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Research Article

Fixed Point Theorem With The CLR's Property and OWC Mappings in Menger Space

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Abstract. In this paper, we prove a fixed point theorem in Menger space by employing the conditions CLR's-property and occasionally weakly compatible mappings, which generalizes the theorem proved by Malviya et al. [5]. Further, this result is justified by a suitable example.

Keywords. CLRs-property; Occasionally weakly compatible mappings; Menger space

Mathematics Subject Classification (2020). 54H25; 47H10

1. Introduction

Menger [6] initiated the theory of Menger space in 1942. The conversion of probabilistic notion into geometry was one of the great efforts. Menger used the notation of new distance distribution function from p to q by Fpq. Schweizer and Sklar [1] introduced a new notion of a probabilistic-norm. This norm naturally generates topology, convergence, continuity and completeness in Menger space. Mishra [7] used compatible mappings and generated many fixed points in Menger space. Altumn and Turkoglu [3] proved some more results of Menger space by utilizing the implicit relation in multivalued mappings. Zhang et al. [12] contributed for the enrichment of Menger space in fixed point theory by employing Schweizer-Sklar t-norm established fuzzy logic

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system. Sehgal and Bharucha-Reid [10] obtained the first result relating theory of fixed points in complete Menger space by applying classical Banach contraction. Recently, Al-Thagafi and Shahzad [2] weakened the notion of weakly compatible mappings by introducing occasionally weakly compatible mappings. Further, Chauhan et al. [4] proved some more theorems by using CLR's-property in fuzzy metric space. Further some more results can be witnessed by using the concepts sub sequentially continuous and semi compatible mappings in Menger space [9], [11].

2. Preliminaries

Definition 2.1 ([5])**.** A continuously t-*norm* is mapping $t:[0,1]\times[0,1]\to[0,1]$ if it has properties:

(t_1) t is abelian and associative.

(t_2) $t(\gamma,1)=\gamma$, for all $\gamma\in[0,1]$.

(t_3) $t(\gamma,\omega)\le t(\alpha,\vartheta)$ for $\gamma\le\alpha$ and $\omega\le\vartheta$, for all $\gamma,\alpha,\vartheta,\omega\in[0,1]$.

Definition 2.2 ([5])**.** A distribution function $F:R\to R$ is non-decreasing and left continuous such that $\inf\{F(t):t\in R\}=0$ and $\sup\{F(t):t\in R\}=1$. We represent the set of all distribution functions as L.

Definition 2.3 ([5])**.** A Menger space is a pair (X,F) having $X\neq\emptyset$ and $F:X\times X\to t$ where L is the set of all distribution functions and the value of F at $(u,v)\in X\times X$ is represented by $F_{u,v}$ and obeys the following conditions:

(I) $F_{u,v}(\alpha)=1$, for all $\alpha>0$ iff $u=v$,

(II) $F_{u,v}(0)=0$,

(III) $F_{u,v}(\alpha)=F_{v,u}(\alpha)$,

(IV) if $F_{u,v}(\alpha)=1$ and $F_{v,w}(\beta)=1$ then $F_{u,w}(\alpha+\beta)=1$, for all u,v,w in X, $\alpha,\beta>0$.

Definition 2.4 ([5])**.** A Menger space is denoted by (X,F,t), in which (X,F) represent probabilistic metric space and t is the t-norm has property

$$F_{u,w}(\alpha+\beta)\ge t(F_{u,v}(\alpha),F_{v,w}(\beta)),\quad \text{for all } u,v,w\in X \text{ and } \alpha,\beta>0.$$

Definition 2.5 ([5])**.** A sequence $\langle x_n\rangle$ converge to β in Menger space (X,F,t) if and only if for each $\epsilon>0$, $t>0$, $\exists\ N(\epsilon)\in N$ implies $F_{x_n,\beta}(\epsilon)>1-t$, for all $n\ge N(\epsilon)$.

Definition 2.6 ([5])**.** A Menger space (X,F,t) is complete if every Cauchy sequence is convergent.

