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Research Article

Effect of Torsional Loading in an Axisymmetric Micro-Isotropic, Micro-Elastic Solid

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Abstract. In this paper, an attempt is made to obtain the solution for the problem of torsional loading in an axisymmetric Micro-isotropic, Micro-elastic half-space under the action of an arbitrary load on its boundary. The components of displacement, microrotation, stress, couple stress and stress moment are obtained. These components are also obtained for a particular type of twist and represented graphically in the positive quadrant.

Keywords. Micro-isotropic & Micro-elastic media; Torsional loading

Mathematics Subject Classification (2020). 74B15

1. Introduction

Classical theory of elasticity is inadequate to describe the modern engineering structures like polycrystalline materials, materials with fibrous or coarse grain. To remove this shortcoming of classical theory Eringen [1, 2] introduced the theory of micromorphic materials which include micro-structure. This theory was simplified by Koh [3] by extending the concept of coincidence of principal directions of stresses and strains of classical theory to the micro-elastic materials and assuming micro-isotropy. He named it as the theory of Micro-isotropic, Micro-elastic materials. Though this theory is a simplified version, still it retains the characteristic features of the micromorphic model. The basic equations of this theory were developed by Koh and Parameswaran [4].

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Kumar and Chadha [5] studied torsional loading problem in micropolar elastic medium. Srinivas *et al*. [7] investigated the general solution of equations of motion of axisymmetric problem of Micro-isotropic, Micro-elastic solid. Renji and Yulan [8] analyzed Torsion problems for cylinder with rectangular hole and a rectangular cylinder with a crack. Vaysfeld and Protserov [9] studied the torsional problem of a multilayered finite cylinder with multiple interface cylinder crack. Rama [6] examined the propagation of Love waves in Micro-isotropic, Micro-elastic layered media.

In this paper the general solution of axisymmetric Micro-isotropic, Micro-elastic half space by applying a torsional loading on its boundary is obtained. Then the obtained components are analyzed graphically by taking a particular case of twist. The stress moment components are represented graphically for the general case.

2. Basic Equations

$$(A_1+A_2-A_3)u_{p,pm}+(A_2+A_3)u_{m,pp}+2A_3\epsilon_{pkm}\phi_{p,k}+\rho f_m=\rho\frac{\partial^2 u_m}{\partial t^2}, \tag{1}$$

$$2B_3\phi_{p,mm}+2(B_4+B_5)\phi_{m,mp}-4A_3(r_p+\phi_p)-\rho l_p=\rho j\frac{\partial^2\phi_p}{\partial t^2}, \tag{2}$$

$$B_1\Phi_{pp,kk}\delta_{ij}+2B_2\emptyset_{(ij),kk}-A_4\emptyset_{pp}\delta_{ij}-2A_5\emptyset_{(ij)}+\rho f_{(ij)}=\frac{1}{2}\rho j\frac{\partial^2\emptyset_{(ij)}}{\partial t^2}. \tag{3}$$

The constitutive equations for micro-isotropic, micro-elastic solid are

$$t_{(km)}=A_1e_{pp}\delta_{km}+2A_2e_{km}, \tag{4}$$

$$t_{[km]}=\sigma_{[km]}=2A_3\epsilon_{pkm}(r_p+\phi_p), \tag{5}$$

$$\sigma_{(km)}=-A_4\emptyset_{pp}\delta_{km}-2A_5\emptyset_{(km)}, \tag{6}$$

$$t_{k(mn)}=B_1\emptyset_{pp,k}\delta_{mn}+2B_2\emptyset_{(mn),k}, \tag{7}$$

$$m_{(kl)}=-2(B_5\phi_{l,k}+B_4\phi_{k,l}+B_5\phi_{p,p}\delta_{kl}), \tag{8}$$

where

$$\left.\begin{aligned}
&A_1=\lambda+\sigma_1, && B_1=\tau_3,\\
&A_2=\mu+\sigma_2, && 2B_2=\tau_7+\tau_{10},\\
&A_3=\sigma_5, && B_3=2\tau_4+2\tau_9+\tau_7-\tau_{10},\\
&A_4=-\sigma_1, && B_4=-2\tau_4,\\
&A_4=-\sigma_2, && B_5=-2\tau_9.
\end{aligned}\right\} \tag{9}$$

Parameshwaran and Koh [4] established the following constraints on micro-isotropic, micro-elastic constants.

