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Research Article

# Company Level Variables in R&D Cooperation Perspective

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**Abstract.** This paper aims to study the importance of cooperation in *Research and Development* (R&D) in the company level variables. The competition intensity and knowledge flow have impacts on those variables. However, these obstacles resulting from market structure and investment relationship may be reduced by expanding R&D relations.

**Keywords.** R&D cooperation; R&D Spillover, Market structure

**MSC.** 91A30; 91A43; 91A80

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## 1. Introduction

Strategic alliances and innovation technologies for industry are based on contracts aimed at improving production, developing industrial technology and reducing the cost of production [2, 3, 5, 10]. However, there are some obstacles that may reduce the role of these cooperative contracts, such as the flow of knowledge among investors and the market environment (e.g., [1, 4, 6, 7, 11]). In this paper, we will focus on the role of corporate cooperation in reducing the negative impacts of market structure and the investment relationship in R&D on company level variables.

We focus on the R&D model by Goyal and Moraga-Gonzalez [7]. They developed a three-stage game model under the network concept to study the R&D strategy within a homogeneous Cournot oligopoly (more than two companies compete by setting their production quantities).

According to their model, companies first choose their partners in R&D, decide upon R&D expenditure, and then compete in the product market. In their model, there is a possibility that the company can free-ride off the knowledge generated by another company in a market. Thus, the effective investment in R&D is divided into individual expenditures and the expenditure of other companies in R&D. The benefit from the expenditure of the other companies depends on an external parameter called an R&D spillover that captures knowledge flow between non-cooperating companies.

The contribution of this paper is mainly based on compensating companies for the expected shortfall in economic features due to increased competition or increased knowledge diffusion. First, this paper analyzes the decisions of companies to build their market relations and their competitiveness, and impact of these decisions on economic components of companies. Second, this paper examines the impact of the knowledge flow (R&D spillover) on the company level variables. We will observe the importance of cooperation in reducing the negative impact of the intensity of competition and in raising the economic situation of the company.

The outcomes of this paper can be summarized as follows. First, the company level variables (investment, production, profit and total welfare) depend on the competition intensity in the market. In a low-competitive market, these variables are very high; while the opposite occurs in the competitive market. The role of the cooperation of companies in R&D to reduce the competition effect depends the competition intensity. If the competitive difference is small, the cooperation has a prominent role in the results and vice versa if the difference is significant. Second, the impact of the R&D spillover on the economic situation of the company depends on the competition intensity in the market. In a market with weak competition, the spillover improves the outcomes while the opposite occurs in a competitive market. To reduce the negative impact of the R&D spillover in the competitive market, companies must establish cooperative agreements in R&D. In the company's view, this finding suggests that the R&D cooperation is a major incentive for increased production and profit.

The paper proceeds as follows. In the next section, we review some issues related to economics. In the third section, we present our results. In the fourth section, we conclude the paper.

## 2. Background

### 2.1 The Model

Consider the following utility function of consumers for $n$ companies in the market where each company produces one product [9]:

$$U = \alpha \sum_{i=1}^{n} q_i - \frac{1}{2}\left(\sum_{i=1}^{n} q_i^2 + 2\delta \sum_{j \neq i} q_i q_j\right) + I. \tag{1}$$

The demand parameters $\alpha > 0$ denotes the willingness of consumers to pay, $q_i$ is the quantity consumed of good $i$ and $I$ measures the consumer's consumption of another product. The parameter $\delta \in [-1, 1]$ captures the marginal rate of differentiation between different goods as follows:

$$\begin{cases} \text{goods are complements (perfect complements)} & \text{if } \delta < 0 \ \ (\delta = -1) \\ \text{goods are independent} & \text{if } \delta = 0 \\ \text{goods are substitutes (homogeneous)} & \text{if } \delta > 0 \ \ (\delta = 1) \end{cases}$$

**Payoffs.** Let $m$ be a consumer's income and $p_i$ be the price of good $i$ produced by company $i$. When consuming $q_i$ of product $i$, the money spent is $p_i q_i$ and the balance is $I = m - p_i q_i$. By substituting into the utility function (1) and calculating $\frac{\partial U}{\partial q_i} = 0$, we find the optimal consumption of good $i$:

$$\alpha - q_i - \delta \sum_{j \neq i} q_j - p_i = 0.$$

Thus, the inverse demand function for each good $i$, $D_i^{-1}$ is

$$p_i = \alpha - q_i - \delta \sum_{j \neq i} q_j, \quad i = 1, \ldots, n.$$

If $c_i$ is the cost of producing good $i$, the profit of company $i$ is

$$\pi_i = (p_i - c_i) q_i = \left( \alpha - q_i - \delta \sum_{j \neq i}^{n} q_j - c_i \right) q_i. \tag{2}$$

The industry surplus represents the total profit of all companies in the market:

$$\Pi = \sum_{i=1}^{n} \pi_i.$$

Consumer surplus is the difference between the price that consumers are willing to pay for a product and the actual market price:

$$CS = \frac{1-\delta}{2} \sum_{i=1}^{n} q_i^2 + \frac{\delta}{2} \left( \sum_{i=1}^{n} q_i \right)^2. \tag{3}$$

