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Research Article

$ng^{\#}$-Closed Sets in an Ideal Nano Topological Space

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Abstract. In this paper, we propose to introduce the new classes of $ng^{\#}$-closed sets, $n\mathfrak{I}_{g^{\#}}$-closed sets, $n\alpha g^{\#}$-closed sets and completely nano codense in ideal an nano topological space. Also, we studied the $n\mathfrak{I}_{g^{\#}}$-closed sets and establish their various characteristic properties.

Keywords. $ng^{\#}$-closed sets; $n\mathfrak{I}_{g^{\#}}$-closed sets; $n\alpha g^{\#}$-closed sets and completely nano codense

Mathematics Subject Classification (2020). 54C05; 54C08; 54C10

1. Introduction

Some new notions in the concept of ideal nano topological spaces were introduced by Parimala *et al*. [5]. Recently, More new classes of sets and it is properties were introduced and investigated by several topologist for some example ([1–3, 8, 9, 12–15]) and [16] in ideal nano topological spaces.

An ideal I [19] on a topological space (X,τ) is a non-empty collection of subsets of X which satisfies the following conditions:

(1) $A \in I$ and $B \subset A$ imply $B \in I$, and

(2) $A \in I$ and $B \in I$ imply $A \cup B \in I$.

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Given a topological space (X,τ) with an ideal I on X. If $\wp(X)$ is the family of all subsets of X, a set operator $(\cdot)^\star : \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subset X$, $A^\star(I,\tau) = \{x \in X : U \cap A \notin I$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$ [6]. The closure operator defined by $cl^\star(A) = A \cup A^\star(I,\tau)$ [20] is a Kuratowski closure operator which generates a topology $\tau^\star(I,\tau)$ called the $\star$-topology finer than τ. The topological space together with an ideal on X is called an ideal topological space or an ideal space denoted by (X,τ,I). We will simply write $A^\star$ for $A^\star(I,\tau)$ and $\tau^\star$ for $\tau^\star(I,\tau)$. In this paper, we propose to introduce the new classes of $ng^{\#}$-closed sets, $n\mathcal{I}_{g^{\#}}$-closed sets, $n\alpha g^{\#}$-closed sets and completely nano codense in ideal an nano topological space. Also, we studied the $n\mathcal{I}_{g^{\#}}$-closed sets and establish their various characteristic properties.

2. Preliminaries

Definition 2.1 ([11])**.** Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.

(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.

(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U} \{R(x) : R(x) \cap X \neq \phi\}$.

(3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not $-X$ with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 ([18])**.** Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(1) U and $\phi \in \tau_R(X)$,

(2) the union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

(3) the intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U,\tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

In the rest of the paper, we denote a nano topological space by $(U,\mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by $n\text{-}int(A)$ and $n\text{-}cl(A)$, respectively.

Definition 2.3 ([18])**.** A subset A of a space $(U,\mathcal{N})$ is called

(1) nano α-open if $A \subseteq n\text{-}int(n\text{-}cl(n\text{-}int(A)))$.

(2) nano semi-open if $A \subseteq n\text{-}cl(n\text{-}int(A))$.

(3) nano pre open set (briefly np-open set) if $A \subseteq n\text{-}int(n\text{-}cl(A))$.

The complements of the above mentioned sets are called their respective closed sets.

Definition 2.4. A subset A of a space $(U,\mathcal{N})$ is called

(1) nano g-closed [4] if $ncl(A) \subseteq G$, whenever $A \subseteq G$ and G is nano open.

(2) nano αg-closed [7] if $n\text{-}\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open.

The complements of the above used sets are called their respective open sets.

Definition 2.5. A subset A of a space $(U,\mathcal{N})$ is called

(1) nano dense (briely n-dense) [17] if $n\text{-}cl(A) = U$.

(2) nano codense (briely n-codense) [2] if $U - A$ is n-dense.

A nano topological space $(U,\mathcal{N})$ with an ideal I on U is called [5] an ideal nano topological space and is denoted by $(U,\mathcal{N},I)$. $G_n(x) = \{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$, denotes the family of nano open sets containing x.