$$\left.\begin{aligned}
&3A_1+2A_2>0,\ \ A_2>0,\ \ A_3>0,\ \ 3A_4+2A_5>0,\ \ A_5>0,\\
&B_3>0,\ \ -B_3<B_4<B_3,\ \ B_3+B_4+B_5>0
\end{aligned}\right\} \tag{10}$$

where ρ is the mass density, j is the micro-inertia, f_m is the body force per unit mass, $f_{(ij)}$ is the symmetric body moment and l_p is the body couple per unit mass. The macro displacement is denoted by u_k, microrotation is denoted by φ_k and micro-strains are denoted by $\emptyset_{km}$.

$$\phi_p=\frac{1}{2}\epsilon_{pkm}\emptyset_{km},\ \ r_p=\frac{1}{2}\epsilon_{pkm}u_{mk}$$

3. Formulation of the Problem

Consider an axisymmetric Micro-isotropic, Micro-elastic half space in cylindrical coordinates. The displacement component and microrotation components will become

$$u_\theta = u_\theta(r,z), \quad \varphi_r = \varphi_r(r,z), \quad \varphi_z = \varphi_z(r,z) \tag{11}$$

and micro-strain will become

$$\emptyset_{\theta\theta} = \emptyset_{\theta\theta}(r,z), \;\; \emptyset_{r\theta} = \emptyset_{r\theta}(r,z), \;\; \emptyset_{z\theta} = \emptyset_{z\theta}(r,z). \tag{12}$$

By substituting (11) and (12) in equations (1) to (3) we get

$$(A_2+A_3)\left[\nabla^2 u_\theta - \frac{u_\theta}{r^2}\right] + 2A_3\left[\frac{\partial\varphi_r}{\partial z} - \frac{\partial\varphi_z}{\partial r}\right] = 0, \tag{13}$$

$$2B_3\left[\nabla^2\varphi_r - \frac{\varphi_r}{r^2}\right] - 4A_3\varphi_r + (2B_4+2B_5)\frac{\partial e}{\partial r} + 2A_3\left[\frac{\partial u_\theta}{\partial z}\right] = 0, \tag{14}$$

$$2B_3\left[\nabla^2\varphi_z\right] - 4A_3\varphi_z + (2B_4+2B_5)\frac{\partial e}{\partial z} - 2A_3\frac{1}{r}\left[\frac{\partial(ru_\theta)}{\partial r}\right] = 0, \tag{15}$$

$$B_1\nabla^2\emptyset_{\theta\theta} + 2B_2\nabla^2\emptyset_{\theta\theta} - A_4\emptyset_{\theta\theta} - 2A_5\emptyset_{\theta\theta} = 0, \tag{16}$$

$$2B_2\nabla^2\emptyset_{(r\theta)} - 2A_5\emptyset_{(r\theta)} = 0, \tag{17}$$

$$2B_2\nabla^2\emptyset_{(z\theta)} - 2A_5\emptyset_{(z\theta)} = 0 \tag{18}$$

where

$$e = \frac{1}{r}\frac{\partial}{\partial r}(r\varphi_r) + \frac{\partial\varphi_z}{\partial z} \;\text{ and }\; \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \tag{19}$$

Here the following the potential functions will be introduced.

$$\varphi_r = \frac{\partial\phi}{\partial r} + \frac{\partial^2\psi}{\partial r\partial z}, \tag{20}$$

$$\varphi_z = \frac{\partial\phi}{\partial z} - \left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)\psi, \tag{21}$$

$$u_\theta = \frac{\partial v}{\partial r}. \tag{22}$$

Substituting (20) to (22) in equations (13) to (15), we get

$$\frac{\partial\phi}{\partial r}\left[(2B_3+2B_4+2B_5)\left(\nabla^2 - \frac{1}{r^2}\right) - 4A_3\right] + \frac{\partial^2\psi}{\partial r\partial z}\left[2B_3\left(\nabla^2 - \frac{1}{r^2}\right) - 4A_3 + (2B_4+2B_5)\left(-\frac{1}{r^2}\right)\right] + 2A_3\frac{\partial^2 v}{\partial r\partial z} = 0, \tag{23}$$

$$\frac{\partial\phi}{\partial z}\left[(2B_3+2B_4+2B_5)\nabla^2 - 4A_3\right] - \left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)\psi\left(2B_3\nabla^2 - 4A_3\right) - 2A_3\left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)v = 0, \tag{24}$$

$$\frac{\partial}{\partial r}\left(\nabla^2 - \frac{1}{r^2}\right)v + \frac{2A_3}{A_2+A_3}\frac{\partial}{\partial r}(\nabla^2)\psi = 0. \tag{25}$$