Total welfare is the sum of the industry surplus and the consumer surplus:

$$TW = CS + \Pi. \tag{4}$$

**Cost Reduction.** The effective amount of investment per company is defined as an individual expenditure on R&D plus partial expenditures of other companies in the market [6]. The benefit from partial expenditures of other companies depends on an external parameter called R&D spillover $\phi$, which captures knowledge flow between non-cooperation companies. In the case of two companies in the industry, the effective investment of company $i$ is defined as follows:

$$S_i = s_i + \phi s_j, \tag{5}$$

where $s_i$ is the amount of investment of company $i$ in R&D and $\phi \in [0, 1)$ is the R&D spillover. The effective investment reduces the marginal production cost of company $i$. If $c_0$ is the marginal cost, then the cost function becomes

$$c_i = c_0 - S_i = c_0 - s_i - \phi s_j. \tag{6}$$

## 2.2 Network

A **network** is a graph consists of a set of objects (called nodes or vertices) that are connected together by edges or links [13]. Let $N = \{i, j, k, \ldots\}$ be a set of all nodes and $E = \{ij, jk, \ldots\}$ be a set of all edges in the network. Then, $G(N,E)$ denotes a network with nodes $N$ and links $E$, and for simplicity the network is denoted by $G$. For the purpose of this study, the focus is on undirected networks where each link between any two nodes runs in both directions (i.e., each two links $ij$ and $ji$ in network $G$ are the same).

A set of **neighbors** of node $i$ consists of all nodes that are linked to it: $N_i = \{j \in N : ij \in E\}$. The length of the neighbors' set of node $i$ is the **degree** of that node i.e., for each node $i \in N$, $\deg(i) = |N_i|$ where $0 \le \deg(i) \le n-1$. If $\deg(i) = k$ for all $i \in N$, the network is called a regular network where we refer to it as $G_k$.

**R&D Network Model.** R&D partnerships between companies can be defined as a network where the nodes represent companies and the links represent cooperation. We assume that the R&D agreement between any two companies requires the approval and participation of both companies. In a network concept, this means that each link between any two companies serves both sides. In the R&D network game, there are three stages as follows [7]:

**The first stage:** Companies freely choose their partners in R&D. At the end of this stage, the cooperation network $G$ will be constructed and companies will identify their locations in that network. In practice, the network $G$ is captured by a symmetric $n \times n$ adjacency matrix $A = (a_{ij})$ where $a_{ij} \in \{0,1\}$. If $a_{ij} = 1$, the two companies $i$ and $j$ are linked (i.e., they cooperate in R&D); otherwise $a_{ij} = 0$.

**The second stage:** Companies choose their amounts of investment (effort) in R&D simultaneously and independently in order to reduce the cost of production. According to the model of Goyal and Moraga-Gonzalez [7], the effective investment in R&D for each company $i$ in the network $G$ is

$$S_i = s_i + \sum_{j \in N_i} s_j + \phi \sum_{k \notin N_i} s_k, \quad i = 1, \ldots, n, \tag{7}$$

where $s_i$ denotes R&D investment of company $i$, $N_i$ is the set of companies participating in R&D with the company $i$ and $\phi \in [0,1)$ is an exogenous parameter that captures knowledge spillovers acquired from companies not engaged in R&D with the company $i$ [7].

If we assume that the marginal cost $c_0$ is constant and equal for all companies, then from equations (6) and (7), we have

$$\begin{aligned} c_i &= c_0 - S_i \\ &= c_0 - s_i - \sum_{j \in N_i} s_j - \phi \sum_{k \notin N_i} s_k, \quad i = 1, \ldots, n, \end{aligned} \tag{8}$$

where the effective investment $S_i$ depends on the network structure.

**The third stage:** Companies compete in the product market by setting quantities (Cournot competition). At this stage, companies choose their levels of production in order to maximize their profits.

The investment in R&D is assumed to be costly and the function of the cost is quadratic. Thus, if company $i$ invests $s_i \in [0, c_0]$, the cost of R&D is $C(s_i) = \mu s_i^2$, where $\mu > 0$ refers to the effectiveness of R&D expenditure [6]. From this, the profit function (2) becomes

$$\pi_i = (p_i - c_i)q_i - C(s_i) = \left(\alpha - c_0 - q_i - \phi \sum_{j \neq i}^{n} q_j + S_i\right) q_i - C(s_i), \tag{9}$$

where the marginal cost satisfies $\alpha > c_0$.

## 2.3 Nash Equilibria

We identify the sub-game perfect Nash equilibrium by using backwards induction. From the profit function (9), we calculate $\frac{\partial \pi_i}{\partial q_i} = 0$ to have the best response function of quantity for good $i$:

$$q_i = \frac{(\alpha - c_0) + S_i - \delta \sum_{j \neq i} q_j}{2}. \tag{10}$$

By substituting the best response functions into each other, we have the symmetric equilibrium output for each good $i$

$$q_i^* = \frac{(2-\delta)(\alpha - c_0) + (2 + (n-2)\delta)S_i + \delta \sum_{j \neq i} S_j}{(2-\delta)((n-1)\delta + 2)}. \tag{11}$$

To find the symmetric equilibrium profit, we substitute the equilibrium output (11) into the profit function (9) which gives

$$\pi_i^* = \left[\frac{(2-\delta)(\alpha - c_0) + (2 + (n-2)\delta)S_i + \delta \sum_{j \neq i} S_j}{(2-\delta)((n-1)\delta + 2)}\right]^2 - C(s_i). \tag{12}$$

For convenience, the profit function can be expressed in the following form

$$\pi_i^* = q_i^{*2} - C(s_i). \tag{13}$$

Now the final list of the equilibria depends on the network structure. By knowing the structure, we have the effective investment of each company $i$. By substituting into the profit function (12) and calculating $\frac{\partial \pi_i^*}{\partial s_i} = 0$, we have the best response function of the R&D investment for each company $i$. By plugging the best response functions into each other, we have the symmetric equilibrium investment $s_i^*$. Then, we use the backwards induction to have the final equilibria. In Appendix A., we provide the final equilibrium equations for regular networks [7].

