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Form Factor Analysis Derived from the Gluon Emission Model Applied to the $\Psi(2S)$ and the $\Upsilon(2S)$

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Abstract. In a recent article it has been shown that form factors may be derived through the use of the Gluon Emission Model (GEM) theoretical structure describing the widths of vector mesons; such form factors, f, are such that $(1-f)$ represents the fraction of the original quark (Q) — anti-quark (Q^*) pair comprising a given vector meson that remains part of the decay scheme, the remainder, f, making a transition to a QQ^* state comprising quarks of the next lightest mass compared to the original (remaining) ones. In conjunction with representative Feynman Diagrams we employ said form factors in order to calculate the various partial widths of the $\Psi(2S)$ and the $\Upsilon(2S)$. Excellent agreement with experiment is reached, and we are able to show that the matrix elements involved in the $\Psi(2S) \to \Psi(1S) + Z$ decay, where Z represents any other product, and in the $\Upsilon(2S) \to \Upsilon(1S) + Z$ decay are roughly four ninths the magnitude of those associated with the $\Psi(2S) \to Z_1 + Z_2$ decay and the $\Upsilon(2S) \to Z_1 + Z_2$ decay, where Z_1 and Z_2 each represent any decay product other than $\Psi(1S)$ or $\Psi(2S)$, respectively.

1. Background

In White (2010) it is shown that the Gluon Emission Model (GEM), first developed by F. Close in 1979 (see F. Close (1979)), represents a theoretical structure which is able to realize excellent agreement with experiment in terms of the widths of all vector mesons (inclusive of the $K^*(892)$) in their ground states and in terms of the evaluation of the strong coupling parameter, α_s, at essentially any energy. The basic premise of the GEM is that vector mesons arise via the spin-flip of one of the quarks comprising a given vector meson at preferred energies, i.e., the energies associated with the masses of the vector mesons. The matrix element, $|V|$, representing the formation of the spin one state which describes a given vector meson is thus descriptive of a spin-spin interaction and is therefore proportional to q_i^2, where q_i represents the charge of a quark of flavor "i". Hence, $|V|^2$ is proportional to q_i^4. Once the basic form for $|V|^2$ is assumed, the width calculations

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proceed strictly upon the principles laid out in Quantum Electrodynamics (QED) but involving certain appropriate alterations. At the present juncture it would be best to exhibit the basic precepts of the GEM as seen, for example, in White (2010):

In all quantum systems in which natural decay occurs between an excited level and the ground state, the integrated absorption cross-section goes as

$$\sigma(\omega) = K\alpha|V|^2(1/m)^2(1/\omega)L(\omega), \tag{1}$$

where K is a constant, ω represents photon frequency, $|V|^2$ represents the square of the matrix element descriptive of the photon emission process, the system has mass m, $L(\omega)$ is a Lorentz Amplitude with a peak at $\omega = \omega_0$ and with a width Γ, and $\alpha = (1/137.036)$ represents the fine structure constant.

Assuming "asymptotic freedom", i.e., that we may ignore the masses of the decay products (light hadron pairs) in relation to the total energy involved in the system under investigation, we may employ Eq. (1) to predict the width of vector mesons by making the following substitutions to take us from a general quantum electrodynamics (QED) to a specific quantum chromodynamics (QCD) process:

We substitute for the photon frequency ω the gluon energy Q_0.

We evaluate the right hand side of Eq. (1) at a specific vector meson mass, m_v, i.e., $Q_0 = m = m_v$. (Hence, the associated Lorentz Amplitude equals unity.)

We require $|V|^2$ to be proportional to $\Sigma_i(q_i)^4$, where q_i = quark charge (in units of electron charge magnitude) associated with the quarks comprising the relevant vector meson.

(The above criterion is consistent with spin-spin interaction [see also R. Dalitz (1977), p. 604] proportional to q_i^2, where i denotes quark flavor, giving rise to spin-flip transitions, and the sum is required only in the case of the ρ, as it comprises both the up quark (u) of charge $q_u = 2/3$ and the down quark (d) of charge $q_d = -1/3$.)

We postulate $|V|^2$ to be proportional to only $\Sigma_i(q_i)^4$, i.e., the precise form of the interaction is universal to all vector mesons in their ground states, except for quark charge differences.

We replace α by α_s, the strong coupling parameter, which has the well-known form from QCD gauge invariance theories (see [2, S. Gasiorowicz and J. L. Rosher, *American Journal of Physics* **49**, 954 and ff (1981)]) of:

$$\alpha_s = B[\ln(Q_0/\Lambda)]^{-1}, \tag{2}$$

where B is a constant and Λ is a parameter to be determined. Again, we emphasize that commensurate with the above replacements is that we must assume that the initial energy involved in the formation of a given vector meson is extremely high, i.e., in the "asymptotically free" region of energy space, where the masses of emerging hadron pairs as decay products can be neglected. Accordingly, then, we find in terms of the above ansatz (normalizing to the ρ)

$$\Gamma_v = A(m_\rho/m_{\Psi(2S)})^3(\Sigma_i(q_i)^4)[\ln(m_v/\Lambda)]^{-1}, \tag{3}$$

where Γ_v represents the width of a given vector meson, v, and A is a constant to be determined.

Equation (1) in the above quotation comes straight from Merzbacher's Quantum Mechanics (E. Merzbacher (1970), p. 486). With the appropriate substitutions it represents the absorption cross-section of a gluon propagating from the collision vertex to the quark (Q) — anti-quark (Q^*) pair comprising the vector meson and, as well, the emission of a gluon signifying the decay of the resonance state. In Close's scheme no distinction is made between a gluon and a virtual photon, except in terms of the couplings to given vertices in a representative Feynman Diagram (FD). Indeed, in Close (1979) one finds many such FDs in which the author depicts a gluon transmuting directly to a virtual photon and vice-versa, all such transmutations described by a coupling magnitude of "1". In the present work we shall therefore represent either a gluon or a virtual photon as "ζ", our symbolic representation of a four-momentum propagator. Since the gluon and the virtual photon are considered within the GEM as two aspects of the same entity, i.e., the four-momentum propagator, in the realm of asymptotic freedom, i.e., the energy, E, associated with a given colliding beams experiment is such that $E > 3000$ Mev, the ratio of lepton production to hadron production associated with a given QQ^* decay must be in the ratio of α/α_s, where α represents the fine structure constant $= (1/137.036)$. The constants, "A" and "Λ", in Eq. (3) of the above quotation may be determined by fitting simultaneously the width of the $\rho(776)$ and the width of the $\phi(1019)$ in accord with Eq. (3) above, and "B" may be determined by setting the ratio of the experimental electron/positron partial width of the $\Upsilon(1S)$ to the GEM's theoretical hadronic width (with "A" and "Λ" determined) of the $\Upsilon(1S)$ equal to α/α_s, as the $\Upsilon(1S)$ exists well into the realm of asymptotic freedom. The representation of the hadronic (H) width of any vector meson, then, takes the form of (see White (2010)):