Definition 2.6 ([5])**.** Let $(U,\mathcal{N},I)$ be a space with an ideal I on U. Let $(\cdot)^\star_n$ be a set operator from $\wp(U)$ to $\wp(U)$ ($\wp(U)$ is the set of all subsets of U). For a subset $A \subseteq U$, $A^\star_n(I,\mathcal{N}) = \{x \in U : G_n \cap A \notin I$, for every $G_n \in G_n(x)\}$ is called the nano local function (briefly, n-local function) of A with respect to I and $\mathcal{N}$. We will simply write $A^\star_n$ for $A^\star_n(I,\mathcal{N})$.

Theorem 2.7. *Let $(U,\mathcal{N},I)$ be a space and A and B be subsets of U. Then*

(1) $A \subseteq B \Rightarrow A^\star_n \subseteq B^\star_n$,

(2) $A^\star_n = n\text{-}cl(A^\star_n) \subseteq n\text{-}cl(A)$ *($A^\star_n$ is a n-closed subset of $n\text{-}cl(A)$),*

(3) $(A^\star_n)^\star_n \subseteq A^\star_n$,

(4) $(A \cup B)^\star_n = A^\star_n \cup B^\star_n$,

(5) $V \in \mathcal{N} \Rightarrow V \cap A^\star_n = V \cap (V \cap A)^\star_n \subseteq (V \cap A)^\star_n$,

(6) $J \in I \Rightarrow (A \cup J)^\star_n = A^\star_n = (A - J)^\star_n$.

Theorem 2.8. *Let $(U,\mathcal{N},I)$ be a space with an ideal I and $A \subseteq A^\star_n$, then $A^\star_n = n\text{-}cl(A^\star_n) = n\text{-}cl(A)$.*

Definition 2.9. Let $(U,\mathcal{N},I)$ be a space. The set operator $n\text{-}cl^\star$ called a nano $\star$-closure is defined by $n\text{-}cl^\star(A) = A \cup A^\star_n$ for $A \subseteq X$.

It can be easily observed that $n\text{-}cl^\star(A) \subseteq n\text{-}cl(A)$.

Theorem 2.10 ([5])**.** *In a space $(U,\mathcal{N},I)$, if A and B are subsets of U, then the following results are true for the set operator $n\text{-}cl^\star$:*

(1) $A \subseteq n\text{-}cl^\star(A)$.

(2) $n\text{-}cl^\star(\phi) = \phi$ *and* $n\text{-}cl^\star(U) = U$.

(3) *If* $A \subset B$, *then* $n\text{-}cl^\star(A) \subseteq n\text{-}cl^\star(B)$.

(4) $n\text{-}cl^\star(A) \cup n\text{-}cl^\star(B) = n\text{-}cl^\star(A \cup B)$.

(5) $n\text{-}cl^\star(n\text{-}cl^\star(A)) = n\text{-}cl^\star(A)$.

Definition 2.11 ([5])**.** A subset A of a space $(U,\mathcal{N},I)$ is said to be nano-I-open (briefly, nI-open) if $A \subseteq n\text{-}int(A^\star_n)$.

Definition 2.12. A subset A of a space $(U,\mathcal{N},I)$ is called

(1) nano $\star$-closed (briefly, $n\star$-closed) [10] if $A_n^* \subseteq A$.

(2) nano I_g-closed (briefly, nI_g-closed) [10] if $A_n^* \subseteq G$ whenever $A \subseteq G$ and G is n-open.

(3) $n\star$-dense [10] if $n\text{-}cl^\star(A) = U$.

(4) $\mathcal{N}$-codense ideal [5] if $\mathcal{N} \cap \mathcal{I} = \{\phi\}$.

3. On $n\mathcal{I}_{g^\#}$-Closed Sets

Definition 3.1. A subset A of an ideal nano topological space $(U,\mathcal{N},\mathcal{I})$ is said to be

(1) $n\mathcal{I}_{g^\#}$-closed if $A_n^\star \subseteq G$ whenever $A \subseteq G$ and G is $n\alpha g$-open.

