



Characterizing Almost Quasi- Γ -ideals and Fuzzy Almost Quasi- Γ -ideals of Γ -semigroups

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Abstract. In this paper, we define the concepts of almost quasi- Γ -ideals and fuzzy almost quasi- Γ -ideals of a Γ -semigroup. Moreover, we give some relationship between almost quasi- Γ -ideals and fuzzy almost quasi- Γ -ideals of Γ -semigroups.

Keywords. Quasi- Γ -ideals; Almost quasi- Γ -ideals; Fuzzy almost quasi- Γ -ideals

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1. Introduction and Preliminaries

An introductory definition of almost ideals of semigroups was launched by Grosek and Satko [2]. They characterized these ideals when a semigroup S contains no proper almost ideals in [2], and afterward, they discovered minimal almost ideals and smallest almost ideals of semigroups in [3] and [4], respectively. Later, Wattanatripop, Chinram and Changphas [10] defined quasi-almost-ideals and fuzzy almost ideals in semigroups, gave the properties of quasi-almost-ideals in semigroups, and provided the relationship between almost ideals and fuzzy almost ideals in semigroups. Furthermore, they defined fuzzy almost bi-ideals in semigroups in [11].

Almost (m, n) -ideals and fuzzy almost (m, n) -ideals in semigroups were studied by Suebsung, Wattanatripop and Chinram in [8].

The concept of a Γ -semigroup has been introduced by Sen in 1981 [6]. In 1986, Sen and Saha [7] changed that definition considering the following more general definition:

Definition 1.1. ([7]) Let S and Γ be nonempty sets. S is called a Γ -semigroup if

- (1) $a\alpha b \in S$ for all $a, b \in S$ and $\alpha \in \Gamma$, and
- (2) $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a, b, c \in S$ and all $\alpha, \beta \in \Gamma$.

Example 1.1. (1) Let (S, \cdot) be a semigroup. We let $\Gamma := \{\cdot\}$. Then (S, Γ) is a Γ -semigroup.

(2) Let (S, Γ) be a Γ -semigroup. For each $\alpha \in \Gamma$, (S, α) is a semigroup.

(3) For each $n \in \mathbb{N}$, we define \cdot_n by $x \cdot_n y = x + y + n$. Let $\Gamma := \{\cdot_n \mid n \in \mathbb{N}\}$. Therefore, (\mathbb{N}, Γ) is a Γ -semigroup.

Recently, Wattanatripop and Changphas defined the concepts of left almost ideals and right almost ideals of a Γ -semigroup in [9]. Moreover, they characterized Γ -semigroups containing no proper left (respectively, right) almost ideals.

Definition 1.2. ([9]) Let S be a Γ -semigroup. A nonempty subset I of S is called

- (1) a *left almost ideal* of S if $(s\Gamma I) \cap I \neq \emptyset$ for all $s \in S$.
- (2) a *right almost ideal* of S if $(I\Gamma s) \cap I \neq \emptyset$ for all $s \in S$.

In [1], some properties of quasi- Γ -ideals in Γ -semigroups were studied.

Definition 1.3. ([1]) A nonempty subset Q of a Γ -semigroup S is called a *quasi- Γ -ideal* of S if $S\Gamma Q \cap Q\Gamma S \subseteq Q$.

In 1965, Zadeh introduced the fundamental fuzzy set concept in [12]. Since then, fuzzy sets are now applied in various fields. A fuzzy subset of a set S is a function from S into the closed interval $[0, 1]$. For any two fuzzy subsets f and g of S ,

1. $f \cup g$ is a fuzzy subset of S defined by

$$(f \cup g)(x) = \max\{f(x), g(x)\} = f(x) \vee g(x)$$

for all $x \in S$,

2. $f \cap g$ is a fuzzy subset of S defined by

$$(f \cap g)(x) = \min\{f(x), g(x)\} = f(x) \wedge g(x)$$

for all $x \in S$ and

3. $f \subseteq g$ if $f(x) \leq g(x)$ for all $x \in S$.

For a fuzzy subset f of a set S , the *support* of f is defined by

$$\text{supp}(f) = \{x \in S \mid f(x) \neq 0\}.$$

The *characteristic mapping* of a subset A of S is a fuzzy subset of S defined by

$$C_A(x) = \begin{cases} 1 & x \in A, \\ 0 & x \notin A. \end{cases}$$

The definition of fuzzy points of a set was given by Pu and Liu [5]. For $x \in S$ and $t \in (0, 1]$, a *fuzzy point* x_t of a set S is a fuzzy subset of S defined by

$$x_t(y) = \begin{cases} t & y = x, \\ 0 & y \neq x. \end{cases}$$

In this paper, we define and study almost quasi- Γ -ideals and fuzzy almost quasi- Γ -ideals in Γ -semigroups. Moreover, we give some relationship between almost quasi- Γ -ideals and fuzzy almost quasi- Γ -ideals of Γ -semigroups.