$$\Gamma_{v-H} \approx (\alpha_s/2\pi)(10,042)(2m_e)(m_\rho/m_v)^3(\Sigma_i(q_i)^4), \tag{1}$$

where m_e represents the electron mass of 0.511 Mev, so that $2m_e = 1.022$ Mev., α_s represents the strong coupling parameter, given by $\alpha_s = 1.2[\ln(m_v/50\,\text{Mev})]^{-1}$, $m_\rho = 776$ Mev represents the mass of the ρ meson, m_v represents the mass of the vector meson with designate "v", and q_i represents the charge of the relevant quark type(s) "i" to undergo the spin flip to form the vector meson under consideration. The q_i involved in ρ formation, for example, are the $q_u = 2/3$ and $q_d = -1/3$, where "u" designates an "up quark" and "d" designates a "down quark". Only $q_s = -1/3$, where "s" designates a "strange quark", is involved in the formation of the kaon branch of the ϕ, whereas q_u, q_d, and q_s are all involved in the formation of the $K^*(892)$ (see White (2008-R and 2008-K)). In addition, as we will see below, the q_i mainly associated with the $J(3097)$ is actually q_s, and that associated with the $\Upsilon(1S)$ is actually $q_c = 2/3$, where "c" is the designate for the "charm quark" (see also White (2008-R)).

From the above it is apparent that the general form for the electron/positron partial width as per the GEM is given by:

$$\Gamma_{v-ee} \approx (\alpha/2\pi)(10,042)(2m_e)(m_\rho/m_v)^3(\Sigma_i(q_i)^4). \quad (2)$$

Representative FDs may be constructed as associated with Eq. (1) and Eq. (2). In their simplest forms, they appear as follows:

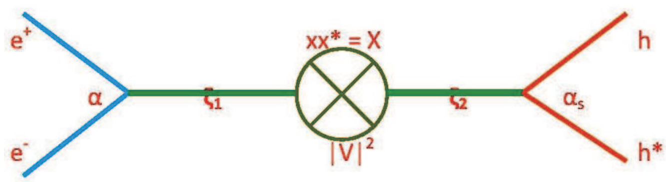

Figure 1. Basic Feynman Diagram for Conventional Vector Meson Formation and Decay into Hadrons (h and h^*) via the GEM.

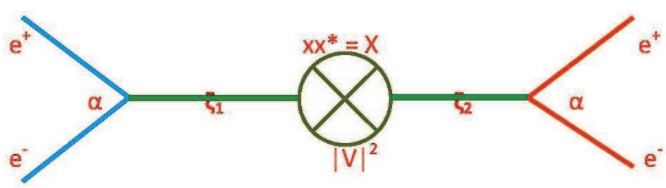

Figure 2. Basic Feynman Diagram for Conventional Vector Meson Formation and Decay into an electron/positron pair via the GEM.

In Figures 1 and 2 above "xx^*" represents the QQ^* associated with vector meson "X", ζ_1 represents the four-momentum propagator which starts out as a virtual photon at the collision vertex and ends up as a gluon absorbed by xx^*. The symbol, ζ_2, represents the gluon emitted in the decay of X. In Figure 1, said gluon couples to hadronic products with coupling strength α_s, whereas in Figure 2 it transmutes to a virtual photon which couples to the electron/positron pair with coupling strength α. In the case of Figure 1 the details of the absorption of ζ_1 are contained in the integrated absorption cross-section, and $|V|^2$, proportional to q_x^4, describes the formation of the spin one resonance. From there ζ_2 (a gluon) is emitted, resulting in coupling to hadrons $(h; h^*)$, the coupling at the latter vertex of magnitude α_s. The calculation of the width of the xx^* state, given the stated mechanism of a spin-flip of one of the "x quarks" due to a spin – spin interaction proportional to q_x^2, proceeds straight along the dictates of standard QED, therefore, except for the replacement of α by α_s at the hh^* vertex. The calculation of leptonic partial widths follows even more directly along the straight-forward lines of QED. In short, to obtain the leptonic and/or hadronic width associated with a given vector meson, one need only construct the relevant FD associated with the decay and then proceed to "calculate the FD" in accord with Eq. (1) and/or Eq. (2).

In Section 2 we consider the $\Psi(2S)$, in terms of its decay directly to dissolution and in terms of its decay involving specifically the $\Psi(1S)$. We will find that the associated FDs are somewhat more complicated than those seen in Figures 1 and 2, and we will see how an associated "form factor" comes into play. We will determine the matrix element associated with the $\Psi(2S) \rightarrow \Psi(1S) + Z$ decay, where "Z" represents any other decay product, and see that it is very likely representative of an electromagnetic interaction, as is $|V|$. In Section 3 we will carry out a similar undertaking as associated with the $\Upsilon(2S)$, whose matrix element for the $\Upsilon(2S) \rightarrow \Upsilon(2S) + Z$ decay will be seen to be strikingly similar to the corresponding one associated with the $\Psi(2S)$. We will close with concluding remarks in Section 4.

