a \le x \le b \\ 1-\beta - \frac{\gamma-\beta}{c-b}(x-b), & b \le x \le c \\ 1-\gamma - \frac{\delta-\gamma}{d-c}(x-c), & c \le x \le d \\ 1-\delta - \frac{\sigma-\delta}{e-d}(x-d), & d \le x \le e \end{cases}$$

The above model is illustrated below.

Illustration 6.1. Suppose a person has been affected by COVID-19 at abnormal stage and is under gone a medical treatment. For this model, we take $a = 4$, $b = 6$, $c = 8$, $d = 10$, $e = 14$ and $\alpha = 0.1$, $\beta = 0.3$, $\gamma = 0.5$, $\delta = 0.7$, $\varepsilon = 1$. The next model and the subsequent Figure 5 demonstrate the health conditions of the patient during the treatment.

$$(D)(x) = \begin{cases} 0.1x - 0.3, & 4 \le x \le 6 \\ 0.1x - 0.3, & 6 \le x \le 8 \\ 0.1x - 0.3, & 8 \le x \le 10 \\ \frac{0.3x-0.2}{4}, & 10 \le x \le 14 \end{cases}$$

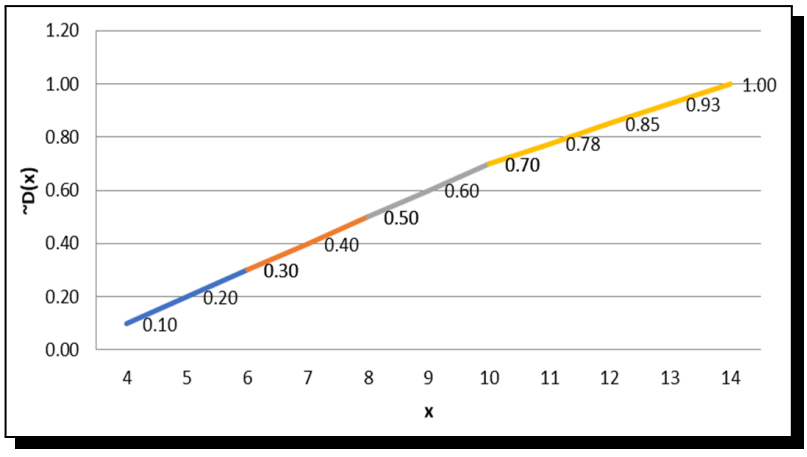

Figure 5

The next model and its graph, Figure 6 explain the decreasing trend of the risk factors of the patient from high pitch to zero pitch.

$$(D)'(x) = \begin{cases} 1.3 - 0.1x, & 4 \le x \le 6 \\ 1.3 - 0.1x, & 6 \le x \le 8 \\ 1.3 - 0.1x, & 8 \le x \le 10 \\ \frac{4.2 - 0.3x}{4}, & 10 \le x \le 14 \end{cases}$$

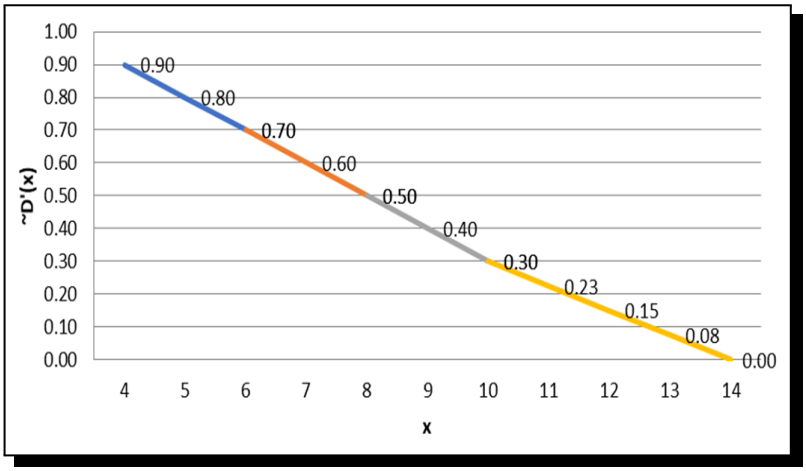

Figure 6

Remark. The fuzzy models depend upon the order of the fuzzy concepts DB, SB, CP, P, $LS\&LM$. Therefore, we can have 125 different models for the virus infection. The expert has to choose the right model for the investigations.

7. Conclusion

Three types of fuzzy models have been given for three levels of COVID-19. The medical Experts can discuss with doctors and nurses by using the models in the absence of patients. This model is useful for online treatment and online medical checkups. Once the model for a patient is constructed, the doctors need not go to the patient's room for further course of action and he/she can decide the treatment using video conferencing. Recently different types of COVID-19 namely α, β, γ, ϵ, δ and λ viruses have been found throughout the world. Researchers in the applications of fuzzy numbers are encouraged to identify the symptoms of these viruses and create fuzzy models for online treatment.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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