Definition 2.7 ([5])**.** Compatible self mappings P and S of a Menger space (X,F,t) are such that $F_{PSx_n,SPx_n}(\beta)\to 1$, for all $\beta>0$ whenever a sequence $\langle x_n\rangle$ in $X\ni Px_n$, $Sx_n\to z$ where z is an element of X as $n\to\infty$.

Definition 2.8 ([9]). Mappings $P : X \to X$, $S : X \to X$ of (X,F,t) are termed as weakly compatible if $Sx = Px \Rightarrow SPx = PSx$, for all $x \in X$.

Definition 2.9 ([2]). Mappings $P : X \to X$, $S : X \to X$ of (X,F,t) are known as OWC (Occasionally Weakly Compatible) if and only if there exists for some x in $X \ni Px = Sx$ implies $PSx = SPx$.

Clearly, two weakly compatible mappings implies OWC mappings but the reverse is need not be true.

Example 2.10. Consider $X = [0,1]$ and d be the metric on X and for each $t_1 \in [0,1]$.

Define

$$F_{u,v}(t_1) = \begin{cases} \dfrac{t_1}{t_1 + |\alpha - \beta|}, & t_1 > 0 \\ 0, & t_1 = 0 \end{cases} \quad \text{for all } \alpha, \beta \text{ in } X \text{ and } t_1 > 0.$$

Define mappings $P, S : X \to X$ as $P(x) = 2x^2$, $x \in [0,1]$ and $S(x) = \dfrac{x}{5}$, $x \in [0,1]$.

We observe that coincidence points for the pair (P,S) are 0, $\dfrac{1}{10}$.

At $x = \dfrac{1}{10}$, $P\left(\dfrac{1}{10}\right) = S\left(\dfrac{1}{10}\right)$ but not $PS\left(\dfrac{1}{10}\right) \neq SP\left(\dfrac{1}{10}\right)$.

At $x = 0$, $P(0) = S(0)$ and $PS(0) = SP(0)$.

Thus P, S are OWC but are non-weakly compatible mappings.

Definition 2.11 ([5]). Mappings $P : X \to X$, $S : X \to X$ of (X,F,t) are mentioned as reciprocally continuous if $PSx_n \to Pz$ and $SPx_n \to Sz$ whenever for the sequence $\langle x_n \rangle \in X$ such that $Px_n, Sx_n \to z$ for some element $z \in X$ as $n \to \infty$.

Definition 2.12 ([8]). Semi compatible self-mappings P, S of a Menger space (X,F,t) are such that if $F_{PSx_n,Sz}(\beta) \to 1$, for all $\beta > 0$ whenever $\langle x_n \rangle \in X$ such that $Px_n, Sx_n \to z$ for some element $z \in X$ as $n \to \infty$.

Definition 2.13 ([4]). Self mappings P and S of a Menger space (X,F,t) are said to satisfying CLR's-property (Common Limit Range property) if there exists a sequence $\langle x_n \rangle \in X \ni Px_n$, $Sx_n \to Sz$, for some element $z \in X$ as $n \to \infty$.

This example shows that the mappings P, S are satisfying CLR's-property but neither semi compatible nor reciprocally continuous.

Example 2.14. Consider $X = [0,2]$ and d is the metric on X, $t \in [0,1]$. Define

$$F_{u,v}(t) = \begin{cases} \dfrac{t}{t + |\alpha - \beta|}, & t > 0 \\ 0, & t = 0 \end{cases} \quad \text{for all } \alpha, \beta \text{ in } X \text{ and } t > 0.$$

Define $P,S : X \to X$ as

$$P(x) = \begin{cases} 1-x, & x \in \left(0, \dfrac{2}{3}\right) \\ x, & x \in \left[\dfrac{2}{3}, 1\right] \end{cases}$$

and

$$S(x) = \begin{cases} 2x, & x \in \left[0, \dfrac{2}{3}\right] \\ 1, & x \in \left(\dfrac{2}{3}, 1\right] \end{cases}$$

Consider a sequence $x_n = \frac{1}{3} - \frac{1}{3n}$ for $n = 1,2,3,\dots$ then

$$Px_n = 1 - \left(\frac{1}{3} - \frac{1}{3n}\right) = \frac{2}{3} + \frac{1}{3n} \to \frac{2}{3}$$

and

$$Sx_n = 2\left(\frac{1}{3} - \frac{1}{3n}\right) = \frac{2}{3} - \frac{2}{9n} \to \frac{2}{3} \text{ as } n \to \infty.$$

$$Px_n, Sx_n \to S\left(\frac{1}{3}\right) = \frac{2}{3} \text{ as } n \to \infty.$$

Thus the mappings P, S are satisfying CLR's-property.

