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# Evidence for Color-by-Color Disengagement from the Process of Lepton Production Associated with the Ψ-Series and Υ-Series Mesons

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**Abstract.** Form factors associated with the $\Psi(1S)$ and $\Upsilon(2S)$ are calculated directly from relevant experimental data in order to verify that they are given by $f_1 = (1-q_s^2) = (8/9)$ in the case of the Ψ-Series mesons and $f_2 = (1-q_c^2) = (5/9)$ in the case of the Υ-series mesons, where $q_s = -1/3$ represents the charge of the strange $(s)$ quark and $q_c = 2/3$ represents the charge of the charm $(c)$ quark. In two recent articles by the author form factors have been shown to represent the fraction of the originally produced quark/anti-quark $(QQ^*)$ state which makes a transition to a $QQ^*$ state of the next lowest mass ... $ss^*$ in the case of the Ψ-Series mesons and $cc^*$ in the case of the Υ-series mesons ... and thus figure prominently into the calculation of the hadronic and leptonic widths of a given meson via the constructs of the Gluon Emission Model (GEM). We undertake to calculate the form factors of the $\Psi(2S)$, the $\Psi(4040)$, the $\Psi(4160)$, and the $\Psi(4415)$ in order to show that $f_1 = (8/9)$ is representative of all Ψ-states listed above, if and only if, it is assumed that one quark color (in the case of the $\Psi(2S)$) or two quark colors (in all other cases) become disengaged from lepton production. A similar set of calculations is performed as to the Υ-series mesons, such illustrating that all three quark colors are functional in lepton production in $\Upsilon(2S)$ decay, fewer than three functional in $\Upsilon(3S)$ and $\Upsilon(4S)$ decay, with likely only one color functioning in $\Upsilon(10860)$ and $\Upsilon(11020)$ decay. For each meson series, then, lepton decay is characterized by the phenomenon of sequential disengagement of quark color from lepton production as a function of increasing mass. In the $\Upsilon(3S)$ and $\Upsilon(4S)$ decay ... and in the $\Psi(3770)$ decay ... it is observed that half-integer color contributions are in force ... possibly leading to a heretofore unobserved quark property: "shade" of color ("light" and "dark").

## 1. Introduction

In White (2010-f) and White (2010-y) it is shown that form factors $(f_i)$, which represent the fraction of quark $(Q)$/anti-quark $(Q^*)$ states, originally produced in the formation of a given vector meson, that make a transition to a $QQ^*$ state comprising quarks of the next lowest mass, figure prominently in the width

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calculations of said vector mesons via the Gluon Emission Model (GEM) theoretical structure describing the formation and decay of vector mesons. Specifically, it is found that for the $\Psi$-Series mesons, characterized by $i = 1$,

$$f_1 = (1 - q_s^2) = (8/9), \tag{1a}$$

where $q_s = -1/3$ represents the charge of the strange ($s$) quark, and for the $\Upsilon$-series mesons more massive than the $\Upsilon(1S)$, characterized by $i = 2$,

$$f_2 = (1 - q_c^2) = (5/9), \tag{1b}$$

where $q_c = 2/3$ represents the charge of the charm ($c$) quark;

$$\text{for the } \Upsilon(1S) \qquad f_3 = 1. \tag{1c}$$

In the present work we will be almost exclusively concerned with the electron/positron ($ee$) partial widths (from which the above form factors are readily obtainable) associated with various vector mesons in the $\Psi$- and $\Upsilon$-series. The role that form factors play in the $ee$ partial width calculations (see White (2010-f) and White (2010-y) for details) is exhibited below. Denoting the $ee$ partial width associated with vector meson "$X$" of mass "$M_X$" by "$\Gamma_{ee}$", we find from White (2010-y), for example, where "$f_x$" represents the appropriate form factor:

$$\begin{aligned}\Gamma_{ee} = f_x[&(\alpha/2\pi)(10,042)(2m_e)(m_\rho/M_X)^3(q_z)^4 \\ &+ (1 - f_x)\{(\alpha/2\pi)(10,042)(2m_e)(m_\rho/M_X)^3(q_x)^4\}].\end{aligned} \tag{2}$$