In a regular R&D network, each company has the same number of cooperative links. For the cooperation network $G_k$, each company has $k$ links called a cooperative activity level. Figure 1 shows some examples of regular networks of size four companies. In the case of the regular network with $n > 2$, we assume that the R&D spillover is zero ($\phi = 0$). Thus, the effective investment function for each company $i$ becomes

$$S_i = s_i + \sum_{j \in N_i} s_j, \quad i = 1, \dots, n. \tag{14}$$

According to Goyal and Moraga-Gonzalez, the effectiveness should satisfy the following:

$$\mu > \max\{\alpha n/4c_0, n/4\} \text{ if } \delta = 0, \tag{15a}$$

$$\mu > \max\{n^2/(n+1)^2, \alpha/4c_0\} \text{ if } \delta = 1. \tag{15b}$$

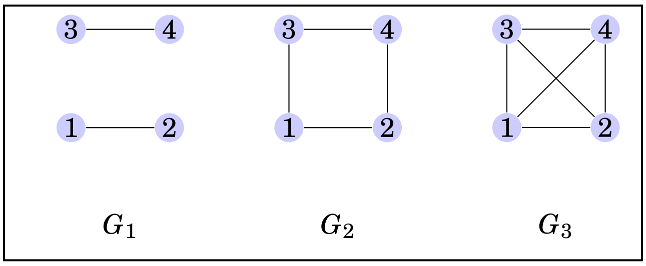


**Figure 1.** A sample of regular networks, each with four nodes

## 3. The Results

### 3.1 Effect of Competition and Compensation of Cooperation

In this section, we examine the role of cooperation of companies in R&D in covering the negative effects of competition on company level variables (investment, production, profit and total welfare). The strength of competition lies in the substitution degree of goods so that competition becomes more intense when the degree increases.

Consider that cooperation of companies in R&D forms a regular network. The following proposition states that if the gap between the differentiated degree is high, the negative impact of the competition intensity on equilibrium outcomes cannot be covered by the expansion of R&D relations.

**Proposition 1.** *Given a regular network $G_k$ with zero spillover such that the effectiveness $\mu$ satisfies* (15)*. If $G_{k+1}$ is a network generated from $G_k$ by adding one activity level, then the outcomes of $G_k$ for independent goods are higher than the outcomes of $G_{k+1}$ for homogeneous goods.*

The proof is given in Appendix B.

**Example 1.** Suppose there are ten companies in the market such that each company produces one product and the cooperation in R&D forms a regular network $G_k$ where $0 \le k \le 9$. For the market structure, assume two values of the differentiated degree $\delta = 0$ (independent goods) and $\delta = 1$ (homogeneous goods).

Figure 2 displays the economic variables in the two market structures for different values of the activity level $k$. As shown in the figure, the economic variables in the independent product market are higher than in the homogeneous market. From a corporate perspective, this conclusion suggests that the market with weak competition is a preferred structure to earn higher profits. In addition, the equilibrium outcomes of $G_k$ in the independent product market are higher than the equilibrium outcomes of $G_{k+1}$ in the homogeneous product market. This suggests that cooperation has no role in covering the negative effects of increased competition among companies. This result can be generalized for any network $G_{k+t}$ generated from $G_k$ by establishing $t$ activity levels such that $k+t<n$.

The previous result can be excluded if the difference between the two differentiated degrees is small. To see this, we present the following example that compares the equilibrium outcomes of the differentiated degrees $\delta = -0.1$ and $\delta = 0.1$.