(2) $n\mathcal{I}_{g^\#}$-open if its complement is $n\mathcal{I}_{g^\#}$-closed.

Definition 3.2. A subset A of a nano topological space $(U,\mathcal{N})$ is said to be nano $g^\#$-closed (briefly $ng^\#$-closed) if $n\text{-}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is $n\alpha g$-open.

The complement of $ng^\#$-closed is called $ng^\#$-open.

Proposition 3.3. *If $(U,\mathcal{N},\mathcal{I})$ is any ideal nano topological space, then every $n\mathcal{I}_{g^\#}$-closed set is $n\mathcal{I}_g$-closed but not conversely.*

Proof. It follows from the fact that every nano open set is $n\alpha g$-open. □

As shown in the following example:

Example 3.4. Let $U = \{a,b,c\}$ with $U/R = \{\{b\},\{a,c\}\}$ and $X = \{b,c\}$ then $\mathcal{N} = \{\phi,U,\{b\},\{a,c\}\}$. Let $\mathcal{I} = \{\phi,\{a,b\}\}$. It is clear that $\{b\}$ is $n\mathcal{I}_g$-closed but not $n\mathcal{I}_{g^\#}$-closed.

Theorem 3.5. *If $(U,\mathcal{N},\mathcal{I})$ is any ideal nano topological space and $A \subseteq U$, then the following are equivalent:*

(1) *A is $n\mathcal{I}_{g^\#}$-closed.*

(2) *$n\text{-}cl^\star(A) \subseteq G$ whenever $A \subseteq G$ and G is $n\alpha g$-open in U.*

Proof. (1)⇒(2): Let $A \subseteq G$ where G is $n\alpha g$-open in U. Since A is $n\mathcal{I}_{g^\#}$-closed, $A_n^\star \subseteq G$ and so $n\text{-}cl^\star(A) = A \cup A_n^\star \subseteq G$.

(2)⇒(1): It follows from the fact that $A_n^\star \subseteq n\text{-}cl^\star(A) \subseteq G$.

Theorem 3.6. *Every $n\star$-closed set is $n\mathcal{I}_{g^\#}$-closed but not conversely.*

Proof. Let A be a $n\star$-closed set. To prove A is $n\mathcal{I}_{g^\#}$-closed, let G be any $n\alpha g$-open set such that $A \subseteq U$. Since A is $n\star$-closed, $A_n^\star \subseteq A \subseteq U$. Thus A is $n\mathcal{I}_{g^\#}$-closed. □

Theorem 3.7. *Let $(U,\mathcal{N},\mathcal{I})$ be an ideal nano topological space. For every $A \in \mathcal{I}$, A is $n\mathcal{I}_{g^\#}$-closed.*

Proof. Let $A \in \mathcal{I}$ and let $A \subseteq G$ where G is $n\alpha g$-open. Since $A \in \mathcal{I}$, $A_n^\star = \phi \subseteq G$. Thus A is $n\mathcal{I}_{g^\#}$-closed. □

Theorem 3.8. *If $(U,\mathcal{N},\mathcal{I})$ is an ideal nano topological space, then $A_n^\star$ is always $n\mathcal{I}_{g^\#}$-closed for every subset A of U.*

Proof. Let $A_n^\star \subseteq G$ where G is $n\alpha g$-open. Since $(A_n^\star)_n^\star \subseteq A_n^\star$, we have $(A_n^\star)_n^\star \subseteq G$. Hence $A_n^\star$ is $n\mathcal{I}_{g^\#}$-closed. □

Theorem 3.9. *Let $(U,\mathcal{N},\mathcal{I})$ be an ideal nano topological space. Then every $n\mathcal{I}_{g^\#}$-closed, $n\alpha g$-open set is $n\star$-closed.*

Proof. Let A be $n\mathcal{I}_{g^\#}$-closed and $n\alpha g$-open. We have $A \subseteq A$ where A is $n\alpha g$-open. Since A is $n\mathcal{I}_{g^\#}$-closed, $A_n^\star \subseteq A$. Thus A is $n\star$-closed. □

Corollary 3.10. *Let $(U,\mathcal{N},\mathcal{I})$ be an ideal nano topological space and A be an $n\mathcal{I}_{g^\#}$-closed set. Consider the following statements:*

(1) *A is a $n\star$-closed set,*

(2) *n-$cl^\star(A) - A$ is a $n\alpha g$-closed set,*

(3) *$A_n^\star - A$ is a $n\alpha g$-closed set.*

Proof. Then (1)⇒(2) and (2)⇒(3) hold.