2. Almost Quasi- Γ -ideals

We begin this section with the following definition of an almost quasi- Γ -ideal of a Γ -semigroup.

Definition 2.1. Let S be a Γ -semigroup. A nonempty subset Q of S is called an *almost quasi- Γ -ideal* if

$$(s\Gamma Q \cap Q\Gamma s) \cap Q \neq \emptyset$$

for all $s \in S$.

Example 2.1. Consider the Γ -semigroup $S = \{a, b, c\}$ with $\Gamma = \{\alpha, \beta\}$ and

α	a	b	c
a	a	a	a
b	b	b	b
c	c	c	c

and

β	a	b	c
a	a	b	c
b	a	b	c
c	a	b	c

The almost quasi- Γ -ideals of S are $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, $\{a, b, c\}$.

Proposition 2.2. Every quasi- Γ -ideal of a Γ -semigroup S is either $s\Gamma Q \cap Q\Gamma s = \emptyset$ for some $s \in S$ or an almost quasi- Γ -ideal of S .

Proof. Assume that Q is a quasi- Γ -ideal of a Γ -semigroup S . Assume that $s\Gamma Q \cap Q\Gamma s \neq \emptyset$ for all $s \in S$. Let $s \in S$. Then

$$s\Gamma Q \cap Q\Gamma s \subseteq S\Gamma Q \cap Q\Gamma S \subseteq Q.$$

That is $(s\Gamma Q \cap Q\Gamma s) \cap Q \neq \emptyset$. Hence Q is an almost quasi- Γ -ideal of S . □

Theorem 2.3. Every almost quasi- Γ -ideal of a Γ -semigroup S is a left almost ideal of S .

Proof. Assume that Q is an almost quasi- Γ -ideal of a Γ -semigroup S . Let $s \in S$. Then

$$\emptyset \neq (s\Gamma Q \cap Q\Gamma s) \cap Q \subseteq s\Gamma Q \cap Q.$$

Hence Q is a left almost ideal of S . □

Similarly, every almost quasi- Γ -ideal of a Γ -semigroup S .

Example 2.4. Consider a Γ -semigroup $S = \{a, b, c, d, e\}$ with $\Gamma = \{\alpha\}$ and

α	a	b	c	d	e
a	a	a	a	d	d
b	a	b	c	d	e
c	a	c	b	d	e
d	d	d	d	a	a
e	d	d	d	a	a

Let $T = \{b, c, d\}$. We have T is a left almost ideal of S but $(e\Gamma T \cap T\Gamma e) \cap T = \emptyset$. Thus T is not an almost quasi- Γ -ideal of S .

This show that a left almost ideal of a Γ -semigroup S need not be an almost quasi- Γ -ideal of S . Similarly, a right almost ideal of S need not be an almost quasi- Γ -ideal of S .

Theorem 2.5. *If Q is an almost quasi- Γ -ideal of a Γ -semigroup S and $Q \subseteq H \subseteq S$, then H is an almost quasi- Γ -ideal of S .*

Proof. Assume that Q is an almost quasi- Γ -ideal of a Γ -semigroup S with $Q \subseteq H \subseteq S$. Let $s \in S$. Then

$$\emptyset \neq (s\Gamma Q \cap Q\Gamma s) \cap Q \subseteq (s\Gamma H \cap H\Gamma s) \cap H.$$

Therefore, H is an almost quasi- Γ -ideal of S . □

Theorem 2.6. *The union of two almost quasi- Γ -ideals of a Γ -semigroup S is an almost quasi- Γ -ideal of S .*

Proof. Let Q_1 and Q_2 be any two almost quasi- Γ -ideals of a Γ -semigroup S . Let $s \in S$. Since $Q_1 \subseteq Q_1 \cup Q_2$, we have

$$\emptyset \neq (s\Gamma Q_1 \cap Q_1\Gamma s) \cap Q_1 \subseteq [s\Gamma(Q_1 \cup Q_2) \cap (Q_1 \cup Q_2)\Gamma s] \cap (Q_1 \cup Q_2).$$

Therefore, $Q_1 \cup Q_2$ is an almost quasi- Γ -ideal of S . □

In Example 2.4, we have $Q_1 = \{a, b\}$ and $Q_2 = \{b, c, d\}$ are almost quasi- Γ -ideals of S , but $Q_1 \cap Q_2 = \{b\}$ is not. Thus we have:

Remark 2.7. The intersection of two almost quasi- Γ -ideals of a Γ -semigroup S need not be an almost quasi- Γ -ideal of S .