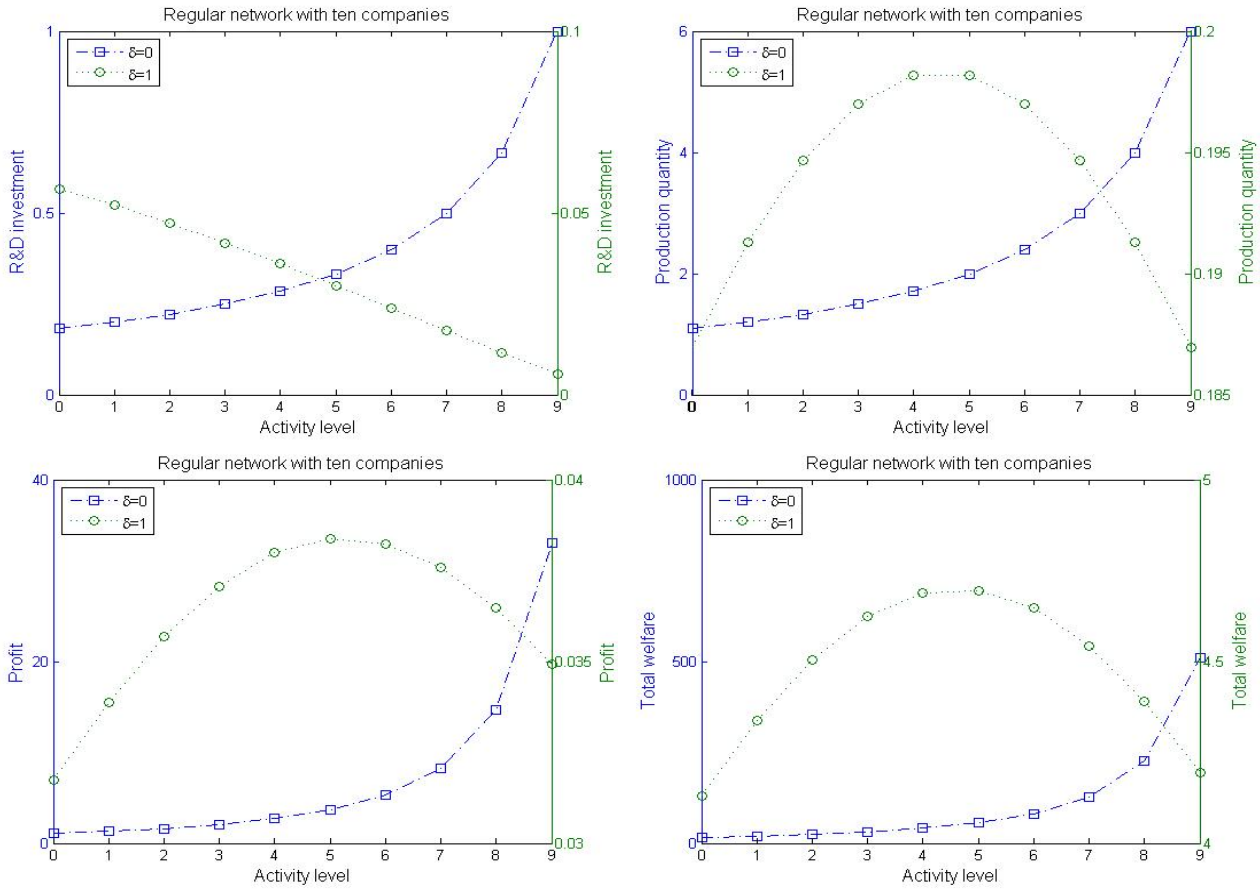


**Figure 2.** The economic outcomes of ten companies form regular network $G_k$ for $\delta = 0$ and $\delta = 1$. The parameters used to plot the figure are $\alpha = 12$, $c_0 = 10$ and $\mu = 3$

**Example 2.** Suppose there are two companies in the market such that each company produces one product. In terms of cooperation in R&D, there are two cases given in Figure 3. Now, consider two structures for the market. The first structure is when companies produce complementary goods with degree $\delta = -0.1$. The second structure is when goods are substitutes where $\delta = 0.1$.

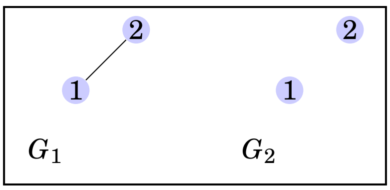


**Figure 3.** R&D cases for two companies in the market. In the network $G_1$, the two companies cooperate in R&D; while in the network $G_2$, they do not

Figure 4 shows the outcomes of company level variables for complementary goods $\delta = -0.1$. It also shows how the results change when goods become substitutes with substitution degree $\delta = 0.1$. As shown in the figure, if the gap between the differentiated degrees is small, the decrease of the outcomes due to the increase in the substitution degree is small under the cooperation case. This finding suggests that if the competition is small, the cooperation of companies in R&D is important to reduce the negative effects of competition.

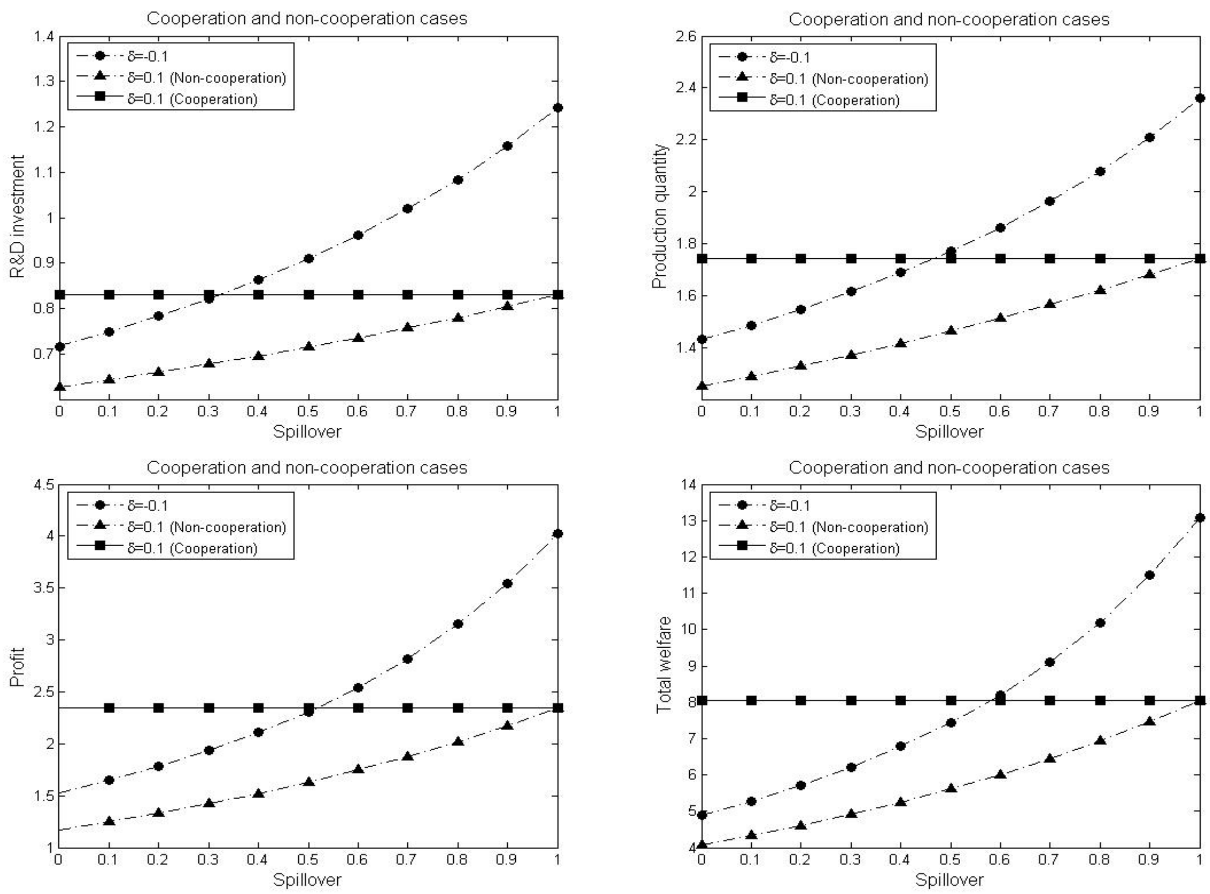


**Figure 4.** The economic variables of companies for the cases given in Figure 3. The parameters used to plot the figure are $\alpha = 12$, $c_0 = 10$, $\delta = -0.1$, $\delta = 0.1$ and $\mu = 1$.