The Hankel transforms defined by

$$\overline{H}(\xi,z) = \int_0^\infty H(r,z)rJ_0(\xi r)dr \quad \text{and} \quad \overline{H}(\xi,z) = \int_0^\infty H(r,z)rJ_1(\xi r)dr, \tag{26}$$

will be applied to equations (23) to (25), we get

$$(D^2 - \xi^2)(D^2 - \xi_2^2)\widehat{\phi} = 0, \tag{27}$$

$$(D^2-\xi^2)(D^2-\xi_1^2)\widehat{\psi}=0, \tag{28}$$

$$2(B_3+B_4+B_5)(D^2-\xi_1^2)\widehat{\phi}+D(D^2-\xi_2^2)\widehat{\psi}=0, \tag{29}$$

where

$$\xi_1^2=\xi^2+k_1^2,\ \xi_2^2=\xi^2+k_2^2 \text{ and } k_1^2=\frac{2A_3}{B_3+B_4+B_5},\ k_2^2=\frac{4A_3}{2B_3}\left(\frac{A_2+2A_3}{A_2+A_3}\right). \tag{30}$$

As various components approach zero as $z\to\infty$ the displacement functions also approaches zero as $z\to\infty$, we choose the solutions of (27) and (28) as follows:

$$\widehat{\phi}=A\exp(-\xi z)+B\exp(-\xi_1 z), \tag{31}$$

$$\widehat{\psi}=C\exp(-\xi z)+D\exp(-\xi_2 z). \tag{32}$$

Substituting (31) and (32) in (29) we get

$$A=\frac{A_2+2A_3}{A_2+A_3}\xi C. \tag{33}$$

By taking inverse transforms to equations (31) and (32) and using (33) we get

$$\phi=\int_0^\infty\left[\frac{A_2+2A_3}{A_2+A_3}\xi C\exp(-\xi z)+B\exp(-\xi_1 z)\right]\xi J_0(\xi r)d\xi, \tag{34}$$

$$\psi=\int_0^\infty[C\exp(-\xi z)+D\exp(-\xi_2 z)]\xi J_0(\xi r)d\xi. \tag{35}$$

Since there acts a torsional loading on the boundary $z=0$ plane, the mathematical equations for boundary conditions are

$$t_{z\theta}=-f(r),\quad m_{zz}=m_{zr}=0\quad \text{at } z=0, \tag{36}$$

$$\varnothing_{\theta\theta}=\varnothing_{(r\theta)}=\varnothing_{(z\theta)}=0\qquad \text{at } r=a. \tag{37}$$

By substituting (34) and (35) in boundary conditions (36), we get

$$2A_3\left[\frac{A_2+2A_3}{A_2+A_3}\xi^2 C+\xi B\right]=\widehat{f}(\xi), \tag{38}$$

$$[(B_3+B_4)\xi^2+2A_3]B+(B_3+B_4)\left(\frac{A_3}{A_2+A_3}\right)\xi^3 C-(B_3+B_4)\xi_2\xi^2 D=0, \tag{39}$$

$$\left[(B_3+B_4)\xi_1 B+(B_3+B_4)\left(\frac{A_3}{A_2+A_3}\right)\xi^2 C\right]-\left[(B_3+B_4)\xi^2+2A_3\left(\frac{A_2+2A_3}{A_2+A_3}\right)\right]D=0, \tag{40}$$

where

$$f(\xi)=\int_0^\infty f(r)rJ_1(\xi r)dr. \tag{41}$$

By solving (38) to (40), we get

$$B=\xi F_1(\xi);\quad C=F_2(\xi)\left(\frac{A_2+A_3}{A_3}\right);\quad D=F_3(\xi);$$

$$F_1(\xi)=\frac{\widehat{f}(\xi)}{2\Delta(A_2+A_3)}(-\xi^2-\epsilon_1+\xi_2\xi);$$

$$F_2(\xi)=\frac{\widehat{f}(\xi)}{2\Delta(A_2+A_3)}\left[\xi_1\xi_2-\xi^2-\epsilon_1\frac{(2A_2+3A_3)}{(A_2+2A_3)}-\frac{2A_3\epsilon_1}{\xi^2(B_3+B_4)}\right];$$

$$F_3(\xi)=\frac{\widehat{f}(\xi)}{2\Delta(A_2+A_3)}\left[\xi(\xi-\xi_1)+\frac{2A_3}{B_3+B_4}\right];$$

$$\epsilon_1 = \frac{2A_3(A_2+2A_3)}{(A_2+A_3)(B_3+B_4)}, \quad \Delta = (\xi^2+\epsilon_1)^2 + \frac{\xi_2\xi^2}{A_2+A_3}[A_3(B_3+B_4)-(A_2+2A_3)\xi_1]. \tag{42}$$

