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Research Article

Bianchi Type I Cosmological Model with Viscous Fluid in the Framework of VSL Theory

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Abstract. The Bianchi type I cosmological model in the framework of *Variable Speed of Light* (VSL) theory is investigated, by taking into account the effect due to viscosity, considering $\overline{p} = p - 3\zeta H$ where $\zeta = \zeta_0 \rho H^{-1}$. The Einstein field equations are solved for variable G, c and Λ in which G, c, Λ and shear parameter σ^2, all are coupled. It is shown that the viscosity term exhibits the influence on the form of solutions, in the framework of varying speed of light theory.

Keywords. VSL theory, Viscous fluid, Dark energy

Mathematics Subject Classification (2020). 83A05; 83C05; 83C15; 83C56

1. Introduction

The *Varying Speed of Light* (VSL) cosmology has been received considerable attention as alternatives to cosmological inflation to provide different basics for resolving the problems of the standard models. Albrecht and Magueijo [1] have investigated possible consequences of a time variation in the velocity of light in vacuum. In particular, it offers new ways of solving the problems of the standard big bang cosmology, distinct from their resolutions in context of

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the inflationary paradigm (Guth [6]) or the pre big bang scenario of low energy string theory (Veneziano [10]). However, a new approach, which has been widely investigated (Kalligas *et al*. [7]), is appealing. In the relativistic cosmological models, generally the energy momentum tensor of matter generated by perfect fluid is considered. For approaching to more realistic model, the consideration of effect of viscosity mechanism is important. In the present paper, we have extended our previous work, Khadekar and Ghogre [8], by introducing a viscosity term as $\bar{p} = p - 3\xi H$.

2. Model and Field Equations

We consider the Bianchi type I space-time with metric

$$ds^2 = -c^2(t)dt^2 + X^2(t)dx^2 + Y^2(t)dy^2 + Z^2(t)dz^2 \tag{2.1}$$

in the comoving co-ordinates $u^i = \delta^i_0$. An average expansion scale factor is defined by $R(t) = (XYZ)^{\frac{1}{3}}$ and the Hubble parameter is $H = \frac{\dot{R}}{R}$. Here G, c and Λ are considered as functions of cosmic time t. We use the field equations in the form

$$R_{ij} - \frac{1}{2}Rg_{ij} = \frac{8\pi G(t)}{c^4(t)}T_{ij} + \Lambda(t)g_{ij}, \tag{2.2}$$

where R_{ij} is the Ricci tensor and R is Ricci scalar. For the physical interpretation of the model, the energy momentum tensor is defined as $T_{ij} = (\rho + \bar{p})u_i u_j + \bar{p}g_{ij}$, where ρ is the energy density, p is the pressure of the cosmic fluid and $\bar{p} = p - 3\xi H$. Here ξ is a viscosity term. We consider viscosity coefficient (Sheykhi and Setare [9]) as $\xi = \xi_0 \rho H^{-1}$, where ξ_0 is constant. As per the procedure given by Belinchón [3], for the metric (2.1), the Einstein field equations can be written as:

$$\frac{\dot{X}\dot{Y}}{XY} + \frac{\dot{Y}\dot{Z}}{YZ} + \frac{\dot{Z}\dot{X}}{ZX} = \frac{8\pi G\rho}{c^2} + \Lambda c^2, \tag{2.3}$$

$$\frac{\ddot{X}}{X} + \frac{\ddot{Y}}{Y} + \frac{\dot{X}\dot{Y}}{XY} - \frac{\dot{c}}{c}\left(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y}\right) = \frac{-8\pi G\bar{p}}{c^2} + \Lambda c^2, \tag{2.4}$$

$$\frac{\ddot{Y}}{Y} + \frac{\ddot{Z}}{Z} + \frac{\dot{Y}\dot{Z}}{YZ} - \frac{\dot{c}}{c}\left(\frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right) = \frac{-8\pi G\bar{p}}{c^2} + \Lambda c^2, \tag{2.5}$$

$$\frac{\ddot{Z}}{Z} + \frac{\ddot{X}}{X} + \frac{\dot{Z}\dot{X}}{ZX} - \frac{\dot{c}}{c}\left(\frac{\dot{Z}}{Z} + \frac{\dot{X}}{X}\right) = \frac{-8\pi G\bar{p}}{c^2} + \Lambda c^2. \tag{2.6}$$

The shear tensor σ_{ij} is defined as

$$\sigma_{ij} = \frac{1}{2}\left(u_{i;j} + u_{j;i} + q_i u_j + q_j u_i\right) - \frac{1}{3}\vartheta h_{ij}, \tag{2.7}$$

where $\vartheta = u^k{}_{;k}$, $h_{ij} = g_{ij} + u_i u_j$ and $q_i = u_{i;j}u^j$.
The magnitude $\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij}$ is obtained as

$$\sigma^2 = \frac{1}{6c^2}\left[\left(\frac{\dot{X}}{X} - \frac{\dot{Y}}{Y}\right)^2 + \left(\frac{\dot{Y}}{Y} - \frac{\dot{Z}}{Z}\right)^2 + \left(\frac{\dot{Z}}{Z} - \frac{\dot{X}}{X}\right)^2\right]. \tag{2.8}$$