## 3.2 Effect of Knowledge Flow and Compensation of Cooperation

In this section, we focus on the negative effects of the R&D spillover on company level variables in a competitive market and on the role of the cooperation in reducing those effects. In terms of the spillover, D'Aspremont and Jacquemin [6] found that if the spillover is high, the expenditure of companies in R&D when they cooperate is higher than when they do not cooperate. Goyal and Moraga-Gonzalez [7] also found that in a competitive market, the equilibrium outcomes decrease with increasing the spillover. Zirulia [17] discussed the effect of the spillover, where the model is based on linking the spillover to technological heterogeneity [17]. The author assumed two technological groups where companies in the same group have the same technology. The results pointed out that the spillover rates have a role in establishing and developing the structure of R&D cooperation.[1]

In a market with two companies, the following proposition compares the equilibrium outcomes of the networks $G_1$ and $G_2$ given in Figure 3. For the R&D investment, the cooperation has no role in reducing the impact of the spillover where the investment in non-cooperation case always high. For the production quantity and total welfare, the cooperation reduces the impact of the spillover if its value is small. For the profit of the companies, the cooperation always improves the results.

[1]There are other authors who discussed the risks associated with R&D activities (e.g., [12, 14–16]).

**Proposition 2.** *Assume two companies in a market where the networks $G_1$ and $G_2$ given in Figure 3 represent cooperation and non-cooperation cases such that the effectiveness $\mu$ satisfies (15). For each market structure such that $\delta \to 1$, we have the following:*

(1) $s^*(G_1) < s^*(G_2)$.

(2) $q^*(G_1) < q^*(G_2)$ *if $\phi$ is not small.*

(3) $\pi^*(G_2) < \pi^*(G_1)$.

(4) $TW^*(G_1) < TW^*(G_2)$ *if $\phi$ is not small.*

The proof is given in Appendix B.

In the following, we examine the effects of the cooperation of companies in R&D on the economic variables.

**Example 3.** Suppose there are two companies producing substitute products where the substitution degree is $\delta = 0.9$. Consider the networks $G_1$ and $G_2$ given in Figure 3.

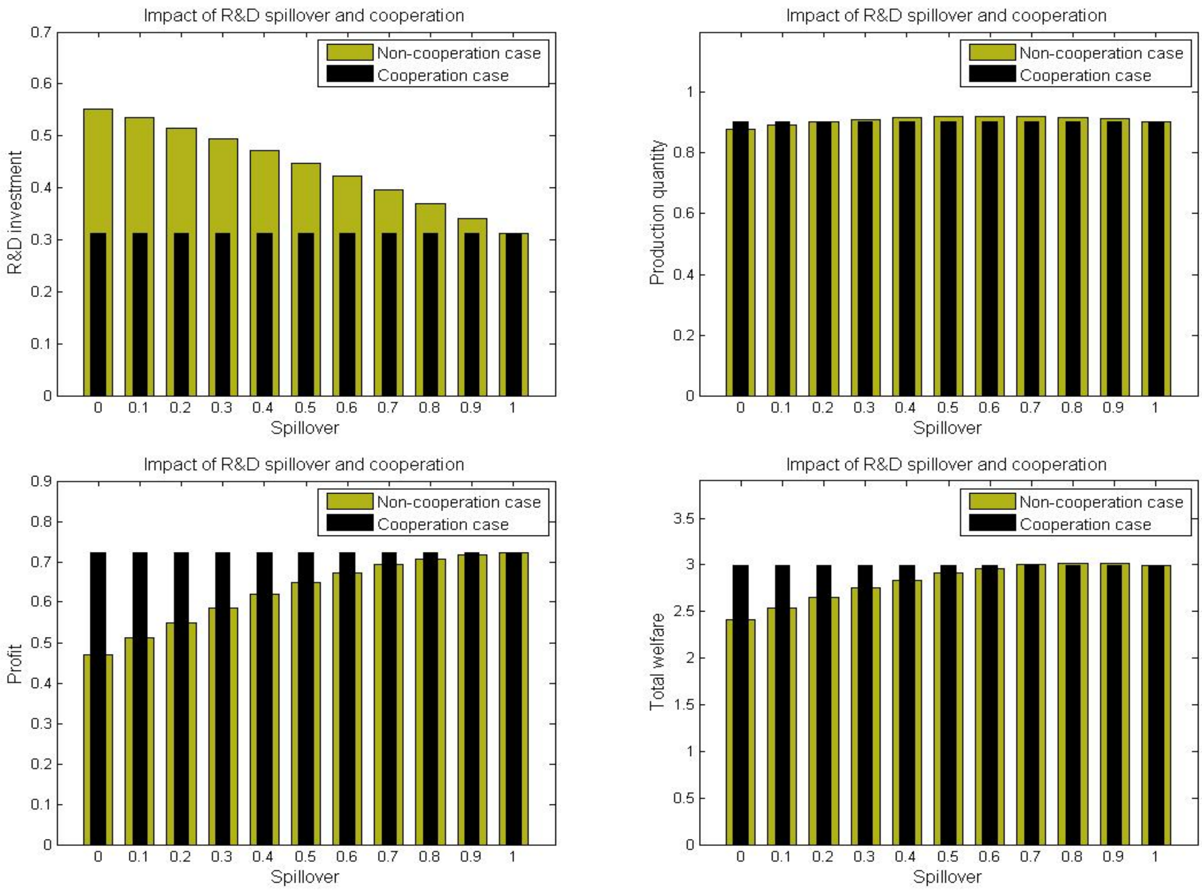


**Figure 5.** The impact of R&D spillover and cooperation on economic variables for the cases given in Figure 3. The parameters used to plot the figures are $\alpha = 12$, $c_0 = 10$, $\delta = 0.9$ and $\mu = 1$