2. The $\Psi(2S)$

As is mentioned in White (2010), as to the $\Psi(1S)$:

Application of the GEM in accord with Figure 1, with $x = c$, seems reasonably straight-forward, but it turns out to be problematic. However, when one sees that the hadronic width of the $J(3097)$, or $\Psi(1S)$, given by the application of Eq. (1) in accord with Figure 1 with $x = c$, is roughly sixteen times too large, as compared to experimental results, coupled with the fact that the hadronic width of the $\Upsilon(1S)$ given by the application of Eq. (1) in accord with Figure 1 with $x = b$ is roughly sixteen times too small, as compared with experimental results, it becomes obvious as to what physically must transpire as regards both the $\Psi(1S)$ and the $\Upsilon(1S)$. Restricting the discussion to the $\Psi(1S)$ for the time being, in what we call "the zeroth order approximation", the basic cc^* structure of the $\Psi(1S)$ must make a point-like transition to an ss^* structure of equal mass, whereupon one of the s quarks undergoes a spin-flip to form the associated resonance (see White (2009-J)). The point-like transition from cc^* to ss^* is instantaneous, thus having no influence on the $\Psi(1S)$'s width. Indeed, the resonance does not even form until an s (or s^*) quark undergoes a spin-flip. That the cc^* to ss^* transition is necessary is quite understandable: The $\Psi(1S)$ is not massive enough for it to be able to decay into hadrons via emission of two c quarks; hence, it must transition to a quark pair of lesser bare mass each. The simplest possible assumption is that the cc^* transitions to the quark pair type characterized by the next smallest mass, viz., the s type. Nothing prevents the cc^* structure from decaying into leptons (e^+e^- and $\mu^+\mu^-$), however. It is found in White (2009-J), in fact, that in order for both the hadronic width of the $\Psi(1S)$ and the leptonic width of the $\Psi(1S)$ as determined via the GEM to match the results of experiment, $(8/9)^{\text{ths}}$ of the cc^* structure must undergo a slightly "un-point-like" transition to ss^*, described by a form factor, $f < 1$, which, in turn, decays into both hadrons and leptons as per Eq. (1) and Eq. (2), respectively, while $(1/9)^{\text{th}}$ of the original cc^* structure remains to decay into leptons exclusively. We may picture the complete details of the $\Psi(1S)$ formation and decay via the following two arrays of FDs, the first such array descriptive of what we may now call "the first order approximation" to the width of the $\Psi(1S)$, the second such array descriptive of what we call "the second order approximation", which follows along the lines of White (2009-J).

Immediately below are reproduced the FD associated with the "first order approximation" (Figure 3) to the width of the $\Psi(1S)$ and that associated with the "second order approximation" (Figure 4) to same:

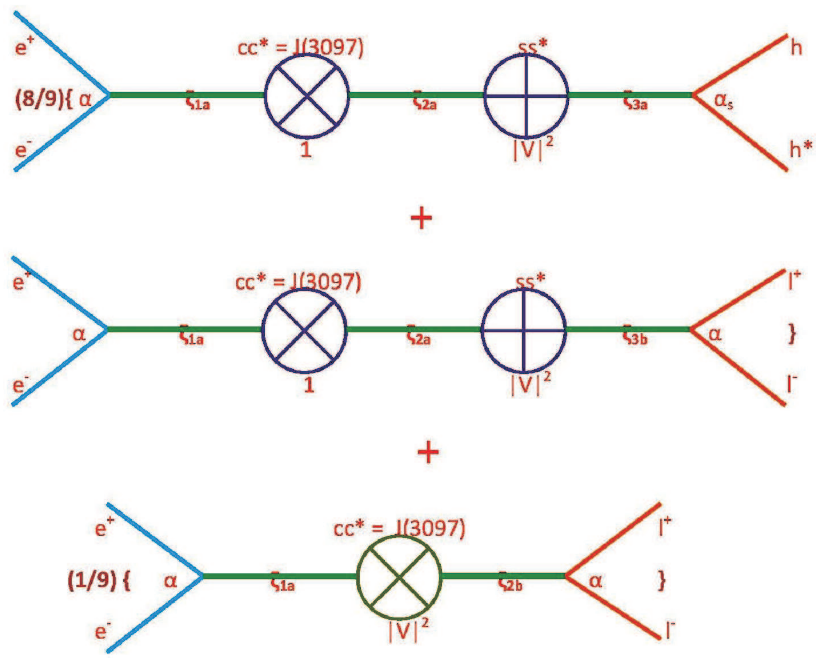

Figure 3. Feynman Diagram Array Characterizing the Formation and Decay of the $J(3097)$ in First Order Approximation via the GEM.

In Figures 3 and 4 above "l" represents a leptonic decay product, ζ_{2a} represents the gluon involved in a point-like transition from cc^* to ss^*, and all other "ζ" designates should be understood from previous discussion. Note that in the second order approximation the form factor $f = (1 - q_c^2) = (8/9)$ multiplies the entire FD array, whereas in the first order approximation it multiplies only the portion of the decay scheme that involves the s or s^* spin-flip. The second order approximation should be a better representation of reality than the first order approximation (and it is) because, logically, it is difficult to imagine how the point-like transition from

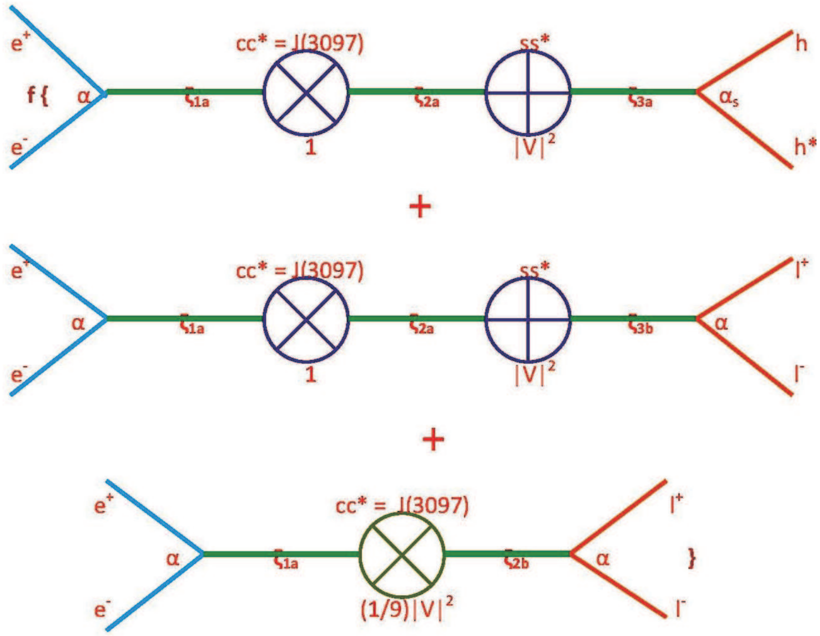

Figure 4. Feynman Diagram Array Characterizing the Formation and Decay of the $J(3097)$ in Second Order Approximation via the GEM.

cc^* to ss^* could take place in exactly the proper proportion every time without some kind of general "cross communication" between the cc^* state and the ss^* state. Accordingly, in second order approximation "f" influences the entire decay scheme, not just a part of it.