Then we get potential functions as

$$\phi = \int_0^\infty \xi^2 \left[\frac{A_2+2A_3}{A_3}F_2(\xi)\exp(-\xi z) + F_1(\xi)\exp(-\xi_1 z)\right] J_0(\xi r)d\xi, \tag{43}$$

$$\psi = \int_0^\infty \xi \left[\frac{A_2+A_3}{A_3}F_2(\xi)\exp(-\xi z) + F_3(\xi)\exp(-\xi_2 z)\right] J_0(\xi r)d\xi. \tag{44}$$

Using (43) and (44) we can obtain the components of displacement, microrotation, stress and couple stress as follows:

$$u_\theta = 2\int_0^\infty \xi^2 \left[F_2(\xi)\exp(-\xi z) + \frac{A_3}{A_2+A_3}F_3(\xi)\exp(-\xi_2 z)\right] J_1(\xi r)d\xi, \tag{45}$$

$$\varphi_r = \int_0^\infty \xi^3 \left[-F_2(\xi)\exp(-\xi z) - F_1(\xi)\exp(-\xi_1 z) + \frac{\xi_2}{\xi}F_3(\xi)\exp(\xi_2 z)\right] J_1(\xi r)d\xi, \tag{46}$$

$$\varphi_z = \int_0^\infty \xi^3 \left[-F_2(\xi)\exp(-\xi z) + \frac{\xi_2}{\xi}F_1(\xi)\exp(-\xi_1 z) - F_3(\xi)\exp(\xi_2 z)\right] J_0(\xi r)d\xi, \tag{47}$$

$$t_{z\theta} = -2(A_2+2A_3)\int_0^\infty \xi^3 \left[F_2(\xi)\exp(-\xi z) - \frac{A_3}{A_2+2A_3}F_1(\xi)\exp(-\xi_1 z)\right] J_1(\xi r)d\xi, \tag{48}$$

$$\begin{aligned} m_{zz} = -2(B_3+B_4)\int_0^\infty \xi^4 \Bigg[&-F_2(\xi)\exp(-\xi z) + \left(1+\frac{A_3}{\xi^2(A_2+2A_3)}\right)F_1(\xi)\exp(-\xi_1 z) \\ &-\frac{\xi_2}{\xi}F_3(\xi)\exp(\xi_2 z)\Bigg] J_0(\xi r)d\xi, \end{aligned} \tag{49}$$

$$\begin{aligned} m_{zr} = -2(B_3+B_4)\int_0^\infty \xi^4 \Bigg[&-F_2(\xi)\exp(-\xi z) + \frac{\xi_1}{\xi}F_1(\xi)\exp(-\xi_1 z) \\ &-\left(1+\frac{A_2+2A_3}{\xi^2(B_3+B_4)(A_2+A_3)}\right)F_3(\xi)\exp(\xi_2 z)\Bigg] J_1(\xi r)d\xi. \end{aligned} \tag{50}$$

4. Numerical Work

To analyze the components obtained in (45) to (50) we take a particular type of twist given by

$$f(r) = \frac{r}{4a^4}\exp\left(-\frac{r^2}{4a^4}\right), \tag{51}$$

where r is the distance of the point from the origin of the coordinate system. By applying Hankel transform (46) to (50) we get

$$\widehat{f}(\xi) = \xi\exp(-a^2\xi^2). \tag{52}$$

Then equations (45) to (50) becomes

$$u_\theta = 2\int_0^\infty \xi^3\exp(-a^2\xi^2)\left[F_2(\xi)\exp(-\xi z) + \frac{A_3}{A_2+A_3}F_3(\xi)\exp(-\xi_2 z)\right] J_1(\xi r)d\xi, \tag{53}$$

$$\varphi_r = \int_0^\infty \xi^4\exp(-a^2\xi^2)\left[-F_2(\xi)\exp(-\xi z) - F_1(\xi)\exp(-\xi_1 z) + \frac{\xi_2}{\xi}F_3(\xi)\exp(\xi_2 z)\right] J_1(\xi r)d\xi, \tag{54}$$

$$\varphi_z = \int_0^\infty \xi^4\exp(-a^2\xi^2)\left[-F_2(\xi)\exp(-\xi z) + \frac{\xi_2}{\xi}F_1(\xi)\exp(-\xi_1 z) - F_3(\xi)\exp(\xi_2 z)\right] J_0(\xi r)d\xi, \tag{55}$$

$$t_{z\theta} = -2(A_2+2A_3)\int_0^\infty \xi^4\exp(-a^2\xi^2)\left[F_2(\xi)\exp(-\xi z) - \frac{A_3}{A_2+2A_3}F_1(\xi)\exp(-\xi_1 z)\right] J_1(\xi r)d\xi, \tag{56}$$

