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Research Article

Best Proximity Points for Cyclic Contractions in CAT(0) Spaces

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Abstract. In this manuscript, we establish best proximity point results for some cyclic contraction maps. We discuss the existence and convergence of best proximity point results for such maps in CAT(0) spaces.

Keywords. Best proximity point; Cyclic contractions; CAT(0) spaces

Mathematics Subject Classification (2020). 47H10; 54H25

1. Introduction

In the classical Banach fixed point theorem, the undertaking operator is necessarily continuous due to contraction inequality. This simple fact brings a natural question: Does a discontinuous contraction mapping possess a fixed point? The answer to this question is affirmative. Indeed, there are various approaches to overcome weakness of the discontinuous mapping for guaranteeing a fixed point. One of the significant results was constructed by Bryant [5] who proved the following result: In a complete metric space, if, for some positive integer $n \geq 2$, the nth iteration of the given mapping forms a contraction, then it possess a unique fixed point. Another outstanding approach was proposed by Kirk, Srinivasan and Veeramani [13] by introducing the notion of cyclic contraction. More precisely, every cyclic contraction in a complete metric space possess a unique fixed point. This statement is plain but significant when we compare with the results of Bryant. A considerable number of authors have investigated densely the concept of the cyclic contractions and bring a variety of the notion and derive a number of interesting results (see, e.g., [1,2,10,11,14–20,22,25–28] and the references therein). Let there be a self-mapping on a metric space (X,d). Suppose that A and B are non-empty subsets of X such that $X = A\cup B$. A self-mapping T on $A\cup B$ is called cyclic [13] if

$$T(A)\subseteq B \text{ and } T(B)\subseteq A.$$

Further, a mapping T is called cyclic contraction [13] if there is a $k\in[0,1)$ such that the following inequality is satisfied:

$$d(Tx,Ty)\le kd(x,y), \quad \text{for all } x\in A, y\in B.$$

After this initial construction, several extensions of cyclic mappings and cyclic contractions have been introduced. In this paper, we follow the notations defined in [11,24].

Let $[0,1]$ be a closed interval in $\mathbb{R}$ and x,y be two points in a metric space (X,d). A geodesic joining x to y is a map $\eta:[0,1]\to X$ such that $\eta(0)=x$, $\eta(1)=y$ and $d(\eta(s),\eta(t))=|s-t|$ for all $s,t\in[0,1]$. The image of η is called a geodesic segment joining x and y which when unique is denoted by $[x,y]$. The space (X,d) is said to be a geodesic space if every two points in X are joined by a geodesic, and X is said to be uniquely geodesic if there is exactly one geodesic joining x and y for each $x,y\in X$. A subset E of X is said to be convex if every pair of points $x,y\in E$ can be joined by a geodesic in X and the image of every such geodesic is contained in E.

A *geodesic triangle* $\Delta(p,q,r)$ in a geodesic space (X,d) consists of three points p,q,r in X and a choice of three geodesic segments $[p,q],[q,r],[r,p]$ joining them. A *comparison triangle* for geodesic triangle $\Delta(p,q,r)$ in X is a triangle $\bar\Delta(\bar p,\bar q,\bar r)$ in the Euclidean plane $\mathbb{R}^2$ such that $d_{\mathbb{R}^2}(\bar p,\bar q)=d(p,q)$, $d_{\mathbb{R}^2}(\bar q,\bar r)=d(q,r)$, and $d_{\mathbb{R}^2}(\bar r,\bar p)=d(r,p)$.

A point $\bar u\in[\bar p,\bar q]$ is called a comparison point for $u\in[p,q]$ if $d(p,u)=d_{\mathbb{R}^2}(\bar p,\bar u)$. Comparison points on $[\bar q,\bar r]$ and $[\bar r,\bar p]$ are defined in the same way.

Definition 1.1. A *geodesic triangle* $\Delta(p,q,r)$ in (X,d) is said to satisfy the CAT(0) *inequality* if for any $u,v\in\Delta(p,q,r)$ and for their comparison points $\bar u,\bar v\in\bar\Delta(\bar p,\bar q,\bar r)$, one has

$$d(u,v)\le d_{\mathbb{R}^2}(\bar u,\bar v).$$

A geodesic space X is said to be a CAT(0) space if all of its geodesic triangles satisfy the CAT(0) inequality. For other equivalent definitions and basic properties of CAT(0) spaces, we refer the readers to standard texts such as [3,7]. It is well known that every CAT(0) space is uniquely geodesic. Notice also that pre-Hilbert spaces, $\mathbb{R}$-trees, and Euclidean buildings are examples of CAT(0) spaces (see [3,4]).