In White (2009-Ψ) we carried out the width calculations associated with the $\Psi(2S)$, but we did so in first order approximation and without the visual aid provided by a relevant FD. We determined in White (2009-Ψ) that the decay scheme of the $\Psi(2S)$ as regards its decay directly to dissolution, i.e., the $\Psi(2S) \rightarrow Z_1 + Z_2$ decay, where Z_1 and Z_2 represent any decay products other than $\Psi(1S)$, is very similar to that of the $\Psi(1S)$. As with the $\Psi(1S)$, a form factor, f', of value slightly less than "1" is associated with the $\Psi(2S)$ decay, but unlike the $\Psi(1S)$, it was determined that the remaining cc^* states (i.e., $(1-f')$ of the original complement) decay into hadrons, as well as leptons. Specifically, we determined that f' is given by

$$f' \approx 1 - (1/4\pi) = 0.9204.$$

However, f' above was derived employing the Meson Table associated with the PDG's 2004 report (PDG (2004)). In their 2008 Meson Table (PDG (2008)) the data associated with the $\Psi(2S)$ has changed significantly, its full width now listed as 13% more than that seen in PDG (2004). A recalculation of the form factor (see White (2009-Ψ, p.65)) now reveals that it is approximately the same as $f = (1 - q_s^2) = (8/9)$ above, the form factor associated with the $\Psi(1S)$. For notational purposes we now designate the $\Psi(2S)$ as "level 2" of the $J(3097)$ resonance, reflecting the assumption that the $\Psi(2S)$ is an excited state of the $J(3097)/\Psi(1S)$ resonance; as well, we designate the $\Psi(1S)$ as "level 1" of the $J(3097)$. Additionally, we designate the state of complete dissolution as "level 0". In terms of the above notation, then, the FD associated with the level 2 to level 0 transition, i.e., complete dissolution of the $\Psi(2S)$, in second order approximation appears as follows:

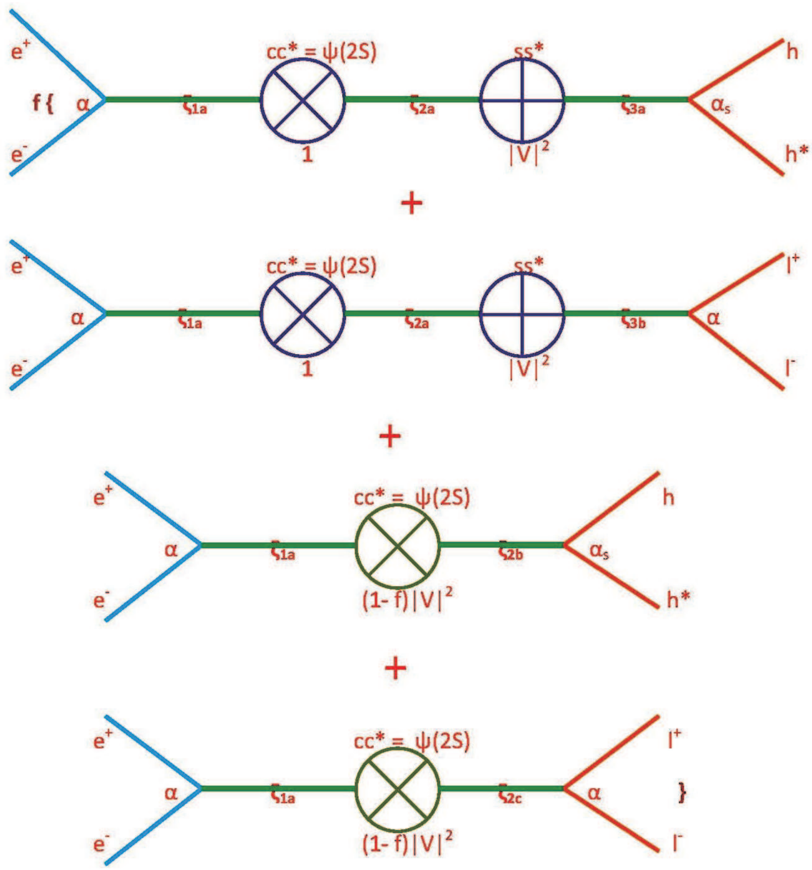

Figure 5. Feynman Diagram Array Characterizing the Formation and Decay of the $\Psi(2S)$ to Complete Dissolution in Second Order Approximation via the GEM.

In accord with Figure 5 we may now employ Eq. (1) and Eq. (2) in order to obtain the partial width of the level 2 to level 0 decay (the $2 \rightarrow 0$ decay) of the $\Psi(2S)$. Denoting said partial width as $\Gamma_{20}(\Psi : \text{GEM})$, we have:

$$\begin{aligned}\Gamma_{20}(\Psi:\text{GEM}) \approx f\{[(\alpha_s/2\pi)(10,042)(2m_e)(m_\rho/m_{\Psi(2S)})^3(q_s)^4)\\ +2.4(\alpha/2\pi)(10,042)(2m_e)(m_\rho/m_{\Psi(2S})^3(q_s^4)]\\ +(1-f')[(\alpha_s/2\pi)(10,042)(2m_e)(m_\rho/m_v)^3(q_c)^4\\ +2.4(\alpha/2\pi)(10,042)(2m_e)(m_\rho/m_{\Psi(2S)})^3(q_c)^4]\}. \quad (3a)\end{aligned}$$