$$m_{zz} = -2(B_3+B_4)\int_0^\infty \xi^5 \exp(-a^2\xi^2)\left[-F_2(\xi)\exp(-\xi z) + \left(1+\frac{A_3}{\xi^2(A_2+2A_3)}\right)F_1(\xi)\exp(-\xi_1 z) - \frac{\xi_2}{\xi}F_3(\xi)\exp(\xi_2 z)\right]J_0(\xi r)d\xi, \tag{57}$$

$$m_{zr} = -2(B_3+B_4)\int_0^\infty \xi^5 \exp(-a^2\xi^2)\left[-F_2(\xi)\exp(-\xi z) + \frac{\xi_1}{\xi}F_1(\xi)\exp(-\xi_1 z) - \left(1+\frac{A_2+2A_3}{\xi^2(B_3+B_4)(A_2+A_3)}\right)F_3(\xi)\exp(\xi_2 z)\right]J_1(\xi r)d\xi. \tag{58}$$

5. Approximation Evaluation of Integrals

As the integrals involved in (53) to (58) are difficult to evaluate, we evaluate them by taking the following approximations. By assuming A_3, k_1^2 and k_2^2 to be small compared to unity we expand ξ_1, ξ_2 and $\frac{1}{\Delta}$ in an infinite series to obtain

$$\xi_1 = \xi + \frac{m_1^2}{2\xi} + o(m_1^4), \quad \xi_2 = \xi + \frac{m_2^2}{2\xi} + o(m_2^4) \text{ and } \Delta = \gamma\epsilon_1 A_1\xi^2, \tag{59}$$

where $A_1 = \frac{1}{B_3+B_4} - \frac{1}{2B_3} - \frac{1}{2(B_3+B_4+B_5)}$.

Then (53) to (58) becomes

$$u_\theta = \frac{1}{A_2+A_3}\int_0^\infty \xi\left(1+\frac{2A_3L_1}{A_1\xi^2}\right)\exp(-a^2\xi^2)\exp(-\xi z)J_1(\xi r)d\xi, \tag{60}$$

$$\varphi_r = \frac{A_3}{A_1(A_2+A_3)(B_3+B_4)}\int_0^\infty (L_4+z\xi L_2)\exp(-a^2\xi^2)\exp(-\xi z)J_1(\xi r)d\xi, \tag{61}$$

$$\varphi_z = \frac{A_3}{A_1(A_2+A_3)(B_3+B_4)}\int_0^\infty (L_3+z\xi L_2)\exp(-a^2\xi^2)\exp(-\xi z)J_0(\xi r)d\xi, \tag{62}$$

$$t_{z\theta} = -\int_0^\infty \xi^2\left(1+\frac{\epsilon_1 L_1(B_3+B_4)}{\xi^2 A_1}\right)\exp(-a^2\xi^2)\exp(-\xi z)J_1(\xi r)d\xi, \tag{63}$$

$$m_{zz} = \frac{-2A_3L_2 z}{A_1(A_2+A_3)}\int_0^\infty \xi^2\exp(-a^2\xi^2)\exp(-\xi z)J_0(\xi r)d\xi, \tag{64}$$

$$m_{zr} = \frac{-2A_3L_2 z}{A_1(A_2+A_3)}\int_0^\infty \xi^3\exp(-a^2\xi^2)J_1(\xi r)d\xi, \tag{65}$$

where

$$\left.\begin{aligned} L_1 &= \frac{1}{2(B_3+B_4+B_5)} + \frac{1}{2B_3} - \frac{2}{B_3+B_4}, \\ L_2 &= \frac{1}{2(B_3+B_4+B_5)} + \frac{1}{2B_3} - \frac{B_3+B_4}{2B_3(B_3+B_4+B_5)}, \\ L_3 &= \frac{1}{B_3+B_4} - \frac{1}{2(B_3+B_4+B_5)}, \\ L_4 &= \frac{1}{B_3+B_4} - \frac{1}{2B_3}. \end{aligned}\right\} \tag{66}$$

The term $\exp(-a^2\xi^2)$ in (60) to (65) is expanded by assuming $a\xi$ is so small that its fourth order terms are negligible and we get

$$u_\theta = \frac{r}{(A_2+A_3)}\left[\frac{1}{\rho_1^3} + \frac{3a^2}{\rho_1^5}\left(1-\frac{5z^2}{\rho_1^2}\right) + \frac{2A_3L_1}{A_1}\left\{\frac{1}{\rho_1+z} - \frac{a^2}{\rho_1^3} + \frac{3a^4}{2\rho_1^5}\left(\frac{5z^2}{\rho_1^2}-1\right)\right\}\right], \tag{67}$$