It is well known that if x,y_1,y_2 are points of CAT(0) space and if y_0 is the midpoint of the segment $[y_1,y_2]$ (that is $y_0=\frac{1}{2}y_1\oplus\frac{1}{2}y_2$), then the CAT(0) inequality implies

$$d(x,y_0)^2\le\frac{1}{2}d(x,y_1)^2+\frac{1}{2}d(x,y_2)^2-\frac{1}{4}d(y_1,y_2)^2 \tag{1}$$

because equality holds in the Euclidean metric. In fact (see [23]), a geodesic metric space is a CAT(0) space if and only if it satisfies inequality (1) above. This inequality is known as the CN inequality of Bruhat and Tits [6]. Some interesting known results in CAT(0) spaces can be found in [8, 9, 21, 24].

2. Preliminaries

In this section we recollect some basic definitions and notions which will be useful and related to our main results.
Define

(1) $P_A(x) = \{y \in X : d(x,y) = d(x,A)\}$;

(2) $dist(A,B) = \inf\{d(x,y) : x \in A, y \in B\}$;

(3) $A_0 = \{x \in A : d(x,b) = dist(A,B)$ for some $b \in B\}$;

(4) $B_0 = \{y \in B : d(a,y) = dist(A,B)$ for some $a \in A\}$.

There are some sufficient conditions which guarantee that A_0 and B_0 are not empty. One such simple condition is that A is compact and B is approximatively compact with respect to A (that is, every sequence $\{x_n\}$ of B such that $d(y,x_n) \to dist(A,B)$ for some y in A should have a convergent subsequence).

The following lemma gives another set of sufficient conditions in reflexive Banach spaces.

Lemma 2.1 (see [12])**.** *Let X be a reflexive Banach space, let A be a nonempty closed, bounded and convex subset of X and let B be a nonempty closed, convex subset of X. Then A_0 and B_0 are nonempty and satisfy $P_B(A_0) \subseteq B_0$ and $P_A(B_0) \subseteq A_0$.*

Definition 2.2 (see [11])**.** A subset K of a metric space X is boundedly compact if each bounded sequence in K has a subsequence converging to a point in K.

Definition 2.3 (see [11])**.** Let A and B be nonempty subsets of a metric space X. A map $T : A \cup B \to A \cup B$ is called a cyclic contraction map if it satisfies:

(1) $T(A) \subseteq B$ and $T(B) \subseteq A$.

(2) For some $k \in (0,1)$ we have $d(Tx,Ty) \le kd(x,y) + (1-k)dist(A,B)$, for all $x \in A, y \in B$.

The main results obtained in [11] are as follows.

Proposition 2.4 (see [11])**.** *Let A and B be nonempty subsets of a metric space X. Suppose $T : A \cup B \to A \cup B$ is a cyclic contraction map. Then starting with any x_0 in $A \cup B$ we have $d(x_n, Tx_n) \to dist(A,B)$, where $x_{n+1} = Tx_n$, $n = 0,1,2,\dots$.*

Proposition 2.5 (see [11])**.** *Let A and B be nonempty subsets of a complete metric space X. Let $T : A \cup B \to A \cup B$ be a cyclic contraction map, let x_0 in A and define $x_{n+1} = Tx_n$, Suppose $\{x_{2n}\}$ has a convergent subsequence in A. Then there exists x in A such that $d(x,Tx) = dist(A,B)$.*

Proposition 2.6 (see [11])**.** *Let A and B be nonempty subsets of a complete metric space X. Let $T : A \cup B \to A \cup B$ be a cyclic contraction map. Then there exists x_0 in $A \cup B$ and $x_{n+1} = Tx_n$, $n = 0,1,2,\dots$, the sequences $\{x_{2n}\}$ and $\{x_{2n+1}\}$ are bounded.*

Theorem 2.7 (see [11]). *Let A and B be nonempty subsets of a metric space (X,d). Let $T: A\cup B \to A\cup B$ be a cyclic contraction map. If either A and B is boundedly compact, then there exists x in $A\cup B$ with $d(x,Tx) = dist(A,B)$.*

Theorem 2.8 (see [11]). *Let A,B be nonempty closed and convex subsets of a uniformly convex Banach space. Suppose $T: A\cup B \to A\cup B$ is a cyclic contraction map, then there exists a unique best proximity point x in A (that is with $\|x-Tx\| = dist(A,B)$). Further, if $x_0 \in A$ and $x_{n+1} = Tx_n$, then $\{x_{2n}\}$ converges to the best proximity point x.*

In this manuscript, motivated and inspired by those above results, we extend and improve the results obtained in [11] to a CAT(0) space.