Also

$$PSx_n = PS\left(\frac{1}{3} - \frac{1}{3n}\right) = P\left(\frac{2}{3} - \frac{2}{9n}\right) = 1 - \left(\frac{2}{3} - \frac{1}{3n}\right) = \frac{1}{3} + \frac{1}{3n} \to \frac{1}{3} \neq \frac{4}{3} = S\left(\frac{2}{3}\right) \text{ as } n \to \infty.$$

Thus the pair (P,S) is not semi compatible.

Further $SPx_n = SP\left(\frac{1}{3} - \frac{1}{3n}\right) = S\left(\frac{2}{3} + \frac{1}{3n}\right) = 1 \to 1$ as $n \to \infty$.

This implies $PSx_n \to \frac{1}{3} \neq \frac{2}{3} = P\left(\frac{2}{3}\right)$ but $SPx_n \to 1 \neq \frac{4}{3} = S\left(\frac{2}{3}\right)$ when $n \to \infty$.

Thus the pair (P,S) is not reciprocally continuous mappings.

Lemma 2.15 ([5])**.** *Let is Menger space (X,F,t) with continuous t-norm if we can find a constant $q \in (0,1)$ such that $F_{u,v}(qt) \geq F_{u,v}(t)$ for all u,v in X and $t > 0$ then $u = v$.*

Lemma 2.16 ([9])**.** *Let (X,F,t) be a Menger space with continuous t-norm $t(\omega,\omega) \geq \omega$ for all $\omega \in [0,1]$, if there exists a constant $\theta \in (0,1)$ such that $F_{u_n,u_{n+1}}(\theta t) \geq F_{u_{n-1},u_n}(t)$, $n = 1,2,3,\dots$ then $\langle u_n \rangle$ is a Cauchy sequence in X.*

The following theorem was proved by P. Malviya et al. in [5].

Theorem 2.17. *Let P, Q, S and T be self mappings on a complete Menger space (X,F,t) with continuous t-norm $t(c,c) \geq c$, for $c \in (0,1)$ satisfying*

(2.3.1) $P(X) \subseteq T(X)$, $Q(X) \subseteq S(X)$.

(2.3.2) (Q,T) *is weakly compatible.*

(2.3.3) $F_{P\alpha\cdot Q\beta}(hx) \geq \min\{F_{S\alpha\cdot T\beta}(x), \{F_{S\alpha\cdot P\alpha}(x)\cdot F_{Q\beta\cdot T\beta}(x)\}, F_{P\alpha\cdot S\alpha}(x)\}$ *for all* α, β *in* X, *and* $h > 1$.

(2.3.4) (P,S) *is semi compatible pair of reciprocally continuous mappings then* P, Q, S *and* T *have unique common fixed point.*

Now, we generalize Theorem 2.17 in the following way.

3. Main Result

Theorem 3.1. *Let* P, Q, S *and* T *be self-mappings on a complete Menger space* (X,F,t) *with continuous* t*-norm* $t(c,c) \geq c$ *for* $c \in (0,1)$ *satisfying:*

(3.1.1) $P(X) \subseteq T(X)$, $Q(X) \subseteq S(X)$

(3.1.2) (Q,T) *is occasionally weakly compatible*

(3.1.3) $F_{P\alpha\cdot Q\beta}(hx) \geq \min\{F_{S\alpha\cdot T\beta}(x), \{F_{S\alpha\cdot P\alpha}(x)\cdot F_{Q\beta\cdot T\beta}(x)\}, F_{P\alpha\cdot S\alpha}(x)\}$ *for all* α, β *in* X, *and* $h > 1$

(3.1.4) *the pair* (P,S) *satisfies CLRs-property*

then P, Q, S *and* T *have unique common fixed point.*

Proof. From (3.1.1), we can construct a sequence $\langle y_n\rangle$ for $n \geq 1$ such that

$$\langle y_{2n}\rangle = Px_{2n} = Tx_{2n+1} \quad\text{and}\quad \langle y_{2n+1}\rangle = Qx_{2n+1} = Sx_{2n+2}.$$

Now our claim $\langle y_n\rangle$ is Cauchy sequence.