$$\varphi_r = \frac{A_3}{A_1(A_2+A_3)(B_3+B_4)}\frac{r}{\rho_1}\left[L_4\left\{\frac{1}{\rho_1+z}-\frac{3a^2z}{\rho_1^4}+\frac{15a^4z}{2\rho_1^6}\left(\frac{7z^2}{\rho_1^2}-3\right)\right\}\right.$$
$$\left.+L_2\frac{z}{\rho_1^2}\left\{1+\frac{3a^2}{\rho_1^2}\left(1-\frac{5z^2}{\rho_1^2}\right)\right\}\right], \tag{68}$$

$$\varphi_z = \frac{A_3}{A_1(A_2+A_3)(B_3+B_4)}\frac{1}{\rho_1}\left[L_3\left\{1+\frac{a^2}{\rho_1^2}\left(1-\frac{3z^2}{\rho_1^2}\right)+\frac{9a^4}{2\rho_1^4}\left(1+\frac{5z^2}{\rho_1^2}\left(\frac{3z^2}{\rho_1^2}-2\right)\right)\right\}\right.$$
$$\left.+L_2\frac{z^2}{\rho_1^2}\left\{1+\frac{3a^2}{\rho_1^2}\left(3-\frac{5z^2}{\rho_1^2}\right)\right\}\right], \tag{69}$$

$$t_{z\theta} = -\frac{r}{\rho_1}\left[\frac{3z}{\rho_1^4}+\frac{15a^4z}{\rho_1^6}\left(3-\frac{7z^2}{\rho_1^2}\right)+(B_3+B_4)\frac{\epsilon_1 L_1}{A_1}\left\{\frac{1}{\rho_1+z}-\frac{3a^2z}{\rho_1^4}+\frac{15a^4z}{2\rho_1^6}\left(\frac{7z^2}{\rho_1^2}-3\right)\right\}\right], \tag{70}$$

$$m_{zz} = \frac{2A_3L_2z}{A_1(A_2+A_3)}\frac{1}{\rho_1^3}\left[1-\frac{3z^2}{\rho_1^2}+\frac{9a^2}{\rho_1^2}\left\{1+\frac{5z^2}{\rho_1^2}\left(\frac{3z^2}{\rho_1^2}-2\right)\right\}\right], \tag{71}$$

$$m_{zr} = \frac{6A_3L_2z^2r}{A_1(A_2+A_3)\rho_1^5}\left[\frac{5z^2}{\rho_1^2}\left(\frac{7z^2}{\rho_1^2}-3\right)-1\right], \tag{72}$$

where $\rho_1^2 = r^2+z^2$.

6. Numerical Results and Analysis

The components of displacement, microrotation, stress and couple stress are calculated in the plane $z=1$ for three different values of $B_3(0.025, 0.050, 0.075)$ in the range $0 \le r \le 4$ and $a=1$, $A_2 = 0.015$, $A_3 = 0.01$, $B_4 = 0.015$, $B_5 = 0.005$.