3. Main Results

In this section we mainly follow the work of Eldred and Veeramani in [11]. We similarly introduce the following definition in a CAT(0) space and give an approximation result.

Definition 3.1. Let A and B be nonempty subsets of a CAT(0) space X. A map $T: A\cup B \to A\cup B$ is called a cyclic contraction map if it satisfies:

(1) $T(A)\subseteq B$ and $T(B)\subseteq A$.

(2) For some $\alpha\in(0,1)$ we have $d(Tx,Ty) \le \alpha d(x,y) + (1-\alpha)dist(A,B)$, for all $x\in A, y\in B$.

Proposition 3.2. *Let A and B be nonempty subsets of a CAT(0) space X. Suppose $T: A\cup B \to A\cup B$ is a cyclic contraction map. Then starting with any x_0 in $A\cup B$ we have $d(x_n,Tx_n)\to dist(A,B)$, where $x_{n+1} = Tx_n$, $n = 0,1,2,\dots$.*

Proof. Now, let $w_n = \frac{1}{2}x_n \oplus \frac{1}{2}x_{n+1} \in A$. By CN inequality and Definition 3.1, we have

$$\begin{aligned}
d^2(x_n,x_{n+1}) &\le 2d^2(x_n,x_{n+2}) + 2d^2(x_{n+1},x_{n+2}) - 4d^2(w_n,x_{n+2})\\
&\le 2d^2(Tx_{n-1},Tx_{n+1}) + 2d^2(Tx_n,Tx_{n+1}) - 4dist^2(A,B)\\
&\le 2[\alpha d(x_{n-1},x_{n+1}) + (1-\alpha)dist(A,B)]^2\\
&\quad + 2[\alpha d(x_n,x_{n+1}) + (1-\alpha)dist(A,B)]^2 - 4dist^2(A,B)\\
&\le 2[\alpha(\alpha d(x_{n-2},x_n) + (1-\alpha)dist(A,B)) + (1-\alpha)dist(A,B)]^2\\
&\quad + 2[\alpha(\alpha d(x_{n-1},x_n) + (1-\alpha)dist(A,B)) + (1-\alpha)dist(A,B)]^2 - 4dist^2(A,B)\\
&= 2[\alpha^2 d(x_{n-2},x_n) + (1-\alpha^2)dist(A,B)]^2\\
&\quad + 2[\alpha^2 d(x_{n-1},x_n) + (1-\alpha^2)dist(A,B)]^2 - 4dist^2(A,B).
\end{aligned}$$

Inductively, we have

$$\begin{aligned}
d^2(x_n,x_{n+1}) &\le 2[\alpha^n d(x_0,x_2) + (1-\alpha^n)dist(A,B)]^2\\
&\quad + 2[\alpha^n d(x_1,x_2) + (1-\alpha^n)dist(A,B))]^2 - 4dist^2(A,B)\\
&\le 2[\alpha^n d(x_0,x_2) + (1-\alpha^n)dist(A,B)]^2\\
&\quad + 2[\alpha^n d(x_1,x_2) + (1-\alpha^n)dist(A,B))]^2 - 3dist^2(A,B).
\end{aligned}$$

This implies, $d^2(x_n,Tx_n)\to dist^2(A,B)$. Therefore

$$d(x_n,Tx_n)\to dist(A,B).$$

□

Now, we give a simple existence result for best proximity point.

Proposition 3.3. *Let A and B be nonempty subsets of a complete CAT(0) space X. Let $T: A\cup B\to A\cup B$ be a cyclic contraction map, let x_0 in A and define $x_{n+1}=Tx_n$. Suppose $\{x_{2n}\}$ has a convergent subsequence in A. Then there exists x in A such that $d(x,Tx)=dist(A,B)$.*