Figure 5 compares between the economic variables before and after cooperation in R&D. As shown in the figure, the spillover has a negative impact on the investment of companies in R&D. For the production quantity and total welfare, the cooperation improves the results if the spillover is small. The outstanding role of cooperation in R&D is appeared in terms of the profit, which is always high in the case of the cooperation.

The results of the previous proposition are not applied if the substitution degree $\delta \to 0$. As shown in Figure 4, when the substitution degree is $\delta = 0.1$, the equilibrium outcomes are higher in the case of cooperation than the case of non-cooperation.

## 4. Conclusion

The paper focused on the importance of R&D cooperation in offsetting the negative impacts of market structure and R&D investment relationship. Under the Cournot competition with differentiated products, the main results are summarized as follows.

While the company level variables are high in a weak competitive market, this market is an appropriate investment environment. As competition increases, these variables decrease, but the R&D collaboration will play a key role in increasing these variables if the competitive difference is small. The effect of knowledge flow between companies on economic characteristics depends on the intensity of competition among companies. In a market with low competition, the role of R&D spillover is important in increasing the results, but the opposite occurs when the competition increases. To cover the negative impact the spillover, companies must establish cooperation in R&D, whose larger role will be to increase profits of companies.

## Appendices

### A. Nash Equilibria

■ **Two companies in the market.**

**(1) Cooperation case:**

$$s^*(G_1) = \frac{(\alpha - c_0)}{\mu(2+\delta)^2 - 2}, \tag{A.1a}$$

$$q^*(G_1) = \frac{\mu(2+\delta)(\alpha - c_0)}{\mu(2+\delta)^2 - 2}, \tag{A.1b}$$

$$\pi^*(G_1) = \frac{\mu\big(\mu(2+\delta)^2 - 1\big)(\alpha - c_0)^2}{(\mu(2+\delta)^2 - 2)^2}, \tag{A.1c}$$

$$TW^*(G_1) = \frac{\mu\big(\mu(2+\delta)^2(3+\delta) - 2\big)(\alpha - c_0)^2}{(\mu(2+\delta)^2 - 2)^2}. \tag{A.1d}$$

**(2) Non-cooperation case:**

$$s^*(G_2) = \frac{(2-\delta\phi)(\alpha - c_0)}{\mu(2+\delta)^2(2-\delta) - (1+\phi)(2-\delta\phi)}, \tag{A.2a}$$

$$q^*(G_2) = \frac{\mu(4-\delta^2)(\alpha - c_0)}{\mu(2+\delta)^2(2-\delta) - (1+\phi)(2-\delta\phi)}, \tag{A.2b}$$

$$\pi^*(G_2) = \frac{\mu\big(\mu(4-\delta^2)^2 - (2-\delta)^2\big)(\alpha - c_0)^2}{\big(\mu(2+\delta)^2(2-\delta) - (1+\phi)(2-\delta\phi)\big)^2}, \tag{A.2c}$$

$$TW^*(G_2) = \frac{\mu\big(\mu(4-\delta^2)^2(3+\delta)-2(2-\delta\phi)^2\big)(\alpha-c_0)^2}{\big(\mu(2+\delta)^2(2-\delta)-(1+\phi)(2-\delta\phi)\big)^2}. \tag{A.2d}$$

■ **Arbitrary number of companies form a regular R&D network.**

**(1) Independent goods:**

$$s^* = \frac{(\alpha-c_0)}{4\mu-k-1}, \tag{A.3a}$$

$$q^* = \frac{2\mu(\alpha-c_0)}{4\mu-k-1}, \tag{A.3b}$$

$$\pi^* = \frac{\mu(4\mu-1)(\alpha-c_0)^2}{(4\mu-k-1)^2}, \tag{A.3c}$$

$$TW^* = \frac{n\mu(6\mu-1)(\alpha-c_0)^2}{(4\mu-k-1)^2}. \tag{A.3d}$$

**(2) Homogeneous goods:**

$$s^* = \frac{(n-k)(\alpha-c_0)}{\mu(n+1)^2-(n-k)(k+1)}, \tag{A.4a}$$

$$q^* = \frac{\mu(n+1)(\alpha-c_0)}{\mu(n+1)^2-(n-k)(k+1)}, \tag{A.4b}$$

$$\pi^* = \frac{(\mu^2(n+1)^2-(n-k)^2)(\alpha-c_0)^2}{(\mu(n+1)^2-(n-k)(k+1))^2}, \tag{A.4c}$$

$$TW^* = \frac{n(\mu^2(n+2)(n+1)^2-2(n-k)^2)(\alpha-c_0)^2}{2(\mu(n+1)^2-(n-k)(k+1))^2}. \tag{A.4d}$$

## B. Proof of Propositions

*Proof of Proposition* 1. Suppose that the effectiveness $\mu$ satisfies (15). We prove the proposal in two steps:

**Step 1:** We show that the equilibrium outcomes in the R&D network $G_k$ decreases with increasing the substitution degree. In particular, we compare the outcomes of the network $G_k$ for independent and homogeneous goods.