For this take $\alpha = x_{2n+1}$, $\beta = x_{2n+2}$ in (3.1.2) we get

$$F_{Px_{2n+1},Qx_{2n+2}}(hx) \geq \min\{F_{Sx_{2n+1}\cdot Tx_{2n+2}}(x), \{F_{Sx_{2n+1}\cdot Px_{2n+1}}(x)\cdot F_{Qx_{2n+2}\cdot Tx_{2n+2}}(x)\}, F_{Px_{2n+1}\cdot Sx_{2n+1}}(x)\} \quad \text{as } n \to \infty.$$

This gives

$$F_{y_{2n+1}\cdot y_{2n+2}}(hx) \geq \min\{F_{y_{2n},y_{2n+1}}(x), \{F_{y_{2n},y_{2n+1}}(x)\cdot F_{y_{2n+2},y_{2n+1}}(x)\}, F_{y_{2n+1},y_{2n}}(x)\}.$$

This results

$$F_{y_{2n+1}\cdot y_{2n+2}}(hx) \geq F_{y_{2n},y_{2n+1}}(x).$$

Similarly

$$F_{y_{2n+2}\cdot y_{2n+3}}(hx) \geq F_{y_{2n+1},y_{2n+2}}(x).$$

In general we have $F_{y_{n+1}\cdot y_n}(hx) \geq F_{y_n,y_{n-1}}(x)$ for $n \geq 1$.

By applying Lemma 2.16, then $\langle y_n\rangle$ is cauchy sequence in complete space X so it has limit point $z \in X$ consequently each subsequence has the same limit point z.

Since the pair (P,S) satisfied CLRs-Property implies a sequence $\langle x_n\rangle$ such that

$$Px_n, Sx_n \to Sz \quad \text{for some } z \text{ in } X \text{ as } n \to \infty.$$

Claim $z = Sz$.

Take $\alpha = x_n$, $\beta = x_{2n+1}$ in (3.1.3), we get

$$F_{Px_n \cdot Qx_{2n+1}}(hx) \geq \min\{F_{Sx_n \cdot Tx_{2n+1}}(x), \{F_{Sx_n \cdot Px_n}(x) \cdot F_{Qx_{2n+1} \cdot Tx_{2n+1}}(x)\}, F_{Px_n \cdot Sx_n}(x)\} \text{ as } n \to \infty.$$

$$F_{Sz \cdot z}(hx) \geq \min\{F_{Sz,z}(x), \{F_{Sz,Sz}(x) \cdot F_{z \cdot z}(x)\}, F_{Sx,Sz}(x)\}.$$

This results

$$F_{Sz \cdot z}(hx) \geq F_{Sz \cdot z}(x).$$

By using Lemma 2.15, implies $z = Sz$.

We claim that $z = Pz$.

Take $\alpha = z$, $\beta = x_{2n+1}$ in (3.1.3), we get

$$F_{Pz \cdot Qx_{2n+1}}(hx) \geq \min\{F_{Sz \cdot Tx_{2n+1}}(x), \{F_{Sz \cdot Pz}(x) \cdot F_{Qx_{2n+1} \cdot Tx_{2n+1}}(x)\}, F_{Pz \cdot Sz}(x)\} \text{ as } n \to \infty.$$

$$F_{Pz \cdot z}(hx) \geq \min\{F_{z,z}(x), \{F_{z,Pz}(x) \cdot F_{z \cdot z}(x)\}, F_{Pz,z}(x)\}.$$

This gives

$$F_{Pz \cdot z}(hx) \geq F_{Pz,z}(x).$$

By applying Lemma 2.15, we obtain $z = Pz$ implies $z = Pz = Sz$. Now, $z = Pz \in P(X) \subseteq T(X)$ by (3.1.1), there exists some $u \in X$ such that $z = Pz = Tu$.