The value of the strong coupling parameter at the $\Psi(2S)$ mass is given by

$$\alpha_s = 1.2[\ln(m_{\Psi(2s)}/50\,\text{Mev})]^{-1} = 1.2[\ln(3686/50)]^{-1} = 0.2791. \quad (3b)$$

Making the appropriate substitutions into Eq. (3a) (mass values are from PDG (2008) and the factor “2.4” in two spots takes into account muon and tauon production (PDG (2008), p. 111)), we obtain:

$$\begin{aligned}\Gamma_{20}(\Psi:\text{GEM}) \approx (8/9)\{[52.52\,\text{Kev}+3.30\,\text{Kev}]\\ +(1/9)[840.24\,\text{Kev}+52.73\,\text{Kev}]\}. \quad (3c)\end{aligned}$$

Hence,

$$\Gamma_{20}(\Psi:\text{GEM}) \approx \{46.68+2.93+82.99+5.21\}\,\text{Kev}. \quad (3d)$$

Separately, the hadronic partial width of the $2 \rightarrow 0$ transition is given by

$$\Gamma_{20-H}(\Psi:\text{GEM}) \approx \{46.68+82.99\}\,\text{Kev} = 129.67\,\text{Kev} \approx 130\,\text{Kev}, \quad (3e)$$

and the leptonic partial width of same is given by

$$\Gamma_{20-L}(\Psi:\text{GEM}) \approx \{2.93+5.21\}\,\text{Kev} = 8.14\,\text{Kev}. \quad (3f)$$

The hadronic partial width of the $\Psi(2S)$ $2 \rightarrow 0$ transition reported by the PDG in PDG (2008), p.111, is

$$\Gamma_{20-H}(\Psi:\text{PDG}) = (132 \pm 4)\,\text{Kev}, \quad (3g)$$

so the GEM produces a match with experiment as regards the hadronic partial width of same. However, the GEM’s result for the leptonic partial width of the $\Psi(2S)$ $2 \rightarrow 0$ transition of 8.14 Kev is 43% higher than the result for the associated leptonic partial width reported in PDG (2008), viz., (5.71 ± 0.10) Kev. Nevertheless, the GEM predicts the full width of the $\Psi(2S)2 \rightarrow 0$ transition as

$$\Gamma_{20-Full}(\Psi:\text{GEM}) \approx \{129.67+8.14\}\,\text{Kev} = 137.81\,\text{Kev} \approx 138\,\text{Kev}, \quad (3h)$$

which represents an exact match to the PDG (2008) report of:

$$\Gamma_{20-Full}(\Psi:\text{PDG}) \approx \{132+5.71\}\,\text{Kev.} \approx 138\,\text{Kev}. \quad (3i)$$

What we find extraordinarily interesting about the above result is that employing more recent data than that used in White (2009-Ψ), the form factor, $f=(1-q_s^2)=(8/9)$, is seen now to apply to both to the $\Psi(2S)$ and the $\Psi(1S)$, in each case yielding the hadronic partial width in decays to complete dissolution essentially exactly.

We now turn our attention to the $\Psi(2S) \rightarrow \Psi(1S) + Z$ decay. Since both the $\Psi(2S)$ and the $\Psi(1S)$ are spin one objects, the mechanism describing the $2 \rightarrow 1$ transition is not the same as that of the $2 \rightarrow 0$ transition, i.e., there is no spin-flip in the $2 \rightarrow 1$ transition. Nevertheless, using the data as to the $2 \rightarrow 1$ transition found in PDG (2008), we may estimate the strength of the interaction by expressing the associated partial width as

$$\Gamma_{21}(\Psi : \text{GEM}) \\ \approx f\{(\alpha_s/2\pi)(10,042)(2m_e)(m_\rho/m_{\Psi(2S)})^2[m_\rho/(m_{\Psi(2S)} - m_{\Psi(1S)})](q_{eff})^4) \\ + 2(\alpha/2\pi)(10,042)(2m_e)(m_\rho/m_{\Psi(2S)}^2[m_\rho/(m_{\Psi(2S)} - m_{\Psi(1S)})](q_{eff}^4)\}, \quad (4)$$

setting $\Gamma_{21}(\Psi : \text{GEM})$ equal to the PDG (2008) result of 182 Kev, and solving for q_{eff}, which represents the effective charge involved in the matrix element representing the $2 \rightarrow 1$ transition. One factor of "$(m_\rho/m_{\Psi(2S)})$" in Eq. (1) must be replaced by "$m_\rho/(m_{\Psi(2S)} - m_{\Psi(1S)})$" to reflect the fact that the final energy associated with the $2 \rightarrow 1$ transition is "$m_{\Psi(1S)}$", and the factor "2.4" appearing in the $2 \rightarrow 0$ calculations must be replaced by "2", as tauons cannot be emitted in the $2 \rightarrow 1$ decay. From here, a measure of $q_{eff} = (2/3)$ would signify that an electromagnetic interaction is most likely involved in the $2 \rightarrow 1$ transition, as all other factors besides the ones just mentioned in Eq. (1) were left unchanged. Accordingly, we find:

$$(8/9)\{26,620 + 1392\}q_{eff}^4 = 182. \quad (5a)$$

Hence,

$$24,900q_{eff}^4 = 182, \quad (5b)$$

from which

$$q_{eff}^4 = 0.007309, \quad (5c)$$

leading to

$$q_{eff} = 0.2924 \approx 0.88|q_s|. \quad (5d)$$

A similar calculation may be performed regarding the $2 \rightarrow 0$ transition.
Specifically, we may express $\Gamma_{20-Full}(\Psi : \text{GEM}) = 138\,\text{Kev}$ as

$$\Gamma_{20-Full}(\Psi : \text{GEM}) \\ \approx f\{[(\alpha_s/2\pi)(10,042)(2m_e)(m_\rho/m_{\Psi(2S)})^3(Q_{eff})^4) \\ + 2.4(\alpha/2\pi)(10,042)(2m_e)(m_\rho/m_{\Psi(2S)})^3(Q_{eff})^4] \\ = 138\,\text{Kev}, \quad (6)$$

where Q_{eff} represents the effective charge associated with the $2 \rightarrow 0$ transition. Accordingly, we find