In Eq. (2) $\alpha = (1/137.036)$ represents the fine structure constant, $m_e = 0.511$ Mev represents the mass of the electron, $m_\rho = 776$ Mev represents the mass of the $\rho$-meson, $q_x$ represents the charge associated with the original $QQ^*$ state comprising $X$, and $q_z$ represents the charge associated with the quark of next lowest mass relative to the original $QQ^*$ structure. Specifically, then, for the $\Psi$-Series mesons

$$\begin{aligned}\Gamma_{ee}(\Psi N) = f_1[&(\alpha/2\pi)(10,042)(2m_e)(m_\rho/M_{\Psi N})^3(q_s)^4 \\ &+ (1 - f_1)\{(\alpha/2\pi)(10,042)(2m_e)(m_\rho/M_{\Psi N})^3(q_c)^4\}],\end{aligned} \tag{3a}$$

where $N$ represents the series designate in terms of an identification as per "$\Psi(NS)$". Similarly,

$$\begin{aligned}\Gamma_{ee}(\Upsilon N) = f_2[&(\alpha/2\pi)(10,042)(2m_e)(m_\rho/M_{YN})^3(q_c)^4 \\ &+ (1 - f_2)\{(\alpha/2\pi)(10,042)(2m_e)(m_\rho/M_{YN})^3(q_b)^4\}],\end{aligned} \tag{3b}$$

where $q_b = -1/3$ represents the charge of the bottom ($b$) quark, describes the $ee$ partial width associated with the $\Upsilon$-series mesons. Hadronic partial widths may also be determined for the $\Psi$-Series mesons by replacing "$\alpha$" by $\alpha_s = 1.2[\ln(M_{\Psi N}/50\,\text{Mev})]^{-1}$, the strong coupling parameter derived via the GEM (see White (2010-f), White (2010-y), White (2009-a)), and carrying out the relevant calculations associated with appropriate Feynman Diagrams. For example (see White (2010-y)), the hadronic width of the $\Psi(1S)$ is determined via the GEM to be 82.0 Kev, which represents nearly an exact match to the most recent report of the Particle Data Group (PDG) of $(81.7 \pm 1.8)$ Kev (see PDG (2009-M)). The $\Upsilon(1S)$ hadronic width is calculated via the GEM (White (2010-y) again nearly exactly in accord with experiment assuming $f_i = f_3 = 1$ and the existence of a two-gluon route of decay, as the GEM yields 50.09 Kev compared to the PDG (2009-M) report of $(49.99 \pm 1.16)$ Kev. Hence, the GEM is, in general, extremely successful in determining the hadronic widths of the Ψ-Series and ϒ-series mesons. The form factors, $f_i$, however, are derived from observation of the relevant leptonic partial widths. We may calculate $f_1$ and $f_2$ directly, in fact, through utilization of Eq. (3a) and (3b), respectively, by inserting the appropriate experimental data into said equations and solving for $f_i$. In each case a quadratic equation is obtained whose solutions are exhibited below:

$$f_1 = (1/32C)[17C + \{289C^2 - 64C\Gamma_{ee}(\mathrm{PDG})\}^{1/2}]; \tag{4a}$$

$$f_2 = (1/2C)[17C - \{289C^2 - 4C\Gamma_{ee}(\mathrm{PDG})\}^{1/2}]. \tag{4b}$$

In Eq. (4) $\Gamma_{ee}$(PDG) represents the $ee$ partial width as reported in PDG (2009-M) and

$$C = (\alpha/2\pi)(10,042)(2m_e)(m_\rho/M_X)^3(1/3)^4. \tag{5}$$

Thus, for $i = 1$ $M_X = M_{\Psi N}$, and for $i = 2$ or 3 $M_X = M_{YN}$.

In Section 2 we will calculate $f_1$ as associated with the $\Psi(1S)$, the $\Psi(2S)$, the $\Psi(4040)$, the $\Psi(4160)$, and the $\Psi(4415)$ assuming, at first, for each one all three quark colors are participating in lepton production. We shall find, however, that $f_1 = (8/9)$ is realized in the case of the $\Psi(2S)$ only if two quark colors are operative in lepton production; in the cases of the $\Psi(4040)$, the $\Psi(4160)$, and the $\Psi(4415)$, $f_1 = (8/9)$ is realized only if one quark color is operative in lepton production. In Section 3 we will calculate $f_2$ as associated with the $\Upsilon(2S)$, the $\Upsilon(3S)$, the $\Upsilon(4S)$, the $\Upsilon(10860)$, and the $\Upsilon(11020)$. The leptonic widths of the ϒ-series mesons are not as precisely determined as those of the Ψ-Series, but we believe we can demonstrate that the same sequential color disengagement from lepton production as seen in the Ψ-Series is at work in the ϒ-series. All experimental data in Sections 2 and 3 (and Section 4) comes from PDG (2009-M).