Using equations (2.4)-(2.6) we get,

$$\left(\frac{\dot{Z}}{Z}-\frac{\dot{X}}{X}\right)^2+\left(\frac{\dot{X}}{X}-\frac{\dot{Y}}{Y}\right)^2+\left(\frac{\dot{Y}}{Y}-\frac{\dot{Z}}{Z}\right)^2=\frac{3\alpha_1^2c^2}{(XYZ)^2}, \tag{2.9}$$

where α_1 is constant. By using eq. (2.8) and (2.9), we get $\sigma^2=\frac{\alpha_1^2}{2(XYZ)^2}$. Hence $\sigma^2 \propto R^{-6}$ or $\sigma=\alpha_2R^{-3}$, where α_2 is proportionality constant. Thus, we get

$$\frac{\dot{\sigma}}{\sigma}=-\left(\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}\right)=-3H. \tag{2.10}$$

Squaring, we get

$$H^2=\frac{1}{9}\left[\frac{\dot{X}^2}{X}+\frac{\dot{Y}^2}{Y}+\frac{\dot{Z}^2}{Z}+\frac{2\dot{X}\dot{Y}}{XY}+\frac{2\dot{Y}\dot{Z}}{YZ}+\frac{2\dot{Z}\dot{X}}{ZX}\right]. \tag{2.11}$$

From eqs. (2.3), (2.8) and (2.11), we can write the analogue of Friedman equation as

$$3H^2=c^2(\sigma^2+\Lambda)+\frac{8\pi G\rho}{c^2}. \tag{2.12}$$

Differentiating eq. (2.3) and using eq. (2.4)-(2.6) in it, we get the conservation equations as:

$$\frac{8\pi G}{c^2}\left[\dot{\rho}+3(p+(1-3\xi_0)\rho)H\right]=\frac{-8\pi\rho}{c^2}\left[\dot{G}-\frac{4\dot{c}}{c}G\right]-c^2\dot{\Lambda}. \tag{2.13}$$

We assume that the conservation of energy momentum tensor of matter holds. i.e. $T^j_{i;j}=0$, which implies

$$\dot{\rho}+3(p+(1-3\xi_0)\rho)H=0. \tag{2.14}$$

Hence we get the relation between G and Λ as

$$\frac{8\pi\rho}{c^2}\left[\dot{G}-4G\frac{\dot{c}}{c}\right]+c^2\dot{\Lambda}=0. \tag{2.15}$$

Assuming $p=\omega\rho$, eq. (2.14) leads to the relation as $\rho=\alpha_3R^{-3(1+\omega-3\xi_0)}$, where α_3 is constant of integration. Using this value of ρ in eq. (2.15), we get

$$\dot{G}-\frac{4\dot{c}}{c}G=-\frac{c^4\dot{\Lambda}R^{3(1+\omega-3\xi_0)}}{8\pi\alpha_3}. \tag{2.16}$$

Now using H^2 from eq. (2.11) in eq. (2.14), it becomes

$$\frac{\dot{\rho}^2}{\rho^3}=3(1+\omega-3\xi_0)^2\left[\frac{8\pi G}{c^2}+\frac{c^2\sigma^2}{\rho}+\frac{c^2\Lambda}{\rho}\right]. \tag{2.17}$$

Differentiating eq. (2.17), and then putting the value of $\dot{\Lambda}$ from eq. (2.15), the value of $\dot{\sigma}$ from eq.(2.10) and $\dot{\rho}$ from eq. (2.14), in the R.H.S. of the resultant equation, we get

$$\frac{2\ddot{\rho}}{\rho}-\frac{3\dot{\rho}^2}{\rho^2}=3c^2(1+\omega-3\xi_0)^2\left[\sigma^2\left(\frac{1-\omega+3\xi_0}{1+\omega-3\xi_0}\right)-\Lambda\right]+\frac{2(1+\omega-3\xi_0)}{H}\frac{\dot{c}}{c}\left[\frac{8\pi G\rho}{c^2}+c^2(\sigma^2+\Lambda)\right]. \tag{2.18}$$

The early highly anisotropic universe is reduced to a smooth present universe during the physical processes. These physical processes are also responsible for bringing down the large value of Λ to its small present value. Hence there must be some relation between the two parameters σ^2 and Λ. Hence we choose Λ such that the R.H.S. of above equation becomes zero, i.e.,

$$3c^2(1+\omega-3\xi_0)^2\left[\sigma^2\left(\frac{1-\omega+3\xi_0}{1+\omega-3\xi_0}\right)-\Lambda\right]-\frac{2(1+\omega-3\xi_0)}{H}\frac{\dot{c}}{c}\left[\frac{8\pi G\rho}{c^2}+c^2(\sigma^2+\Lambda)\right]=0. \tag{2.19}$$