(1)⇒(2): By (1) A is $n\star$-closed. Hence $A_n^\star \subseteq A$ and n-$cl^\star(A) - A = (A \cup A_n^\star) - A = \phi$ which is a $n\alpha g$-closed set.

(2)⇒(3): n-$cl^\star(A) - A = A_n^\star \cup A - A = (A_n^\star \cup A) \cap A^c = (A_n^\star \cap A^c) \cup (A \cap A^c) = (A_n^\star \cap A^c) \cup \phi = A_n^\star - A$ which is a $n\alpha g$-closed set by (2). □

Theorem 3.11. *Let $(U,\mathcal{N},\mathcal{I})$ be an ideal nano topological space. Then every $ng^\#$-closed set is a $n\mathcal{I}_{g^\#}$-closed set but not conversely.*

Proof. Let A be a $ng^\#$-closed set. Let G be any $n\alpha g$-open set such that $A \subseteq G$. Since A is $ng^\#$-closed, n-$cl(A) \subseteq G$. So, $A_n^\star \subseteq n$-$cl(A) \subseteq G$ and thus A is $n\mathcal{I}_{g^\#}$-closed. □

Example 3.12. Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and $X = \{a,b\}$ then $\mathcal{N} = \{\phi, U, \{a\}, \{b,d\}, \{a,b,d\}\}$. Let $\mathcal{I} = \{\phi, \{d\}\}$. It is clear that $\{d\}$ is $n\mathcal{I}_{g^\#}$-closed but not $ng^\#$-closed.

Theorem 3.13. *If $(U,\mathcal{N},\mathcal{I})$ is an ideal topological space and A is a $n\star$-dense in itself, $n\mathcal{I}_{g^\#}$-closed subset of U, then A is $ng^\#$-closed.*

Proof. Let $A \subseteq G$ where G is $n\alpha g$-open. Since A is $n\mathcal{I}_{g^\#}$-closed, $A_n^\star \subseteq G$. As A is $n\star$-dense in itself, n-$cl(A) = A_n^\star$. Thus n-$cl(A) \subseteq G$ and hence A is $ng^\#$-closed. □

Corollary 3.14. *If $(U,\mathcal{N},\mathcal{I})$ is any ideal nano topological space where $\mathcal{I} = \{\phi\}$, then A is $n\mathcal{I}_{g^\#}$-closed if and only if A is $ng^\#$-closed.*

Proof. In $(U,\mathcal{N},\mathcal{I})$, if $\mathcal{I} = \{\phi\}$ then $A_n^\star = n$-$cl(A)$ for the subset A. A is $n\mathcal{I}_{g^\#}$-closed ⇔ $A_n^\star \subseteq G$ whenever $A \subseteq G$ and G is $n\alpha g$-open ⇔ n-$cl(A) \subseteq G$ whenever $A \subseteq G$ and G is $n\alpha g$-open ⇔ A is $ng^\#$-closed. □

Corollary 3.15. *In an ideal nano topological space $(U,\mathcal{N},\mathcal{I})$, where $\mathcal{I}$ is n-codense, if A is a nano semi-open and $n\mathcal{I}_{g^{\#}}$-closed subset of U, then A is $ng^{\#}$-closed.*

Proof. A is $n\star$-dense in itself.
Therefore A is $ng^{\#}$-closed. □

Remark 3.16. We have the following implications for the subsets stated above:

$$\begin{array}{ccccc} n\text{-closed} & \longrightarrow & ng^{\#}\text{-closed} & \longrightarrow & ng\text{-closed} \\ \downarrow & & \downarrow & & \downarrow \\ n\star\text{-closed} & \longrightarrow & n\mathcal{I}_{g^{\#}}\text{-closed} & \longrightarrow & n\mathcal{I}_{g}\text{-closed} \end{array}$$

None of the above implications are reversible.