Proof. Let $\{x_{2n_k}\}$ be subsequence of $\{x_{2n}\}$ converging to some $x\in A$. Now let $w_n=\frac{1}{2}x\oplus\frac{1}{2}x_{2n_k-1}\in A$, by CN inequality and Proposition 3.2, we have

$$\begin{aligned} dist^2(A,B)\le d^2(x,x_{2n_k-1}) &\le 2d^2(x,x_{2n_k})+2d^2(x_{2n_k},x_{2n_k-1})-4d^2(w_n,x_{2n_k})\\ &\le 2d^2(x,x_{2n_k})+2d^2(Tx_{2n_k-1},x_{2n_k-1})-dist^2(A,B).\end{aligned}$$

This implies

$$d^2(x,x_{2n_k-1})\to dist^2(A,B).$$

Hence

$$d(x,x_{2n_k-1})\to dist(A,B).$$

Since

$$dist(A,B)\le d(x_{2n_k},Tx)\le d(x_{2n_k-1},x),$$

we have

$$d(x,Tx)=dist(A,B).$$

□

Next, we investigate our main results of this paper to give an existence and convergence theorem for best proximity points. We state the convergence lemma which forms the basis for our main result.

Lemma 3.4. *Let A be a nonempty closed and convex subset and B be a nonempty closed subset of a CAT(0) space. Let $\{x_n\}$ and $\{z_n\}$ be sequences in A and $\{y_n\}$ be a sequence in B satisfying:*

(i) $d(z_n,y_n)\to dist(A,B.)$

(ii) *For every $\epsilon>0$ there exists N_0 such that for all $m>n\ge N_0$, $d(x_m,y_n)\le dist(A,B)+\epsilon$.*

Then, for every $\epsilon>0$ there exists N_1 such that for all $m>n\ge N_1$, $d(x_m,z_n)\le\epsilon$.

Proof. Given $\epsilon>0$, choose N_0 from our assumptions, such that

$$d^2(x_m,y_n)<dist^2(A,B)+\frac{\epsilon^2}{4},\quad m>n\ge N_0. \tag{2}$$

And choose $N_1>N_0$ such that

$$d^2(z_n,y_n)<dist^2(A,B)+\frac{\epsilon^2}{4},\quad n\ge N_1. \tag{3}$$

Let $w_n=\frac{1}{2}x_m\oplus\frac{1}{2}z_n$, by CN inequatlity, we have

$$d^2(w_n,z_n)\le\frac{1}{2}d^2(x_m,y_n)+\frac{1}{2}d^2(z_n,y_n)-\frac{1}{4}d^2(x_m,z_n).$$

Equivalently,

$$d^2(x_m,z_n) \le 2d^2(x_m,y_n)+2d^2(z_n,y_n)-4d^2(w_n,y_n),$$
$$\le 2d^2(x_m,y_n)+2d^2(z_n,y_n)-4dist^2(A,B).$$

From (2), (3), and above inequality, we have

$$d^2(x_m,z_n) \le \left(2dist^2(A,B)+\frac{\epsilon^2}{2}\right)+\left(2dist^2(A,B)+\frac{\epsilon^2}{2}\right)-4dist^2(A,B) \le \epsilon^2.$$

Hence

$$d(x_m,z_n) \le \epsilon.$$ □

Lemma 3.5. *Let A be a nonempty closed and convex subset and B be a nonempty closed subset of a CAT(0) space. Let $\{x_n\}$ and $\{z_n\}$ be sequence in A and $\{y_n\}$ be a sequence in B satisfying:*

(i) $d(x_n,y_n) \to dist(A,B)$.

(ii) $d(z_n,y_n) \to dist(A,B)$.

Then

$$d(x_n,z_n) \to 0.$$

Proof. Let $w_n = \frac{1}{2}x_n \oplus \frac{1}{2}z_n$. Then, by CN inequality, we have

$$d^2(w_n,y_n) \le \frac{1}{2}d^2(x_n,y_n)+\frac{1}{2}d^2(z_n,y_n)-\frac{1}{4}d^2(x_n,z_n).$$

We can rewrite it as

$$d^2(x_n,z_n) \le 2d^2(x_n,y_n)+2d^2(z_n,y_n)-4d^2(w_n,z_n)$$
$$\le 2d^2(x_n,y_n)+2d^2(z_n,y_n)-4dist^2(A,B).$$

By our assumptions, we have

$$d^2(x_n,z_n) \to 0.$$

Hence

$$d(x_n,z_n) \to 0.$$ □

Corollary 3.6. *Let A be a nonempty closed and convex subset and B be a nonempty closed subset of a CAT(0) space. Let $\{x_n\}$ be a sequence in A and $y_0 \in B$ such that $d(x_n,y_0) \to dist(A,B)$. Then $x_n \to P_A(y_0)$.*