Here we consider $\frac{G}{c^2}$ as a constant α_0, (Belinchón [4]), $c = c_0 R^n$ (Barrow [2]) and value of ρ as $\rho = \alpha_3 R^{-3(1+\omega-3\xi_0)}$ in eq. (2.10), where c_0 is constant, we get

$$\alpha_4\sigma^2-\alpha_5\Lambda = AR^{-\alpha_5}, \tag{2.20}$$

where $\alpha_4 = 3-3\omega+9\xi_0-2n$, $\alpha_5 = 3+3\omega-9\xi_0+2n$, $A = \frac{16n\pi\alpha_0\alpha_3}{c_0^2}$. Using the value of R from $\sigma^2 \propto R^{-6}$ or $\sigma = \alpha_2 R^{-3}$, we solve eq. (2.20) to get the value of Λ as:

$$\Lambda = B_1\sigma^2 A_1\sigma^{\frac{-\alpha_5}{3}} \tag{2.21}$$

where $B_1 = \frac{\alpha_4}{\alpha_5}$, $A_1 = \frac{A}{\alpha_5(\alpha_1)^{\frac{\alpha_5}{3}}}$. Using this value of Λ in eq. (2.18), we get $\frac{2\ddot{\rho}}{\dot{\rho}} = \frac{3\dot{\rho}}{\rho}$, after solving which we get,

$$\rho = Dt^{-2}, \tag{2.22}$$

where $D = \frac{4}{d_2^2}$ and d_2 is constant of integration. Using ρ from $\rho = \alpha_3 R^{\frac{-3(1+\omega)}{1+3\xi_0}}$ and eq. (2.22), we get the scale factor R as

$$R = at^{\frac{2}{3(1+\omega-3\xi_0)}}, \tag{2.23}$$

where $a = \frac{\alpha_3(d_2)^2}{4}$. By using the value of R from the above in eq. (2.10), we get

$$\sigma = B_3 t^{\frac{-2}{(1+\omega-3\xi_0)}}, \tag{2.24}$$

where $B_3 = \frac{d_0}{a^3}$.
Using this value in eq. (2.21), we get the value of Λ as:

$$\Lambda = B_4 t^{\frac{-4}{(1+\omega-3\xi_0)}} - B_5 t^{\frac{-2(3+3\omega-9\xi_0+2n)}{3(1+\omega-3\xi_0)}}, \tag{2.25}$$

where $B_4 = B_1 B_3^2$ and $B_5 = A_1 B_3^{\frac{\alpha_5}{3}}$. Substituting this value of Λ in eq. (2.16), and then solving the resultant linear differential equation in G, we get

$$G = G_0 t^{\frac{-8n}{3(1+\omega-3\xi_0)}}\left[B_6 t^{\frac{-2(1-\omega+3\xi_0)+4n}{(1+\omega-3\xi_0)}} + B_7 t^{\frac{2(3+3\omega-9\xi_0)+8n}{3(1+\omega-3\xi_0)}}\right], \tag{2.26}$$

where $B_6 = \frac{B_4 C_0^4(1+\omega-3\xi_0)a^{3+3\omega-9\xi_0+4n}}{4\pi\alpha_3(1+\omega-3\xi_0)(1-\omega+3\xi_0)}$, $B_7 = \frac{B_5\alpha_5 C_0^4(1+\omega-3\xi_0)a^{3+3\omega-9\xi_0+4n}}{8\pi\alpha_3(3+3\omega-9\xi_0-\alpha_5)}$, G_0 is constant of integration. Also using $q = -\frac{R\ddot{R}}{\dot{R}^2}$ we get the deceleration parameter as

$$q = \frac{1+3\omega-9\xi_0}{2}. \tag{2.27}$$

3. Conclusion

We have extended the previous work of Vishwakarma [11] in the framework of VSL theory proposed by Albrecht and Magueijo [1], by taking into account the viscosity mechanism. The effect of variable c on the dynamics and the evolution of Bianchi type I universe is essentially determined by G, c and Λ. Observational existence of the variation of the speed of light seems to be suggested by the variation of the fine structure contact in the spectra of quasars

(Drinkwater *et al*. [5], and Webb *et al*. [12]). Here we have obtained the solutions for Bianchi type I cosmological model, for the viscous fluid by considering $\bar{p} = p - 3\xi H$, where ξ is the viscosity coefficient defined as $\xi = \xi_0 \rho H^{-1}$. We have obtained solutions of the field equations under the assumption that $\frac{G}{c^2} = \alpha_0 = constant$ and varying speed of light c is proportional to the expansion rate of the universe i.e. $c_0 R^n$, where c_0, n are constants. The resultant equations (2.23)-(2.26) clearly exhibit the essential influence of the viscous term on the values of σ, Λ and G. For $\xi_0 = 0$, the values of cosmological constants approach to the results discussed earlier in Khadekar and Ghogre [8]. From this study of the Bianchi I model, it is observed that such models are compatible with the results of recent observations.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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