**Step 2:** We show that the outcomes of $G_k$ for independent goods are higher than the outcomes of $G_{k+1}$ for homogeneous goods.

(1) We want to prove that $s^*_{\delta=0}(G_k) > s^*_{\delta=1}(G_{k+1})$. First, we compare between the R&D investment in $G_k$ for independent and homogeneous goods:

$$s^*_{\delta=0}(G_k) - s^*_{\delta=1}(G_k) = \frac{\mu(\alpha-c_0)\big((n+1)^2-4(n-k)\big)}{\big(4\mu-k-1\big)\big(\mu(n+1)^2-(n-k)(k+1)\big)}.$$

Since $(n+1)^2-4(n-k)>0$ for each $0\le k\le n-1$, then $s^*_{\delta=0}(G_k)-s^*_{\delta=1}(G_k)>0$.

Now, we want to prove that $s^*_{\delta=0}(G_k) > s^*_{\delta=1}(G_{k+1})$. By calculating the difference between the two equilibria, we have

$$s^*_{\delta=0}(G_k) - s^*_{\delta=1}(G_{k+1}) = \frac{(\alpha-c_0)\big(\mu(n+1)^2-(n-(k+1))(4\mu+1)\big)}{\big(4\mu-k-1\big)\big(\mu(n+1)^2-(n-(k+1))(k+2)\big)}.$$

The term $(n-(k+1))$ takes its highest value when $k=0$, but since the effectiveness $\mu$ is high, then $\mu(n+1)^2 > (n-(k+1))(4\mu+1)$ and this implies $s^*_{\delta=0}(G_k) - s^*_{\delta=1}(G_{k+1}) > 0$.

(2) We want to prove that $q^*_{\delta=0}(G_k) > q^*_{\delta=1}(G_{k+1})$. In the first step, we show that for each regular network $G_k$, $q^*_{\delta=0}(G_k) > q^*_{\delta=1}(G_k)$. By calculating the difference between the two equilibria, we have

$$q^*_{\delta=0}(G_k) - q^*_{\delta=1}(G_k) = \frac{\mu(\alpha-c_0)\big(4\mu n(n+1) - 3(n-k)(k+1)\big)}{\big(4\mu-k-1\big)\big(\mu(n+1)^2-(n-k)(k+1)\big)}.$$

For sufficient values of $\mu$, the expression $4\mu n(n+1) - 3(n-k)(k+1) > 0$ and this implies $q^*_{\delta=0}(G_k) - q^*_{\delta=1}(G_k) > 0$.

In the second step, we want to prove that $q^*_{\delta=0}(G_k) > q^*_{\delta=1}(G_{k+1})$. From equations (A.3b) and (A.4b), we have

$$q^*_{\delta=0}(G_k) - q^*_{\delta=1}(G_{k+1}) = \frac{\mu(\alpha-c_0)\big((n+1)(4\mu n+(k+1)) - 4(n-(k+1))(k+2)\big)}{\big(4\mu-k-1\big)\big(\mu(n+1)^2-(n-(k+1))(k+2)\big)}.$$

The term $(n-(k+1))(k+2)$ depends on the activity level $k$ more than the network size $n$, but since $\mu$ is sufficiently large and $0 \le k \le n-1$, then $q^*_{\delta=0}(G_k) - q^*_{\delta=1}(G_{k+1}) > 0$.

(3) We want to prove that $\pi^*_{\delta=0}(G_k) - \pi^*_{\delta=1}(G_{k+1}) > 0$. First, we compare between the profit in $G_k$ for independent and homogeneous goods. From our calculation, we have

$$\text{sign}\Big(\pi_{\delta=0}(G_k) - \pi_{\delta=1}(G_k)\Big) = \text{sign}\Big(4(4\mu-1)\big(\mu(n+1)^2-(n-k)(k+1)\big)^2 - (4\mu-(k+1))^2\big(\mu(n+1)^2-(n-k)^2\big)\Big).$$

Note that as $k$ approaches $n-1$, we have $4(4\mu-1) \gg (4\mu-(k+1))^2$. Also, as $k$ approaches 0, we have $\big(\mu(n+1)^2-(n-k)(k+1)\big)^2 \gg \big(\mu(n+1)^2-(n-k)^2\big)$. In both cases, we have $\pi^*_{\delta=0}(G_k) - \pi^*_{\delta=1}(G_k) > 0$.

Now, we want to compare between $\pi^*_{\delta=0}(G_k)$ and $\pi^*_{\delta=1}(G_{k+1})$. When calculating the difference between the two equilibria, we have

$$\text{sign}\Big(\pi^*_{\delta=0}(G_k) - \pi^*_{\delta=1}(G_{k+1})\Big) = \text{sign}\Big(\mu(\alpha-c_0)^2\Big[4(4\mu-1)\big(\mu(n+1)^2-(n-(k+1))(k+2)\big)^2 - \big(4\mu+(k-1)\big)^2\big(\mu(n+1)^2-(n-(k+1))^2\big)^2\Big]\Big).$$

The sign depends on values of $n$, $k$ and $\mu$. Therefore, for each market size $n$ such that the effectiveness $\mu$ satisfies (15), we have that if $k \to n-1$, then $4(4\mu-1) \gg (4\mu+(k+1))^2$ and if $k \to 0$, we have

$$\big(\mu(n+1)^2-(n-(k+1))(k+2)\big) \gg \big(\mu(n+1)^2-(n-(k+1))^2\big)^2.$$

Thus, $\pi^*_{\delta=0}(G_k) > \pi^*_{\delta=1}(G_{k+1})$.