Proof. Since $dist(A,B) \le d(y_0,P_A(y_0)) \le d(y_0,x_n)$, we have $d(y_0,P_A(y_0)) = dist(A,B)$. Now, take $y_n = y_0$ and $z_n = P_A(y_0)$ in Lemma 3.5, then we have

$$d(x_n,y_0) \to dist(A,B)$$

and

$$d(P_A(y_0),y_0) \to dist(A,B).$$

Therefore, we have

$$d(x_n,P_A(y_0)) \to 0.$$

This means

$$x_n \to P_A(y_0). \qquad \square$$

Theorem 3.7. *Let A,B be nonempty closed and convex subsets of a $CAT(0)$ space. Suppose $T : A\cup B \to A\cup B$ is a cyclic contraction map, then there exists a best proximity point x of T in A (that is with $d(x,Tx) = dist(A,B)$). Further, if $x_0 \in A$ and $x_{n+1} = Tx_n$, then $\{x_{2n}\}$ converges to the best proximity point x.*

Proof. Suppose $dist(A,B) = 0$, then $A\cap B \neq \emptyset$ and the theorem follows from Banach contraction theorem, as T is a contraction map on $A\cap B$. Therefore assume that $dist(A,B) \neq 0$.
Since

$$d(x_{2n},Tx_{2n}) \to dist(A,B)$$

and

$$d(T^2x_{2n},Tx_{2n}) \to dist(A,B).$$

By Lemma 3.5, we have

$$d(x_{2n},x_{2(n+1)}) = d(x_{2n},T^2x_{2n}) \to 0.$$

Similarly, we can show that

$$d(Tx_{2n},Tx_{2(n+1)}) \to 0.$$

We next show that for every $\epsilon > 0$, there exists N_0 such that $m > n \geq N_0$,

$$d(x_{2m},Tx_{2n}) \leq dist(A,B)+\epsilon.$$

Suppose not, then there exists $\epsilon > 0$ such that for all $k \in \mathbb{N}$ there exists $m_k > n_k \geq k$ for which

$$d(x_{2m_k},Tx_{2n_k}) > dist(A,B)+\epsilon$$

this m_k can be chosen such that it is the least integer greater than n_k to satisfy the above inequality. Now

$$\begin{aligned}
(dist(A,B)+\epsilon)^2 &< d^2(x_{2m_k},Tx_{2n_k}) \\
&\leq 2d^2(x_{2m_k},Tx_{2m_k}) + 2d^2(Tx_{2n_k},Tx_{2m_k}) - 4d^2(w_n,Tx_{2m_k}) \\
&\leq 2d^2(x_{2m_k},Tx_{2m_k}) + 2d^2(Tx_{2n_k},Tx_{2m_k}) - 4dist^2(A,B) \\
&\leq 2d^2(x_{2m_k},Tx_{2m_k}) + 2[\alpha d(x_{2n_k},x_{2m_k}) + (1-\alpha)dist(A,B)]^2 - 4dist^2(A,B) \\
&= 2d^2(x_{2m_k},Tx_{2m_k}) + 2[\alpha d(x_{2n_k},x_{2(n_k+1)}) + (1-\alpha)dist(A,B)]^2 - 4dist^2(A,B),
\end{aligned}$$

where $w_n = \frac{1}{2}x_{2m_k} \bigoplus \frac{1}{2}Tx_{2n_k}$. Consequently,

$$\begin{aligned}
(dist(A,B)+\epsilon)^2 &< 2dist^2(A,B) + 2[\alpha.0 + (1-\alpha)dist(A,B)]^2 - 4dist^2(A,B) \\
&= -2\alpha dist^2(A,B) < 0,
\end{aligned}$$

which is a contradiction. Therefore, by Lemma 3.4, $\{x_{2n}\}$ is a Cauchy sequence and hence $x_{2n} \to x$ in A. From Proposition 3.3, it follows that $d(x,Tx) = dist(A,B)$. Hence the proof is finished. $\square$

4. Conclusion and Future Scope

In this manuscript, we have obtained some best proximity point results for cyclic contractions in the setting of CAT(0) spaces. Our results generalize and improve the recent results of Eldred and Veeramani [11] as well as some other results in the literature. The future scope of our results, ones may investigate the existence and convergence of best proximity point theorems for cyclic generalized multi-valued contraction mappings in CAT(0) spaces.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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