Theorem 3.17. *Let $(U,\mathcal{N},\mathcal{I})$ be an ideal nano topological space and $A \subseteq U$. If $A \subseteq B \subseteq A_n^{\star}$, then $A_n^{\star} = B_n^{\star}$ and B is $n\star$-dense in itself.*

Proof. Since $A \subseteq B$, then $A_n^{\star} \subseteq B_n^{\star}$ and since $B \subseteq A_n^{\star}$, then $B_n^{\star} \subseteq (A_n^{\star})_n^{\star} \subseteq A_n^{\star}$. Therefore $A_n^{\star} = B_n^{\star}$ and $B \subseteq A_n^{\star} \subseteq B_n^{\star}$. Hence proved. □

Theorem 3.18. *Let $(U,\mathcal{N},\mathcal{I})$ be an ideal nano topological space. Then every subset of U is $n\mathcal{I}_{g^{\#}}$-closed if and only if every $n\alpha g$-open set is $n\star$-closed.*

Proof. Suppose every subset of U is $n\mathcal{I}_{g^{\#}}$-closed. Let G be $n\alpha g$-open in U. Then $G \subseteq G \subseteq U$ and G is $n\mathcal{I}_{g^{\#}}$-closed by assumption. It implies $G_n^{\star} \subseteq G$. Hence G is $n\star$-closed.

Conversely, let $A \subseteq U$ and G be $n\alpha g$-open such that $A \subseteq G$. Since G is $n\star$-closed by assumption, we have $A_n^{\star} \subseteq G_n^{\star} \subseteq U$. Thus A is $n\mathcal{I}_{g^{\#}}$-closed. □

Theorem 3.19. *Let $(U,\mathcal{N},\mathcal{I})$ be an ideal nano topological space and $A \subseteq U$. Then A is $n\mathcal{I}_{g^{\#}}$-open if and only if $F \subseteq n\text{-}int^{\star}(A)$ whenever F is $n\alpha g$-closed and $F \subseteq A$.*

Proof. Suppose A is $n\mathcal{I}_{g^{\#}}$-open. If F is $n\alpha g$-closed and $F \subseteq A$, then $U\text{-}A \subseteq U\text{-}F$ and so $n\text{-}cl^{\star}(U\text{-}A) \subseteq U\text{-}F$. Therefore $F \subseteq U-(n\text{-}cl^{\star}(U-A)) = n\text{-}int^{\star}(A)$. Hence $F \subseteq n\text{-}int^{\star}(A)$.

Conversely, suppose the condition holds. Let G be a $n\alpha g$-open set such that $U-A \subseteq G$. Then $U-G \subseteq A$ and so $U-G \subseteq n\text{-}int^{\star}(A)$. Therefore $n\text{-}cl^{\star}(U-A) \subseteq G$. So $U-A$ is $n\mathcal{I}_{g^{\#}}$-closed. Hence A is $n\mathcal{I}_{g^{\#}}$-open. □

The following Theorem gives a characterization of normal spaces in terms of $n\mathcal{I}_{g^{\#}}$-open sets.

Definition 3.20. A subset A of a nano topological space $(U,\mathcal{N})$ is said to be a completely nano codense (briely completely n-codense) if $NPO(X) \cap I = \{\phi\}$, where $NPO(X)$ is the family of all nano preopen sets.

Theorem 3.21. *Let $(U,\mathcal{N},\mathcal{I})$ be an ideal nano topological space where $\mathcal{I}$ is completely n-codense. Then the following are equivalent:*

(1) *U is normal,*

(2) *for any disjoint nano closed sets A and B, there exist disjoint $n\mathcal{I}_{g^{\#}}$-open sets U and V such that $A \subseteq G$ and $B \subseteq H$,*

(3) *for any nano closed set A and nano open set H containing A, there exists an $n\mathcal{I}_{g^{\#}}$-open set G such that $A \subseteq G \subseteq n\text{-}cl^{\star}(G) \subseteq H$.*

Proof. (1)⇒(2): The proof follows from the fact that every nano open set is $n\mathcal{I}_{g^{\#}}$-open.