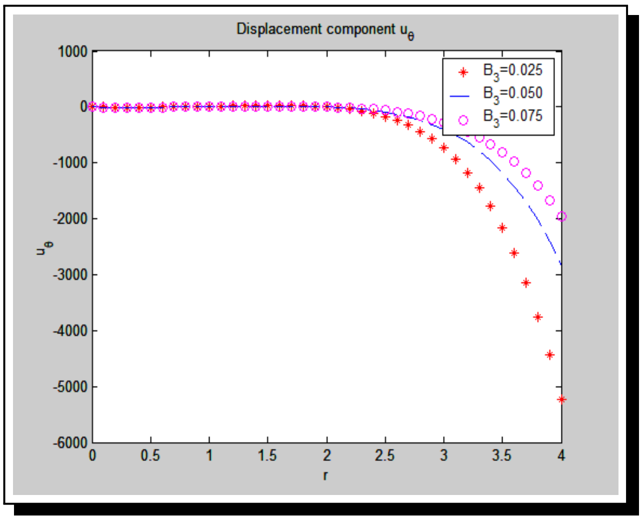

Figure 1

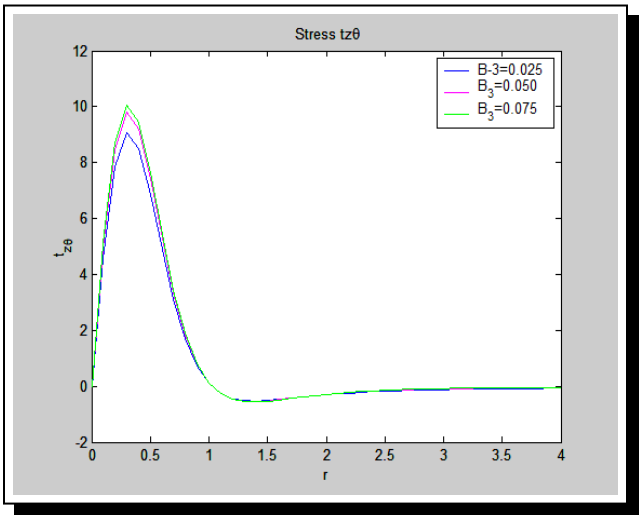

Figure 2

It is observed from Figure 1 that displacement u_θ curve is falling down when the distance from the origin $r > 2$. From Figure 2 it is clear that the stress component $t_{z\theta}$ increases rapidly when distance $r < 0.3$ then it decreases rapidly when $0.3 < r < 1$ and constant almost when $r > 1$.

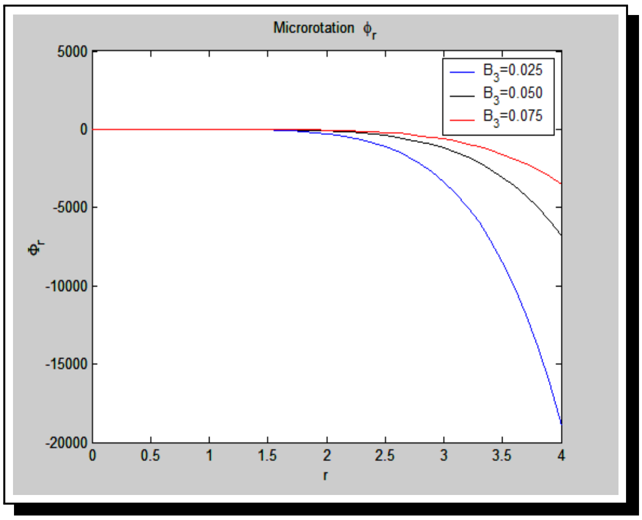

Figure 3

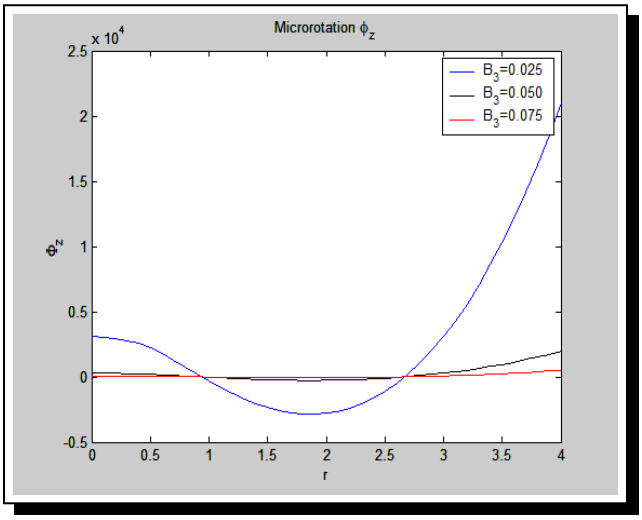

Figure 4

It is observed from Figure 3 that the microrotation component φ_r is constant for various values of B_3 when $r < 1.5$ and decreases gradually for $r > 1.5$. Figure 4 shows that microrotation component φ_z decreases when $r < 1.7$ and increases from there.