## 2. Calculation of $f_1$, the Form Factor of the Ψ-Series

### 2.A. *The* $\Psi(1S)$

Assuming that all quark colors are operative in the leptonic decay mode of the $\Psi(1S)$, as the GEM as presented so far does, and inserting appropriate experimental quantities (with $M_{\Psi 1} = 3097$ Mev) into Eq. (3a) yields

$$\Gamma_{ee}(\mathrm{GEM}) = 5.72\,\mathrm{Kev},$$

where $\Gamma_{ee}(\text{GEM})$, here and onward, represents the relevant *ee* partial width as determined via the GEM. The PDG (2009-M) reports

$$\Gamma_{ee}(\text{PDG}) = (5.55 \pm 0.16)\,\text{Kev}.$$

Thus, $\Gamma_{ee}(\text{GEM})$ is just out of the experimental range of the currently accepted *ee* partial width of the $\Psi(1S)$. Turning now to Eq. (4a), in which $C = 2.3149$, we find:

$$f_1(\Psi 1) = 0.8951 = (8/9)\{1.0070\}. \tag{6}$$

Hence, the form factor associated with the $\Psi(1S)$, viz., $f_1(\Psi 1)$, is seen to be calculated as very nearly the GEM-theoretical value of $f_1 = (8/9)$, thus indicating that, indeed, all quark colors do participate in the leptonic decay mode of the $\Psi(1S)$.

2.B. *The* $\Psi(2S)$

Again, assuming that all quark colors are operative in the leptonic decay of the $\Psi(2S)$ and inserting the appropriate experimental quantities into Eq. (3a) (with $M_{\Psi 2} = 3686\,\text{Mev}$), we obtain

$$\Gamma_{ee}(\text{GEM}) = 3.39\,\text{Kev},$$

whereas $\Gamma_{ee}(\text{PDG}) = (2.36 \pm 0.04)\,\text{Kev}$.

We first note that if we were to assume that each quark color contributes the same amount to $\Gamma_{ee}$ and that one color has disengaged from the process of lepton production, then

$$\Gamma_{ee}(\text{GEM}) \to (2/3)3.39\,\text{Kev} = 2.26\,\text{Kev},$$

which, again, lies just outside the PDG's range. Now, again assuming a two color contribution to $\Gamma_{ee}$, $C = (2/3)[1.3731] = 0.9154$, from which we find

$$f_1(\Psi 2) = 0.8792 = (8/9)\{0.9891\}. \tag{7}$$

Basing our confidence level on the relative closeness of $f_1(\Psi 2)$ to "(8/9)", we are $\sim 99\%$ sure that

$$f_1(\Psi 2) = f_1 = (8/9)$$

and, in turn, $\sim 99\%$ sure that one quark color has disengaged from lepton production as associated with the $\Psi(2S)$.

2.C. *The* $\Psi(4040)$ [*Our Designation:* $\Psi(3S)$]

Again, assuming that all quark colors are operative in the leptonic decay of the $\Psi(3S)$ and inserting the appropriate experimental quantities into Eq. (3a) (with $M_{\Psi 3} = 4039\,\text{Mev}$), we obtain

$$\Gamma_{ee}(\text{GEM}) = 2.58\,\text{Kev},$$

whereas $\Gamma_{ee}(\text{PDG}) = (0.86 \pm 0.07)\,\text{Kev}$.

If we now assume that two colors are now inoperative, i.e., only one color participates in lepton production associated with the $\Psi(3S)$,

$$\Gamma_{ee}(\mathrm{GEM}) \to (1/3)2.58\,\mathrm{Kev} = 0.86\,\mathrm{Kev},$$

which represents an exact match to $\Gamma_{ee}(\mathrm{PDG})$.
Commensurately, $C = (1/3)[1.0437] = 0.3479$, which translates to

$$f_1(\Psi 3) = 0.8886 = (8/9)\{0.9997\}. \tag{8}$$

We are essentially 100% confident, therefore, that now two quark colors have disengaged from lepton production by the time we reach the (assumed) "3*S*" level.