Theorem 2.8. *A Γ -semigroup S has no proper almost quasi- Γ -ideal if and only if for any $a \in S$ there exists s_a such that $s_a\Gamma(S \setminus \{a\}) \cap (S \setminus \{a\})\Gamma s_a \subseteq \{a\}$.*

Proof. Assume that $S \setminus \{a\}$ is not an almost quasi- Γ -ideal of S . Then there exists $s_a \in S$ such that

$$[s_a\Gamma(S \setminus \{a\}) \cap (S \setminus \{a\})\Gamma s_a] \cap (S \setminus \{a\}) = \emptyset.$$

Therefore, $s_a\Gamma(S \setminus \{a\}) \cap (S \setminus \{a\})\Gamma s_a \subseteq \{a\}$.

Conversely, assume that for any $a \in S$ there exists s_a such that

$$s_a \Gamma(S \setminus \{a\}) \cap (S \setminus \{a\}) \Gamma s_a \subseteq \{a\}.$$

Then $[s_a \Gamma(S \setminus \{a\}) \cap (S \setminus \{a\}) \Gamma s_a] \cap (S \setminus \{a\}) = \emptyset$. Hence $S \setminus \{a\}$ is not an almost quasi- Γ -ideal of S . Let A be a proper almost quasi- Γ -ideal of S . Then $A \subseteq S \setminus \{a\} \subseteq S$ for some $a \in S$, this is a contradiction. Therefore, S has no proper almost quasi- Γ -ideal. \square

3. Fuzzy almost quasi- Γ -ideals

Let S be a Γ -semigroup and $\mathcal{F}(S)$ be the set of all fuzzy subset of S . For each $\alpha \in \Gamma$, define a binary operation \circ_α on $\mathcal{F}(S)$ by

$$(f \circ_\alpha g)(x) = \begin{cases} \sup_{x=aa'b} \{\min\{f(a), g(b)\}\} & \text{if } x \in SaS, \\ 0 & \text{otherwise.} \end{cases}$$

Let $\Gamma^* := \{\circ_\alpha \mid \alpha \in \Gamma\}$. Then $(\mathcal{F}(S), \Gamma^*)$ is a Γ -semigroup.

Next, we define fuzzy almost quasi- Γ -ideals in semigroups and give some relationship between almost quasi- Γ -ideals and fuzzy almost quasi- Γ -ideals of Γ -semigroups.

Definition 3.1. Let f be a fuzzy subset of a Γ -semigroup S such that $f \neq 0$. f is called a *fuzzy almost quasi- Γ -ideal* of S if for all fuzzy point x_t of S , there exist $\alpha, \beta \in \Gamma$ such that $[(f \circ_\alpha x_t) \cap (x_t \circ_\beta f)] \cap f \neq 0$.

Theorem 3.1. Let f be a fuzzy almost quasi- Γ -ideal of a semigroup S and g be a fuzzy subset of S such that $f \subseteq g$. Then g is a fuzzy almost quasi- Γ -ideal of S .

Proof. Assume that f is a fuzzy almost quasi- Γ -ideal of a Γ -semigroup S and g is a fuzzy subset of S such that $f \subseteq g$. Then for all a fuzzy point x_t of S , there exist $\alpha, \beta \in \Gamma$ such that $[(f \circ_\alpha x_t) \cap (x_t \circ_\beta f)] \cap f \neq 0$. We have

$$[(f \circ_\alpha x_t) \cap (x_t \circ_\beta f)] \cap f \subseteq [(g \circ_\alpha x_t) \cap (x_t \circ_\beta g)] \cap g.$$

This implies $[(g \circ_\alpha x_t) \cap (x_t \circ_\beta g)] \cap g \neq 0$. Therefore g is a fuzzy almost quasi- Γ -ideal of S . \square

Corollary 3.2. Let f and g be fuzzy almost quasi- Γ -ideals of a Γ -semigroup S . Then $f \cup g$ is a fuzzy almost quasi- Γ -ideal of S .