$$(8/9)\{4254 + 267\}Q_{eff}^4 = 138. \quad (7a)$$

Hence,

$$4019Q_{eff}^4 = 138, \tag{7b}$$

from which

$$Q_{eff}^4 = 0.034337, \tag{7c}$$

leading to

$$Q_{eff} = 0.4305 \approx 1.29|q_s|. \tag{7d}$$

As noted above, the $2 \rightarrow 1$ transition does not involve a spin-flip, so it takes place via the emission of a longitudinal gluon, whereas the $2 \rightarrow 0$ transition takes place via the emission of a transverse gluon. Characterizing the square of the effective matrix element in the $2 \rightarrow 1$ transition as $\langle |V_{21}(\Psi)|^2\rangle$ and that of the $2 \rightarrow 0$ transition as $\langle |V_{20}(\Psi)|^2\rangle$, we see that

$$\langle |V_{21}(\Psi)|^2\rangle / \langle |V_{20}(\Psi)|^2\rangle = q_{eff}^4/Q_{eff}^4 = 0.007309/0.034337 = 0.2129, \tag{8}$$

indicating that the interaction potential matrix element in the longitudinal gluon emission is roughly 46% that of the transverse gluon emission.

3. **The $\Upsilon(2S)$**

In White (2010) it is shown that the width of the $\Upsilon(1S)$ is fully explained by assuming that (1) all bb^* (b represents the bottom quark of charge $q_b = -1/3$) states comprising the original structure of the $\Upsilon(1S)$ make a point-like transition to a cc^* structure, which then decays via a spin-flip, and (2) double gluon emission is involved in the decay. Condition 1 means that the relevant form factor, f_1, for the decay is equal to "1", while condition 2 suggests an additional route for decay not seen as associated with the less massive vector mesons. The FD array associated with the $\Upsilon(1S)$ decay appears in Figure 6.

Calculation of the above FD yields 54.02 Kev as the full width of the $\Upsilon(1S)$ . . . an exact match to experiment as reported in PDG (2008), p. 119. Also in White (2010) it is seen that the decay of the $\Upsilon(1S)$ contains the form factor, f_2, in exact analogy to the form factor, $f = 1 - q_s^2$, associated with $\Psi(1S)$ and $\Psi(2S)$ decays, i.e., $f_2 = 1 - q_c^2 = 5/9$. In exact analogy to the $J(3097)$, as well, the bb^* states decay only into leptons, and the double gluon emission route is in force, in analogy to the $\Upsilon(1S)$ decay. The FD associated with the "level 2", i.e., the $\Upsilon(2S)$ state, to "level 0", i.e., complete dissolution is thus as seen in Figure 7.

Defining $\Gamma_{20-Full}(Y : \text{GEM})$ as the full width of the $\Upsilon(2S)$ associated with the $2 \rightarrow 0$ transition, in analogy to Eq. (3a) we obtain:

$$\begin{aligned}\Gamma_{20}(Y : \text{GEM}) \approx f_2\{[(\alpha_s/2\pi)(10,042)(2m_e)(m_\rho/m_{Y(2S)})^3(q_c)^4 \\ + 3(\alpha/2\pi)(10,042)(2m_e)(m_\rho/m_{Y(2S)})^3(q_c^4)\end{aligned}$$

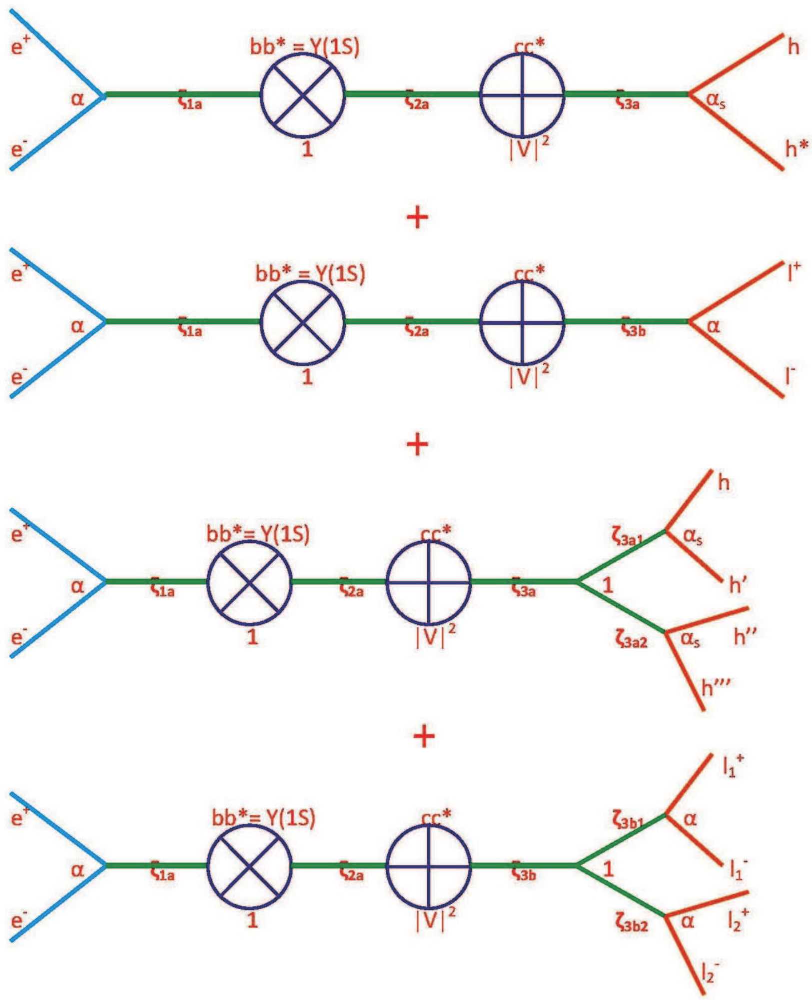

Figure 6. Basic Feynman Diagram for $\Upsilon(1S)$ Formation and Decay into Hadrons (h, h', h'', and h''') and leptons ($l_1^{\pm}$ and $l_2^{\pm}$) via the GEM.