2.D. *The* $\Psi(4160)$ [*Our designation:* $\Psi(4S)$]

Again, assuming that all quark colors are operative in the leptonic decay of the $\Psi(4S)$ and inserting the appropriate experimental quantities into Eq. (3a) (with $M_{\Psi 4} = 4153\,\mathrm{Mev}$), we obtain

$$\Gamma_{ee}(\mathrm{GEM}) = 2.37\,\mathrm{Kev},$$

whereas $\Gamma_{ee}(\mathrm{PDG}) = (0.83 \pm 0.07)\,\mathrm{Kev}$.

If we again assume that only one color participates in lepton production associated with the $\Psi(4S)$,

$$\Gamma_{ee}(\mathrm{GEM}) \to (1/3)2.37\,\mathrm{Kev} = 0.79\,\mathrm{Kev},$$

which represents a statistical match to $\Gamma_{ee}(\mathrm{PDG})$.
Commensurately, $C = (1/3)[0.9600] = 0.3200$, which translates to

$$f_1(\Psi 4) = 0.8778 = (8/9)\{0.9876\}. \tag{9}$$

We are therefore $\sim$ 99% confident that only one color participates in lepton production as associated with the $\Psi(4S)$.

2.E. *The* $\Psi(4415)$ [*Our designation:* $\Psi(5S)$]

Again, assuming that all quark colors are operative in the leptonic decay of the $\Psi(5S)$ and inserting the appropriate experimental quantities into Eq. (3a) (with $M_{\Psi 5} = 4421\,\mathrm{Mev}$), we obtain

$$\Gamma_{ee}(\mathrm{GEM}) = 1.96\,\mathrm{Kev},$$

whereas $\Gamma_{ee}(\mathrm{PDG}) = (0.58 \pm 0.07)\,\mathrm{Kev}$.

If we again assume that only one color participates in lepton production associated with the $\Psi(5S)$,

$$\Gamma_{ee}(\mathrm{GEM}) \to (1/3)1.96\,\mathrm{Kev} = 0.65\,\mathrm{Kev},$$

which again represents a statistical match to $\Gamma_{ee}(\mathrm{PDG})$.
Commensurately, $C = (1/3)[0.7958] = 0.2653$, which translates to

$$f_1(\Psi 5) = 0.9128 = (8/9)\{1.0269\}. \tag{10}$$

We are therefore $\sim$ 97% confident that only one color participates in lepton production as associated with the $\Psi(5S)$.

2.F. *Short Summary of Section* 2

Of great interest is the fact that the average number multiplying "(8/9)" in Eqs. (6)-(10) is given by (to three significant figures)

$$\langle n \rangle = 1.00 \pm 0.01\,, \tag{11}$$

which suggests with very high confidence that (1) $f_1 = (8/9)$ for all $\Psi$-series objects and that (2) sequential quark color disengagement from the process of lepton production takes place with increasing energy in the $\Psi$-series mesons. Table 1 below illustrates the phenomenon as per Section 2.

**Table 1.** Color participation in lepton production in the $\Psi$-series.

| Meson | Mass (Mev) | $\Gamma_{ee}$(GEM) | $\Gamma_{ee}$(PDG) | Number of colors operative |
|---|---|---|---|---|
| $\Psi(1S)$ | 3097 | 5.72 | $5.55 \pm 0.16$ | 3 |
| $\Psi(2S)$ | 3686 | 2.26 | $2.36 \pm 0.04$ | 2 |
| $\Psi(3S)$ | 4039 | 0.86 | $0.86 \pm 0.07$ | 1 |
| $\Psi(4S)$ | 4153 | 0.79 | $0.83 \pm 0.07$ | 1 |
| $\Psi(5S)$ | 4421 | 0.65 | $0.58 \pm 0.07$ | 1 |

## 3. Calculation of $f_2$, the Form Factor of the $\Upsilon$-Series

For the exposition in Section 3 to follow, it proves to be convenient to add a subscript, $j = 1, 2$, or 3, to certain previously-defined variables, where "$j$" indicates the number of quark colors assumed to be operative in lepton decay. For example, "$\Gamma_{ee1}$(GEM)" indicates a leptonic width per the GEM theory assuming only one color participates in *ee* production, and "$f_{21}$" indicates the associated form factor, $f_2$, under such condition. All calculations follow along the lines seen in Section 2 with Eq. (3a) and (4a) supplanted by Eq. (3b) and (4b), respectively. In addition, as the methodology for determining leptonic widths and form factors is by now clear, our presentation in the present section will not feature the entire set of detailed information put forth in Section 2.