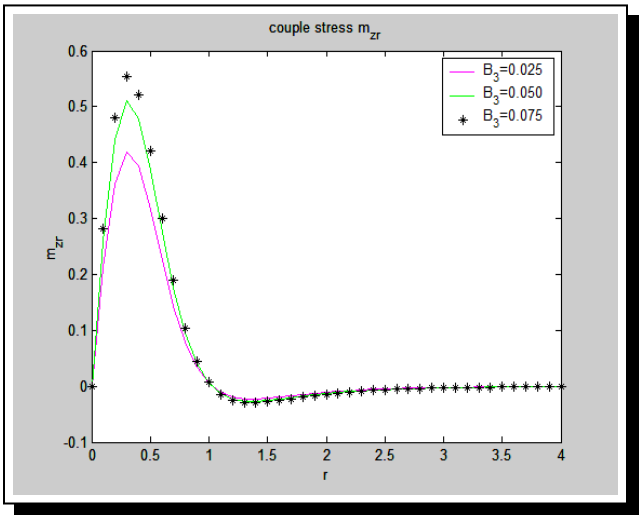

Figure 5

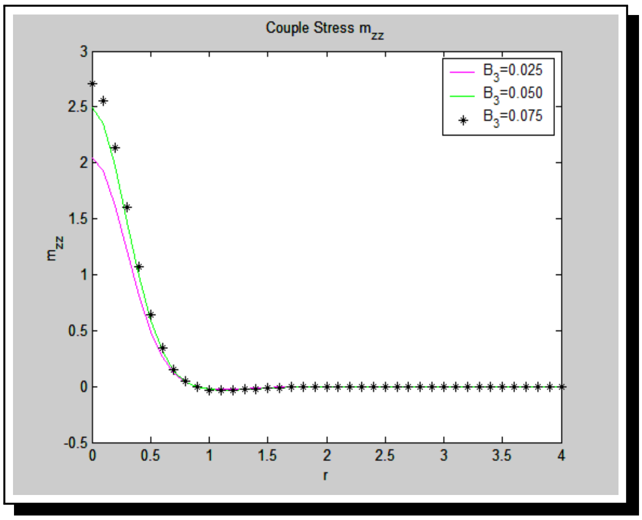

Figure 6

It is observed from Figure 5 that couple stress component m_{zr} rapidly increases $r < 0.3$ and rapidly decreases between $0.3 < r < 1$ and then it is constant almost when distance is greater than 1. Similarly other couple stress component m_{zz} decreases rapidly when $r < 1$ and almost constant from there onwards in Figure 6.

7. Evaluation of Micro-strains

Equations (16) to (18) can be written as

$$[\nabla^2 - l_1^2]\varnothing_{\theta\theta} = 0 \tag{73}$$

$$[\nabla^2 - l_2^2]\varnothing_{(r\theta)} = 0 \tag{74}$$

$$[\nabla^2 - l_2^2]\varnothing_{(z\theta)} = 0 \tag{75}$$

where $l_1^2 = \frac{A_4+2A_5}{B_1+2B_2}$ and $l_2^2 = \frac{A_5}{B_2}$.

The solutions of equations (73) to (75) can be assumed in the form of

$$\varnothing_{\theta\theta} = E\exp(-\xi z)J_1(l_1 r) \tag{76}$$

$$\varnothing_{(r\theta)} = F\exp(-\xi z)J_1(l_2 r) \tag{77}$$

$$\varnothing_{(z\theta)} = G\exp(-\xi z)J_1(l_2 r) \tag{78}$$

where E, F and G are arbitrary constants to be determined using the boundary conditions given in (37). Then we get

$$E = \frac{J_1(l_1 a)}{l_1 a J_0(l_1 a)} \quad \text{and} \quad F = G = \frac{J_1(l_2 a)}{l_2 a J_0(l_2 a)}$$

Figure 7 shows the curves of Micro-strains for $A_4 = 0.05$, $A_5 = 0.025$, $B_1 = 0.03$, $B_2 = 0.02$. It is observed from the graph that $\phi_{r\theta}$ and $\phi_{z\theta}$ are the same.

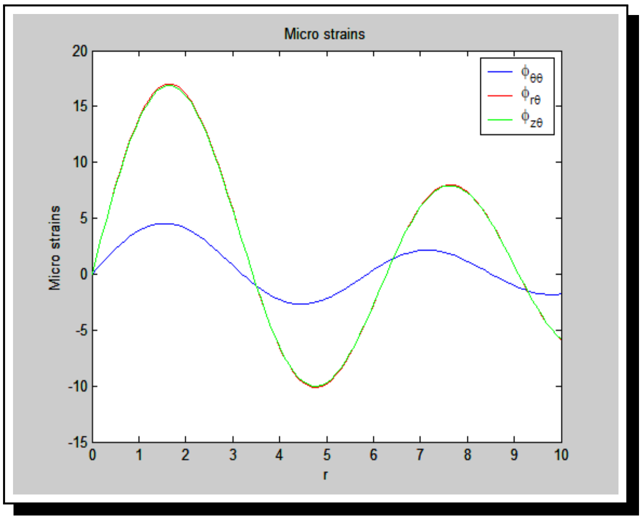

Figure 7

8. Conclusions

Thus various components have been calculated for the general torsional problem in Micro-isotropic, Micro-elastic solid. It is observed that for the taken twist except couple stress φ_z all the remaining components are decreasing. It is also observed that by assuming $A_2 = \frac{\mu}{2}$, $A_3 = \frac{\kappa}{2}$ and $B_3 = \frac{\gamma}{2}$, $B_4 = \frac{\beta}{2}$, $B_5 = \frac{\alpha}{2}$ the result of Kumar and Chadha [7] can be obtained. Again by assuming $\alpha \to 0$, $\beta \to 0$ and $\gamma \to 0$ the classical result can also be obtained.

Competing Interests

The author declares that she has no competing interests.

Authors' Contributions

The author wrote, read and approved the final manuscript.

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