$$
\begin{aligned}
&+(\alpha_s^2/2\pi)(10,042)(2m_e)(m_\rho/m_{Y(2S)})^3(q_c)^4\\
&+3(\alpha^2/2\pi)(10,042)(2m_e)(m_\rho/m_{Y(2S})^3(q_c^4)]\\
&+(1-f_2)[3(\alpha/2\pi)(10,042)(2m_e)(m_\rho/m_{Y(2S)})^3(q_b)^4]\}. \qquad (9a)
\end{aligned}
$$

The value of the strong coupling parameter at the $\Upsilon(2S)$ mass is given by

$$\alpha_s = 1.2[\ln(m_{Y(2s)}/50\,\text{Mev})]^{-1} = 1.2[\ln(10,023/50)]^{-1} = 0.2264. \quad (9b)$$

Making the appropriate substitutions into Eq. (3a) (mass values are from PDG (2008) and the factor “3” in three spots takes into account muon and tauon

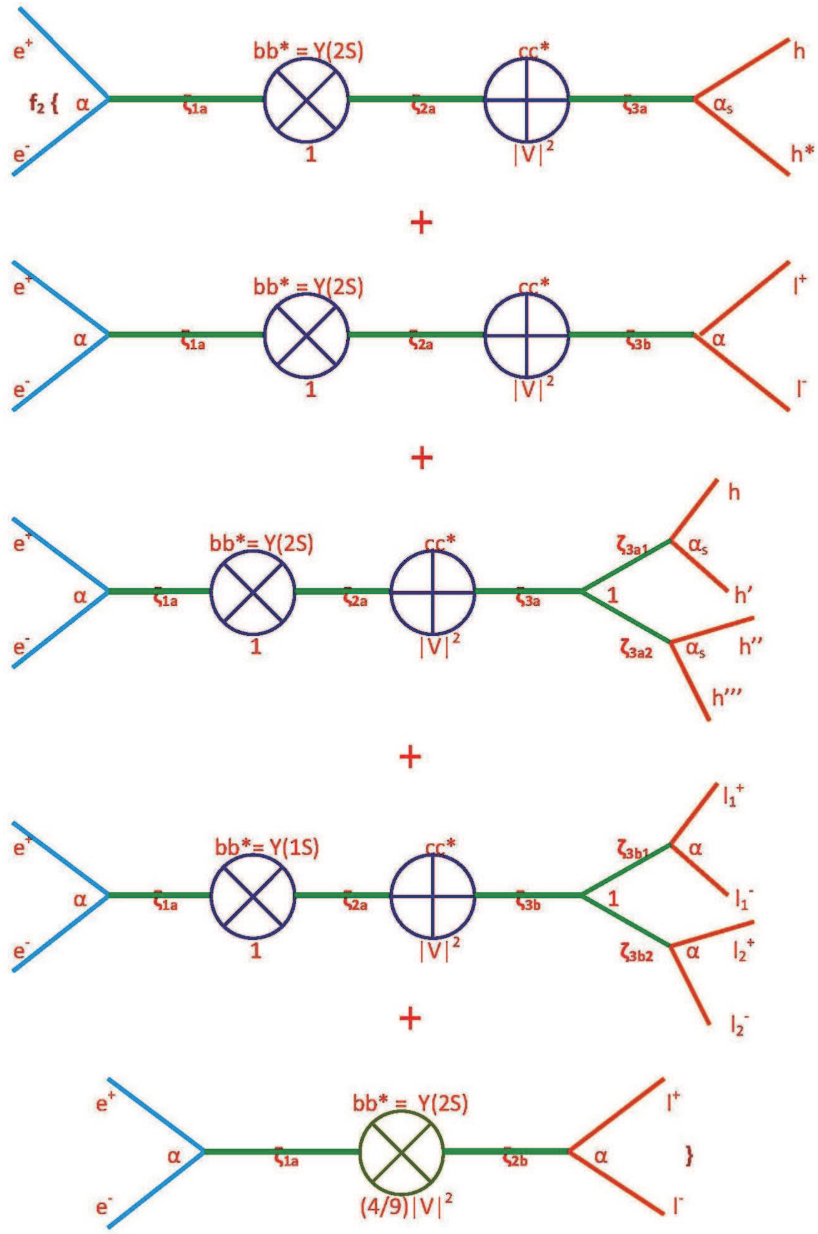

Figure 7. Basic Feynman Diagram for $\Upsilon(1S)$ Formation and Decay into Hadrons (h, h', h'', and h''') and leptons ($l_1^{\pm}$ and $l_2^{\pm}$) via the GEM.

production in accord with e-μ-τ universality), we obtain:

$$\Gamma_{20}(Y : \text{GEM}) \approx (5/9)\{[33.90 + 3.28 + 7.67 + 0.02] + (4/9)[0.20]\}\,\text{Kev.} \quad (9c)$$

Separating the hadronic (H) component from the leptonic (L) component of $\Gamma_{20}(Y : \text{GEM})$, we find

$$\Gamma_{20}(Y : \text{GEM}) \approx (23.09 + 1.88)\,\text{Kev}. \tag{9d}$$

Hence,

$$\Gamma_{20-H}(Y : \text{GEM}) \approx 23.09\,\text{Kev}, \tag{9e}$$

and

$$\Gamma_{20-L}(Y : \text{GEM}) \approx 1.88\,\text{Kev}. \tag{9f}$$

Both $\Gamma_{20-H}(Y : \text{GEM})$ and $\Gamma_{20-L}(Y : \text{GEM})$ are in excellent agreement with experiment, as the PDG (2008) report on p. 121 lists

$$\Gamma_{20-H}(Y : \text{PDG}) = 23.09\,\text{Kev} \quad \text{(an exact match)} \tag{9g}$$

and

$$\Gamma_{20-L}(Y : \text{PDG}) = (1.84 \pm 0.03)\,\text{Kev}. \tag{9h}$$

By analogy to the $2 \to 1$ transition associated with the $\Psi(2S)$ in the above section, we may determine the effective charge (q_{eff}) involved in the longitudinal gluon emission associated with the $2 \to 1$ transition of the $\Upsilon(2S)$. Accordingly, we have:

$$\begin{aligned}\Gamma_{21}&(Y : \text{GEM})\\ &\approx f_2\{(\alpha_s/2\pi)(10,042)(2m_e)(m_\rho/m_{Y(2S)})^2[m_\rho/(m_{Y(2S)} - m_{\Upsilon(1S)})](q_{eff})^4\\ &\quad + 2(\alpha/2\pi)(10,042)(2m_e)(m_\rho/m_{\Upsilon(2S})^2[m_\rho/(m_{Y(2S)} - m_{\Upsilon(1S)})](q_{eff}^4)\}.\end{aligned} \tag{10}$$