3.A. *The* $\Upsilon(2S)$

Inserting $M_{Y2} = 10023$ Mev into Eq. (3b), we obtain

$$\Gamma_{ee3}(\text{GEM}) = 0.624\,\text{Kev},$$

a figure just outside of the experimental range, given by

$$\Gamma_{ee}(\text{PDG}) = (0.612 \pm 0.011)\,\text{Kev}.$$

The resulting form factor is found from Eq. (4b) to be

$$f_{23} = 0.5446 = (5/9)\{0.9803\}. \tag{12}$$

We are thus $\sim 98\%$ sure that three colors are operative in the leptonic decay of the $\Upsilon(2S)$ and that $f_2 = (5/9)$.

3.B. *The* $\Upsilon(3S)$

Inserting $M_{Y3} = 10355$ Mev into Eq. (3b), we obtain

$$\Gamma_{ee3}(\text{GEM}) = 0.566\,\text{Kev},$$

a figure well outside of the experimental range, given by

$$\Gamma_{ee}(\text{PDG}) = (0.443 \pm 0.008)\,\text{Kev}.$$

We find $\Gamma_{ee2}(\text{GEM}) = 0.377$ Kev, which is well below the above experimental range of values. However, we do note that

$$\langle \Gamma_{ee3+2}(\text{GEM}) \rangle = (1/2)[\Gamma_{ee3}(\text{GEM}) + \Gamma_{ee2}(\text{GEM})] = 0.471\,\text{Kev},$$

a figure which is only $\sim 6\%$ discrepant with experiment. Furthermore, we find

$$\begin{aligned} \langle f_2 \rangle &= (1/2)[f_{23} + f_{22}] = (1/2)[0.4217 + 0.6565] \\ &= 0.5441 = (5/9)\{0.9794\}. \end{aligned} \tag{13}$$

Hence, we obtain the same confidence level as to $f_2$ from the $\Upsilon(3S)$ data as we did from the $\Upsilon(2S)$ data if it is assumed that the leptonic decay of the $\Upsilon(3S)$ takes place with an even mix of two and three color participation. Viewed in such manner, the $\Upsilon(3S)$ would represent the very threshold for the onset of color non-participation in lepton decay associated with the $\Upsilon$-series mesons.

3.C. *The* $\Upsilon(4S)$

Inserting $M_{Y4} = 10579$ Mev into Eq. (3b), we obtain

$$\Gamma_{ee3}(\text{GEM}) = 0.531\,\text{Kev},$$

a figure well outside of the experimental range, given by

$$\Gamma_{ee}(\text{PDG}) = (0.272 \pm 0.029)\,\text{Kev}.$$

Assuming only two colors operative, we obtain $\Gamma_{ee2}(\text{GEM}) = 0.354$ Kev, and assuming one operative color, we obtain $\Gamma_{ee1}(\text{GEM}) = 0.177$ Kev. We now note that

$$\langle \Gamma_{ee2+1}(\text{GEM}) \rangle = (1/2)[\Gamma_{ee2}(\text{GEM}) + \Gamma_{ee1}(\text{GEM})] = 0.266\,\text{Kev},$$

which represents a statistical match to $\Gamma_{ee}(\text{PDG})$ above. Thus, by assuming $f_2 = (5/9)$ and that lepton decay of the $\Upsilon(4S)$ takes place with an even mix of one and two color participation, we realize excellent agreement with experiment.

3.D. *The* $\Upsilon(10860)$ [*Our Designation:* $\Upsilon(5S)$]

Inserting $M_{Y5} = 10860$ Mev into Eq. (3b), we obtain

$$\Gamma_{ee3}(\text{GEM}) = 0.490\,\text{Kev},$$

a figure well outside of the experimental range, given by

$$\Gamma_{ee}(\text{PDG}) = (0.31 \pm 0.07)\,\text{Kev}.$$

We note, however, that $\Gamma_{ee2}(\text{GEM}) = 0.33$ Kev with an associated $f_{22} = 0.5271 = (5/9)\{0.9488\}$, thus indicating $\sim 95\%$ confidence that two colors are operating in the leptonic decay of the $\Upsilon(5S)$ and that $f_2 = (5/9)$.