(2)⇒(3): Suppose A is nano closed and V is a nano open set containing A. Since A and $X-V$ are disjoint nano closed sets. There exist disjoint $n\mathcal{I}_{g^{\#}}$-open sets G and W such that $A \subseteq G$ and $U-H \subseteq W$. Since $U-H$ is $n\alpha g$-closed and W is $n\mathcal{I}_{g^{\#}}$-open, $U-H \subseteq n\text{-}int^{\star}(W)$. Then $U-(n\text{-}int^{\star}(W)) \subseteq H$. Again $G \cap W = \phi$ which implies that $G \cap n\text{-}int^{\star}(W) = \phi$ and so $G \subseteq U-(n\text{-}int^{\star}(W)$. Then $n\text{-}cl^{\star}(G) \subseteq U-(n\text{-}int^{\star}(W) \subseteq H$ and thus G is the required $n\mathcal{I}_{g^{\#}}$-open sets with $A \subseteq G \subseteq n\text{-}cl^{\star}(G) \subseteq H$.

(3)⇒(1): Let A and B be two disjoint nano closed subsets of U. Then A is a nano closed set and $U-B$ is a nano open set containing A. By hypothesis, there exists a $n\mathcal{I}_{g^{\#}}$-open set G such that $A \subseteq G \subseteq n\text{-}cl^{\star}(G) \subseteq U-B$. Since G is $n\mathcal{I}_{g^{\#}}$-open and A is $n\alpha g$-closed we have, $A \subseteq n\text{-}int^{\star}(G)$. Since $\mathcal{I}$ is completely n-codense, $\mathcal{N}^{\star} \subseteq \mathcal{N}^{\alpha}$ and so $n\text{-}int^{\star}(G)$ and $G-(n\text{-}cl^{\star}(G)) \in \mathcal{N}^{\alpha}$. Hence $A \subseteq n\text{-}int^{\star}(G) \subseteq n\text{-}int(n\text{-}cl(n\text{-}int(n\text{-}int^{\star}(G)))) = U$ and $B \subseteq U\text{-}(n\text{-}cl^{\star}(U)) \subseteq n\text{-}int(n\text{-}cl(n\text{-}int(U-(n\text{-}cl^{\star}(G))))) = H$. G and H are the required disjoint nano open sets containing A and B respectively, which proves (1). □

Definition 3.22. A subset A of a nano topological space $(U,\mathcal{N})$ is said to be a $n\alpha g^{\#}$-closed set if $n\text{-}cl_{\alpha}(A) \subseteq G$ whenever $A \subseteq G$ and G is $n\alpha g$-open. The complement of a $n\alpha g^{\#}$-closed set is said to be a $n\alpha g^{\#}$-open set.

If $\mathcal{I}$=$\mathcal{N}$, it is not difficult to see that $n\mathcal{I}_{g^{\#}}$-closed sets coincide with $n\alpha g^{\#}$-closed sets and so we have the following corollary:

Corollary 3.23. *Let $(U,\mathcal{N},\mathcal{I})$ be an ideal nano topological space where $\mathcal{I}$=$\mathcal{N}$. Then the following are equivalent:*

(1) *U is normal,*

(2) *for any disjoint nano closed sets A and B, there exist disjoint $n\alpha g^{\#}$-open sets G and H such that $A \subseteq G$ and $B \subseteq H$,*

(3) *for any nano closed set A and nano open set H containing A, there exists a $n\alpha g^{\#}$-open set G such that $A \subseteq G \subseteq n\text{-}cl_{\alpha}(G) \subseteq H$.*

4. Conclsuion

The notions of sets in an ideal nano topological space is extensively developed and used in data mining, computational topology for geometric design and molecular design, computer-aided design, computer-aided geometric design, digital topology, information systems, particle physics and quantum physics etc.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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