(4) We want to prove that $TW^*_{\delta=0}(G_k) - TW^*_{\delta=1}(G_{k+1}) > 0$. In the first step, we show that for each regular network $G_k$, $TW^*_{\delta=0}(G_k) > TW^*_{\delta=1}(G_k)$. By calculating the difference between the two equilibria, we have

$$\text{sign}\Big(TW^*_{\delta=0}(G_k) - TW^*_{\delta=1}(G_k)\Big) = \text{sign}\Big(4(6\mu-1)\Big(\mu(n+1)^2-(n-k)(k+1)\Big)^2 - (4\mu-(k+1))^2\Big(\mu(n+1)^2(n+2)-2(n-k)^2\Big)\Big).$$

As $k$ approaches $n-1$, we have $4(6\mu-1) \gg (4\mu-(k+1))^2$ also, as $k$ approaches 0, we have

$$\left(\mu(n+1)^2-(n-k)(k+1)\right)^2 \gg \left(\mu(n+1)^2(n+2)-2(n-k)^2\right).$$

Thus, we have $TW^*_{\delta=0}(G_k)-TW^*_{\delta=1}(G_k)>0$.

In the second step, we want to prove that $TW^*_{\delta=0}(G_k)>TW^*_{\delta=1}(G_{k+1})$. From our calculation, we have

$$\operatorname{sign}\left(TW^*_{\delta=0}(G_k)-TW^*_{\delta=1}(G_{k+1})\right)=\operatorname{sign}\Big(4(6\mu-1)\big(\mu(n+1)^2-(n-(k+1))(k+2)\big)^2 - (4\mu-(k+1))^2\big(\mu(n+1)^2(n+2)-2(n-k)^2\big)\Big).$$

In the previous expression, $k\to n-1$ implies that $4(6\mu-1)\gg(4\mu-(k+1))^2$ also, $k\to 0$, implies that

$$\left(\mu(n+1)^2-(n-(k-1))(k+2)\right)^2 \gg \left(\mu(n+1)^2(n+2)-2(n-k)^2\right).$$

Thus, $TW^*_{\delta=0}(G_k)>TW^*_{\delta=1}(G_{k+1})$. □

*Proof of Proposition* 2. Suppose that the effectiveness $\mu$ satisfies (15). We prove the proposition by comparing the equilibria in the networks given in Figure 3.

(1) We want to prove that $s^*(G_2)-s^*(G_1)>0$.

$$s^*(G_2)-s^*(G_1)=\frac{(\alpha-c_0)\left((2-\delta\phi)(3+\phi)+\delta\mu(2+\delta)^2(1-\phi)\right)}{\left(\mu(2+\delta)^2(2-\delta)-(1+\phi)(2-\delta\phi)\right)\left(\mu(2+\delta)^2-2\right)}.$$

For $\delta\to 1$, the previous fraction is positive and this implies $s^*(G_2)>s^*(G_1)$

(2) We want to prove that $q^*(G_2)-q^*(G_1)>0$ if $\delta\to 1$ and $\phi$ is not small.

$$q^*(G_2)-q^*(G_1)=\frac{\mu(\alpha-c_0)\left[\left((1-\phi)(\delta(2+\phi)-2)\right)\right]}{\left(\mu(2+\delta)^2(2-\delta)-(1+\phi)(2-\delta\phi)\right)\left(\mu(2+\delta)^2-2\right)}.$$

Note that if $\phi$ is small i.e., $\delta(2+\phi)-2<0$ and this implies $\phi<2(1-\delta)/\delta$, then $q^*(G_1)>q^*(G_2)$. However, if $\phi$ is not small, we have $q^*(G_1)<q^*(G_2)$.

(3) We want to prove that $\pi^*(G_1)-\pi^*(G_2)>0$.

$$\operatorname{sign}\left(\pi^*(G_1)-\pi^*(G_2)\right)=\operatorname{sign}\Big((\mu(2+\delta)^2-1)(\mu(2+\delta)^2(2-\delta)-(1+\phi)(2-\delta\phi))^2 - (\mu(4-\delta^2)^2-(2-\delta)^2)(\mu(2+\delta)^2-2)^2.$$

If the effectiveness $\mu$ satisfies the condition (15),

$$(\mu(2+\delta)^2(2-\delta)-(1+\phi)(2-\delta\phi))^2 \gg (\mu(2+\delta)^2-2)^2.$$

This implies $\pi^*(G_1)-\pi^*(G_2)>0$.

(4) We want to prove that $TW^*(G_2)-TW^*(G_1)>0$ if $\delta\to 1$ and $\phi$ is not small.

$$\operatorname{sign}\left(TW^*(G_1)-TW^*(G_2)\right)=\operatorname{sign}\Big(\big(\mu(2+\delta)^2(3+\delta)-2\big)\big(\mu(2+\delta)^2(2-\delta)-(1+\phi)(2-\delta\phi)\big)^2 - \big(\mu(4-\delta^2)^2(3+\delta)-2(2-\delta\phi)^2\big)\big(\mu(2+\delta)^2-2\big)^2\Big).$$

For each value of the differentiation degree, the term $(1+\phi)(2-\delta\phi)$ is affected by value of $\phi$. If it is small, the previous term is small, which allow the expression

$\left(\mu(2+\delta)^2(2-\delta)-(1+\phi)(2-\delta\phi)\right)^2$ to be large and then $TW^*(G_1)-TW^*(G_2)>0$. The opposite occurs if $\phi$ is not small. □

## Competing Interests

The author declares that he has no competing interests.

## Authors' Contributions

The author wrote, read and approved the final manuscript.

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