Proof. Since $f \subseteq f \cup g$, by Theorem 3.1, $f \cup g$ is a fuzzy almost quasi- Γ -ideal of S . \square

Example 3.3. Consider the Γ -semigroup \mathbb{Z}_5 where $\Gamma = \{\bar{0}\}$ and $\bar{a}\gamma\bar{b} = \bar{a} + \gamma + \bar{b}$. Let $f : \mathbb{Z}_5 \rightarrow [0, 1]$ defined by

$$f(\bar{0}) = 0, f(\bar{1}) = 0.4, f(\bar{2}) = 0, f(\bar{3}) = 0.2, f(\bar{4}) = 0.3$$

and $g : \mathbb{Z}_5 \rightarrow [0, 1]$ defined by

$$g(\bar{0}) = 0, g(\bar{1}) = 0.3, g(\bar{2}) = 0.5, g(\bar{3}) = 0, g(\bar{4}) = 0.9.$$

We have f and g are fuzzy almost quasi- Γ -ideals of \mathbb{Z}_5 but $f \cap g$ is not a fuzzy almost quasi- Γ -

ideal of \mathbb{Z}_5 .

Theorem 3.4. *Let Q be a nonempty subset of a Γ -semigroup S . Then Q is an almost quasi- Γ -ideal of S if and only if C_Q is a fuzzy almost quasi- Γ -ideal of S .*

Proof. Assume that Q is an almost quasi- Γ -ideal of a Γ -semigroup S and let x_t be a fuzzy point of S . Then $[(Q\Gamma x) \cap (x\Gamma Q)] \cap Q \neq \emptyset$. Thus there exists $y \in (Q\Gamma x) \cap (x\Gamma Q)$ and $y \in Q$. Thus $y \in (Q\alpha x) \cap (x\beta Q)$ for some $\alpha, \beta \in \Gamma$. So $[(C_Q \circ_\alpha x_t) \cap (x_t \circ_\beta C_Q)](y) \neq 0$ and $C_Q(y) = 1$. Hence $[(C_Q \circ_\alpha x_t) \cap (x_t \circ_\beta C_Q)] \cap C_Q \neq 0$. Therefore C_Q is a fuzzy almost quasi- Γ -ideal of S .

Conversely, assume that C_Q is a fuzzy almost quasi- Γ -ideal of S . Let $s \in S$. Then

$$[(C_Q \circ_\alpha s_1) \cap (s_1 \circ_\beta C_Q)] \cap C_Q \neq 0$$

for some $\alpha, \beta \in \Gamma$. Then there exists $x \in S$ such that

$$[(C_Q \circ_\alpha s_1) \cap (s_1 \circ_\beta C_Q) \cap C_Q](x) \neq 0.$$

Hence $x \in [(Q\Gamma s) \cap (s\Gamma Q)] \cap Q$. So $[(Q\Gamma s) \cap (s\Gamma Q)] \cap Q \neq \emptyset$. Consequently, Q is an almost quasi- Γ -ideal of S . □

Theorem 3.5. *Let f be a fuzzy subset of a Γ -semigroup S . Then f is a fuzzy almost quasi- Γ -ideal of S if and only if $\text{supp}(f)$ is an almost quasi- Γ -ideal of S .*

Proof. Assume that f is a fuzzy almost quasi- Γ -ideal of a Γ -semigroup S . Let $s \in S$ and $t \in (0, 1]$. Then $[(f \circ_\alpha s_t) \cap (s_t \circ_\beta f)] \cap f \neq 0$. Hence there exists $x \in S$ such that

$$[(f \circ_\alpha s_t) \cap (s_t \circ_\beta f) \cap f](x) \neq 0.$$

So there exist $y_1, y_2 \in S$ such that $x = y_1\alpha s = s\beta y_2, f(x) \neq 0, f(y_1) \neq 0$ and $f(y_2) \neq 0$. That is $x, y_1, y_2 \in \text{supp}(f)$. Thus $[(C_{\text{supp}(f)} \circ_\alpha s_t) \cap (s_t \circ_\beta C_{\text{supp}(f)})](x) \neq 0$ and $C_{\text{supp}(f)}(x) \neq 0$. Therefore $[(C_{\text{supp}(f)} \circ_\alpha s_t) \cap (s_t \circ_\beta C_{\text{supp}(f)})] \cap C_{\text{supp}(f)} \neq 0$. Hence $C_{\text{supp}(f)}$ is a fuzzy almost quasi- Γ -ideal of S . By Theorem 3.4, $\text{supp}(f)$ is an almost quasi- Γ -ideal of S .