3.E. *The* $\Upsilon(11020)$ [*Our Designation:* $\Upsilon(6S)$]

Inserting $M_{Y6} = 11019$ Mev into Eq. (3b), we obtain

$$\Gamma_{ee3}(\text{GEM}) = 0.470\,\text{Kev},$$

a figure well outside of the experimental range, given by

$$\Gamma_{ee}(\text{PDG}) = (0.130 \pm 0.030)\,\text{Kev}.$$

We note, however, that $\Gamma_{ee1}(\text{GEM}) = 0.157$ Kev, which is in the experimental range of $\Gamma_{ee}(\text{PDG})$ above, thus indicating that only one color is operating in the leptonic decay of the $\Upsilon(6S)$ and that $f_2 = (5/9)$.

3.F. *Short Summary of Section* 3

There is clear indication of color disengagement from lepton production in conjunction with $f_2 = (5/9)$ as associated with the $\Upsilon$-series mesons. Assuming the color participation outlined below in Table 2 along with assuming that $f_2 = (5/9)$ throughout, the average relative discrepancy between $\Gamma_{ee}(\text{GEM})$ and $\Gamma_{ee}(\text{PDG})$ associated with the $\Upsilon$-series mesons is only 7.5%, whereas the data set itself has an average imprecision of 12.0%.

**Table 2.** Color participation in lepton production in the $\Upsilon$-series.

| Meson | Mass (Mev) | $\Gamma_{ee}$(GEM) | $\Gamma_{ee}$(PDG) | Number of colors operative |
|---|---|---|---|---|
| $\Upsilon(2S)$ | 10023 | 0.624 | $0.612 \pm 0.11$ | 3 |
| $\Upsilon(3S)$ | 10355 | 0.471 | $0.443 \pm 0.008$ | $2\frac{1}{2}$ |
| $\Upsilon(4S)$ | 10579 | 0.266 | $0.272 \pm 0.029$ | $1\frac{1}{2}$ |
| $\Upsilon(5S)$ | 10860 | 0.33 | $0.31 \pm 0.07$ | 2 |
| $\Upsilon(6S)$ | 11019 | 0.157 | $0.13 \pm 0.03$ | 1 |

## 4. **Concluding Remarks**

We find the phenomenon of "color shutdown" as to the process of lepton production in the Ψ-Series and Υ-series mesons extremely interesting. The confidence level is seen to be extremely high (at least regarding the Ψ-Series mesons) that the phenomenon exists, but what causes it? Because of the color-by-color sequential nature of the color turn-off with increasing mass within the subset of integer number of colors operative in lepton decay, it is unlikely that the phenomenon has its basis in color screening effects. The phenomenon we have explored above looks much too quantized for that. As interesting and, frankly, mysterious, as the color-by-color turn-off phenomenon is, the half-integer turn-offs are far more so. There is even an example of possibly a "three-quarter color" turn-off not yet explored in the present work, to which we turn presently:

The Ψ(3770)($M_{\Psi(3770)} = 3773\,\text{Mev}$) has an anomalously small *ee* partial width, viz.,

$$\Gamma_{ee}(\text{PDG}) = (0.265 \pm 0.018)\,\text{Kev}.$$

Assuming $f_1 = (8/9)$ and that $\Gamma_{ee}$ is due to an even mix of zero color participation in the Ψ(3770) lepton decay and one-half color participation in same, the GEM yields via Eq. (4a) $\Gamma_{ee}(\text{GEM}) = 0.264\,\text{Kev}$, a figure representing nearly an exact match to experiment. On average, then, the Ψ(3770) has only $\frac{1}{4}$ of a color participating in lepton production. We have characterized the half-integer color participations in Section 3 as representations of even mixes in contribution to lepton decay from "$k$" colors operative and "$k-1$" colors operative. If such representation is accurate, then in some instances it must be the case that one quark of the di-quark meson system has "$k$" colors operative in lepton decay, while the other has "$k-1$" colors operative. Such would explain the half-integer color contributions exhibited in Section 3. The case of the Ψ(3770), however, suggests something else entirely, as its leptonic width result must be explained by an even mix of zero color contribution to lepton production and one-half color contribution to same. Unless the case of the Ψ(3770) represents merely a numerical coincidence, an alternative explanation would have to be that we have uncovered a new feature associated with quarks … that of "shade", in that each color must possess two "shades" ("light"? and "dark"?) in order to explain the quarter-integer color contribution to lepton production of the Ψ(3770).