Now we claim $z = Qu$.

Take $\alpha = z$, $y = u$ in (3.1.3), gives

$$F_{Pz,Qu}(hx) \geq \min\{F_{Sz \cdot Tu}(x), \{F_{Sz \cdot Pz}(x) \cdot F_{Qu \cdot Tu}(x)\}, F_{Pz \cdot Sz}(x)\}.$$

By using $z = Pz = Sz = Tu$ we get

$$F_{z \cdot Qu}(hx) \geq \min\{F_{z,z}(x), \{F_{z,z}(x) \cdot F_{Qu \cdot z}(x)\}, F_{z,z}(x)\}$$

this gives

$$F_{z \cdot Qu}(hx) \geq F_{z \cdot Qu}(x).$$

By Lemma 2.15, we get $z = Qu$.

This gives $z = Qu = Tu$.

Again since the pair (Q,T) is OWC means if u in X such that $Qu = Tu$ implies $QTu = TQu$ and this results $Qz = Tz$.

Claim $z = Qz$.

Take $\alpha = z$, $y = z$ in (3.1.3), we get

$$F_{Pz,Qz}(hx) \geq \min\{F_{Sz \cdot Tz}(x), \{F_{Sz \cdot Pz}(x) \cdot F_{Qz \cdot Tz}(x)\}, F_{Pz \cdot Sz}(x)\}.$$

By using $z = Pz = Sz$, $Qz = Tz$ we get

$$F_{z \cdot Qz}(hx) \geq \min\{F_{z,z}(x), \{F_{z,z}(x) \cdot F_{Qz \cdot Qz}(x)\}, F_{z,z}(x)\}.$$

This gives

$$F_{z,Qz}(hx) \geq F_{z,z}(x).$$

By Lemma 2.15, we get $z = Qz$.

Thus $z = Pz = Sz = Qz = Tz$.

Therefore z is a common fixed point for the mappings P, Q, S and T.

Uniqueness

If possible z_1 is another fixed point for the mappings P, Q, S and T then $z_1 = Pz_1 = Sz_1 = Qz_1 = Tz_1$.

Put $\alpha = z$, $\beta = z_1$ in (3.1.3), we have

$$F_{Pz,Qz_1}(hx) \geq \min\{F_{Sz\cdot Tz}(x), \{F_{Sz\cdot Pz}(x)\cdot F_{Qz_1,Tz_1}(x)\}, F_{Pz\cdot Sz}(x)\}.$$

This results

$$F_{z\cdot z_1}(hx) \geq \min\{F_{z\cdot z}(x), \{F_{z\cdot z}(x)\cdot F_{z_1\cdot z_1}(x)\}, F_{z\cdot z}(x)\}.$$

This results $F_{z\cdot z_1}(hx) \geq 1$ and this gives $F_{z\cdot z_1}(hx) = 1$.

This implies $z = z_1$.

Therefore z is the unique common fixed point of the four mappings P, Q, S and T. □

Now we provide a supporting example to justify the theorem.

4. Example

Consider $X = [0,1]$ is general metric on X and each $t \in [0,1]$.

Define

$$F_{u,v}(t) = \begin{cases} \dfrac{t}{t+|\alpha-\beta|}, & t > 0 \\ 0, & t = 0 \end{cases} \quad \text{for all } \alpha, \beta \text{ in } X,\ t > 0.$$

Define mappings P, S, T and $Q : X \to X$ as

$$P(x) = Q(x) = \begin{cases} 1-3x, & x \in \left[0, \frac{1}{3}\right] \\ \frac{4}{5}, & x \in \left(\frac{1}{3}, \frac{2}{5}\right) \\ x, & x \in \left[\frac{2}{5}, 1\right] \end{cases}$$

and

$$S(x) = T(x) = \begin{cases} 2x, & x \in \left[0, \frac{1}{3}\right] \\ \frac{3}{5}, & x \in \left(\frac{1}{3}, \frac{2}{5}\right] \\ x^2, & x \in \left(\frac{2}{5}, 1\right] \end{cases}$$

Now $P(X) = Q(X) = [0,1]$ and $S(X) = T(X) = [0,1]$.