Conversely, assume that $\text{supp}(f)$ is an almost quasi- Γ -ideal of S . By Theorem 3.4, $C_{\text{supp}(f)}$ is a fuzzy almost quasi- Γ -ideal of S . Then for each fuzzy point s_t of S , we have

$$[(C_{\text{supp}(f)} \circ_\alpha s_t) \cap (s_t \circ_\beta C_{\text{supp}(f)})] \cap C_{\text{supp}(f)} \neq 0$$

for some $\alpha, \beta \in \Gamma$. Then there exists $x \in S$ such that

$$[(C_{\text{supp}(f)} \circ_\alpha s_t) \cap (s_t \circ_\beta C_{\text{supp}(f)}) \cap C_{\text{supp}(f)}](x) \neq 0.$$

Hence $[(C_{\text{supp}(f)} \circ_\alpha s_t) \cap (s_t \circ_\beta C_{\text{supp}(f)})](x) \neq 0$ and $C_{\text{supp}(f)}(x) \neq 0$. Then there exist $y_1, y_2 \in S$ such that $x = y_1\alpha s = s\beta y_2, f(x) \neq 0, f(y_1) \neq 0$ and $f(y_2) \neq 0$. This means $[(f \circ_\alpha s_t) \cap (s_t \circ_\beta f)] \cap f \neq 0$. Therefore f is a fuzzy almost quasi- Γ -ideal of S . □

Next, we define minimal fuzzy almost quasi- Γ -ideals in Γ -semigroups and give some relationship between minimal almost quasi- Γ -ideals and minimal fuzzy almost quasi- Γ -ideals of Γ -semigroups.

Definition 3.2. A nonzero fuzzy almost quasi- Γ -ideal f is called *minimal* if for each fuzzy almost quasi- Γ -ideal g of S such that $g \subseteq f$, we have $\text{supp}(g) = \text{supp}(f)$.

Theorem 3.6. Let Q be a nonempty subset of a Γ -semigroup S . Then Q is a minimal almost quasi- Γ -ideal of S if and only if C_Q is a minimal fuzzy almost quasi- Γ -ideal of S .

Proof. Assume that Q is a minimal almost quasi- Γ -ideal of a Γ -semigroup S . By Theorem 3.4, C_Q is a fuzzy almost quasi- Γ -ideal of S . Let g be a fuzzy almost quasi- Γ -ideal of S such that $g \subseteq C_Q$. Then $\text{supp}(g) \subseteq \text{supp}(C_Q) = Q$. Since $g \subseteq C_{\text{supp}(g)}$, we have $C_{\text{supp}(g)}$ is a fuzzy almost quasi- Γ -ideal of S . By Theorem 3.4, $\text{supp}(g)$ is an almost quasi- Γ -ideal of S . Since Q is minimal, $\text{supp}(g) = Q = \text{supp}(C_Q)$. Therefore, C_Q is minimal.

Conversely, assume that C_Q is a minimal fuzzy almost quasi- Γ -ideal of S . Let Q' be an almost quasi- Γ -ideal of S such that $Q' \subseteq Q$. Then $C_{Q'}$ is a fuzzy almost quasi- Γ -ideal of S such that $C_{Q'} \subseteq C_Q$. Hence $Q' = \text{supp}(C_{Q'}) = \text{supp}(C_Q) = Q$. Therefore, Q is minimal. \square

Corollary 3.7. Let Q be a nonempty subset of a Γ -semigroup S . Then Q has no proper almost quasi- Γ -ideal of S if and only if for all fuzzy almost quasi- Γ -ideal f of S , $\text{supp}(f) = S$.

4. Conclusion

A Γ -semigroup is an algebraic structure that it is a one of generalization of semigroups. Many authors investigated interesting results in this algebraic system. Also, the concept of fuzzy subsets is interesting to play with it. In this paper, we define almost quasi- Γ -ideals and fuzzy almost quasi- Γ -ideals of Γ -semigroups. The union of two almost quasi- Γ -ideals [fuzzy almost quasi- Γ -ideals] is also an almost quasi- Γ -ideal [a fuzzy almost quasi- Γ -ideal]. However, the intersection of two almost quasi- Γ -ideals [fuzzy almost quasi- Γ -ideals] need not be an almost quasi- Γ -ideal [a fuzzy almost quasi- Γ -ideal]. Moreover, we investigate some relationship between almost quasi- Γ -ideals and fuzzy almost quasi- Γ -ideals in Theorem 3.4, 3.5 and 3.6.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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