In any event we may estimate the threshold for the complete turn-off of lepton production in the Ψ-Series and in the Υ-series by plotting the number of operative colors versus $(M_{\Psi N}/M_{\Psi 1})$ and $(M_{\Upsilon N}/M_{\Upsilon 2})$, respectively, omitting the Ψ-(3770) outlier case, and estimating where on said plots $N_C = 0.5$ (our definition for the above-mentioned threshold), where $N_C$ represents the number of operative colors from Tables 1 and 2. From Figure 1 we note that

$$N_C \approx 3x_1^{-3.5}, \quad \text{where} \quad x_1 = (M_{\Psi N}/M_{\Psi 1}). \tag{14a}$$

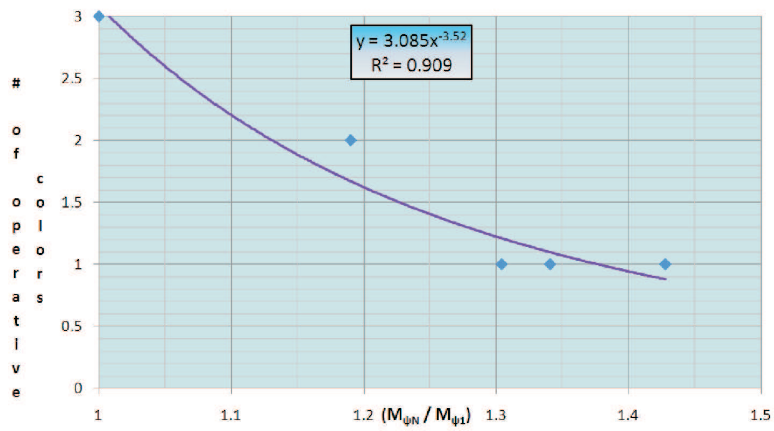


**Figure 1.** Number of operative colors in Lepton Decay (Ψ-Series).

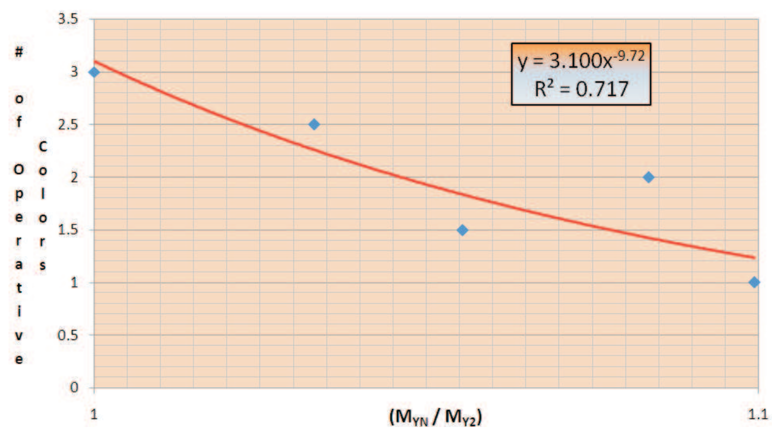


**Figure 2.** Number of operative colors in Lepton Decay (ϒ-series).

Hence, $x_1 \approx (5/3)$ when $N_c = \frac{1}{2}$, thus indicating a lepton shut-down threshold for the Ψ-Series at about 5170 Mev. From Figure 2 we note that

$$N_C \approx 3x_2^{-9.75}, \quad \text{where } \; x_2 = (M_{YN}/M_{Y2}). \tag{14b}$$

Hence, $x_2 \approx (6/5)$ when $N_c = \frac{1}{2}$, thus indicating a lepton shut-down threshold for the Ψ-Series at about 12050 Mev. Comparing the color shut-down behaviors, as exhibited by Eqs. (14a) and (14b), it is apparent that the ϒ-series shutdown with increasing energy is much more rapid, relatively speaking, than is that of the Ψ-Series. As a final word, we remark that whether quark colors each can be either "light" or "dark" . . . or not . . . the color shut-down phenomenon is extremely interesting and should prove to be the subject of future research in high energy physics theoretical work.

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