So that $P(X) \subseteq T(X)$ and $Q(X) \subseteq S(X)$.

Clearly, $\frac{1}{5}$ and 1 are coincidence points for the mappings Q, T.

At $x=\frac{1}{5}$, $Q\left(\frac{1}{5}\right)=T\left(\frac{1}{5}\right)$ but $QT\left(\frac{1}{5}\right)=Q\left(\frac{2}{5}\right)=\frac{2}{5}\neq\frac{3}{5}=T\left(\frac{2}{5}\right)=TQ\left(\frac{1}{5}\right)$.

At $x=1$, $Q(1)=T(1)$ and $QT(1)=Q(1)=1=T(2)=TQ(1)$.

Thus Q, T are OWC mappings but are not weakly compatible.

If $x_n=1-\frac{2}{3n}$ for $n\geq 1$, then

$$Px_n=P\left(1-\frac{2}{3n}\right)=1-\frac{2}{3n}\to 1$$

and

$$Sx_n=S\left(1-\frac{2}{3n}\right)=\left(1-\frac{2}{3n}\right)^2\to 1 \text{ as } n\to\infty.$$

This implies Px_n, $Sx_n\to S(1)$ as $n\to\infty$.

This gives pair (P,S) satisfies CLR's-property.

Further if consider a sequence $x_n=\frac{1}{5}-\frac{1}{3n}$ for $n\geq 1$, then

$$Px_n=P\left(\frac{1}{5}-\frac{1}{3n}\right)=1-3\left(\frac{1}{5}-\frac{1}{3n}\right)=\frac{2}{5}+\frac{1}{3n}\to\frac{2}{5}$$

and

$$Sx_n=S\left(\frac{1}{5}-\frac{1}{3n}\right)=2\left(\frac{1}{5}-\frac{1}{3n}\right)=\frac{2}{5}-\frac{2}{3n}\to\frac{2}{5} \text{ as } n\to\infty.$$

Now

$$PSx_n=PS\left(\frac{1}{5}-\frac{1}{3n}\right)=P\left(\frac{2}{5}-\frac{2}{3n}\right)=\frac{4}{5}\to\frac{4}{5}$$

and also

$$SPx_n=SP\left(\frac{1}{5}-\frac{1}{3n}\right)=S\left(\frac{2}{5}+\frac{1}{3n}\right)=\left(\frac{2}{5}+\frac{1}{3n}\right)^2\to\frac{4}{25} \text{ as } n\to\infty.$$

$$P\left(\frac{2}{5}\right)=\frac{2}{5},\quad S\left(\frac{2}{5}\right)=\frac{3}{5}.$$

Thus $Px_n,Sx_n\to S\left(\frac{1}{5}\right)=\frac{2}{5}$ as $n\to\infty$.

Thus pair (P,S) satisfies CLRs-property

$$PSx_n\to\frac{4}{5}\neq\frac{2}{5}=S\left(\frac{2}{5}\right) \text{ as } n\to\infty.$$

So the mappings P, S are not semi compatible

$$PSx_n\to\frac{4}{5}\neq\frac{2}{5}=P\left(\frac{2}{5}\right)$$

and

$$SPx_n\to\frac{4}{25}\neq\frac{3}{5}=S\left(\frac{2}{5}\right) \text{ as } n\to\infty.$$

Thus mappings P and S not reciprocally continuous.

Now, we conclude that the pair (P,S) satisfies CLRs-property but neither semi compatible nor reciprocally continuous mappings.

Thus the mappings P, Q, S and T satisfied all the conditions in Theorem 3.1 and containing the unique common fixed point at 1.

5. Conclusion

We proved fixed point theorem using weaker condition as the pair (Q,T) occasionally weakly compatible instead of weakly compatible and the pair (P,S) CLRs-property instead of semi-compatible reciprocally continuous.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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