Setting $\Gamma_{21}(Y : \text{GEM})$ equal to $\Gamma_{21}(Y : \text{PDG}) = 7.05\,\text{Kev}$ and making the appropriate substitutions yields:

$$(5/9)\{3055 + 197\}q_{eff}^4 = 7.05. \tag{11a}$$

Thus,

$$1807 q_{eff}^4 = 7.05. \tag{11b}$$

Hence,

$$q_{eff}^4 = 0.003901, \tag{11c}$$

leading to

$$q_{eff} = 0.2499 \approx 0.75|q_b|. \tag{11d}$$

The corresponding effective charge (Q_{eff}) associated with the $2 \to 0$ transition of the $\Upsilon(2S)$ is determined via

$$\begin{aligned}&f_2\{[(\alpha_s/2\pi)(10,042)(2m_e)(m_\rho/m_{\Psi(2S)})^3(Q_{eff})^4\\ &\quad + 3(\alpha/2\pi)(10,042)(2m_e)(m_\rho/m_{\Psi(2S})^3(Q_{eff})^4] = 24.93\,\text{Kev}.\end{aligned}$$

Thus,

$$(5/9)\{1716+166\}Q_{eff}^4 = 24.93. \tag{12a}$$

Hence,

$$1046Q_{eff}^4 = 24.93, \tag{12b}$$

leading to

$$Q_{eff}^4 = 0.023834. \tag{12c}$$

Therefore,

$$Q_{eff} = 0.3929 \approx 1.18|q_b|. \tag{12d}$$

Analogous to the $\Psi(2S)$, characterizing the square of the effective matrix element in the $2 \to 1$ transition as $\langle |V_{21}(Y)|^2\rangle$ and that of the $2 \to 0$ transition as $\langle |V_{20}(Y)|^2\rangle$, we see that

$$\begin{aligned}\langle |V_{21}(Y)|^2\rangle/\langle |V_{20}(Y)|^2\rangle = q_{eff}^4/Q_{eff}^4 &= 0.003901/0.023834\\ &= 0.1637,\end{aligned} \tag{13}$$

indicating that the interaction potential matrix element in the longitudinal gluon emission is roughly 40% that of the transverse gluon emission in the case of the $\Upsilon(2S)$.

4. **Concluding Remarks**

We may summarize some of the important findings of the present work in Table 1, seen below, in which we list the type of meson from lightest to heaviest, the associated form factor, f_i, which also represents the fraction of the meson's original QQ^* state which makes a transition to the QQ^* state associated the next lowest mass, the decay mode (if any) associated with the fraction, $(1 - f_i)$, of the original QQ^* state which does not make the above transition, and whether or not there is a two-gluon mode of decay (Y = "yes"; N = "no").

Table 1. Summary of results as regards form factors in the present work.

Index	Meson	f_i	Original QQ^* state's decay mode	Two-gluon mode?
1	$\Psi(1S)$	$(1-q_s^2) = (8/9)$	cc^*: leptons	N
2	$\Psi(2S)$	$(1-q_s^2) = (8/9)$	cc^*: leptons and hadrons	N
3	$\Upsilon(1S)$	1		Y
4	$\Upsilon(2S)$	$(1-q_c^2) = (5/9)$	bb^*: leptons	Y

We note that f_i is either 1 or $(1-q_z^2)$, where q_z represents the charge of the quark of the next lowest mass from the type which originally forms either the $\Psi(1S)$ or the $\Upsilon(1S)$. The non-zero form factors arise, we believe, because of the impossibility in three cases . . . or the great difficulty in the case of the $\Psi(2S)$ . . . to be able to decay via two hadrons bearing the Q and Q^* of the original construction of the given meson. The lightest charm-bearing meson, for example, is the D of mass 1870 Mev. The $\Psi(1S)$ has a mass of 3097 Mev, which is only 1.66 times the mass of the D. The $\Psi(2S)$, on the other hand, has a mass (3686 Mev) of 1.97 times the mass of the D. Because of the Uncertainty Principle, therefore, some decays involving two Ds may at least be virtually possible for the $\Psi(2S)$, which explains why some of the cc^* states associated with the $\Psi(2S)$ do decay into hadrons. The B meson, however, has a mass of 5366 Mev, 57% as massive as the $\Upsilon(1S)$ and 54% as massive as the $\Upsilon(2S)$. Hence the original bb^* states making up either the $\Upsilon(1S)$ or the $\Upsilon(2S)$ do not decay into hadrons.

The form factor analysis presented above yields, in second order approximation, either exact or nearly exact matches with experiment as to hadronic partial widths and full widths associated with complete dissolution of all mesons listed above calculated via the GEM (see White (2010) in addition to the present article). Of the four mesons listed above, for only the $\Psi(2S)$ is the partial leptonic width associated with the decay to complete dissolution noticeably discrepant from experimental results as calculated via the GEM (8.14 Kev via the GEM vs. 5.71 Kev as per the PDG). However, we note in the latter connection that $\Gamma_{20-L}(\Psi : \text{PDG})$ has risen by 15% from 2004 (PDG (2004)) to 2008 (PDG(2008)), viz., from 4.96 Kev to 5.71 Kev. At that rate of increase, in another dozen years or so there may be another match for the GEM even there.

Our findings herein also include some interesting results regarding the $2 \to 1$ transition, the transition involving longitudinal gluons of much less energy than those transverse gluons involved in the $2 \to 0$ transition. We summarize such results in Table 2 below, in which we list the meson, the type of transition, the associated effective square of the relevant matrix element and its associated effective charge, and the ratio of the "$|V|^2$" associated with the $2 \to 1$ transition to that associated with the $2 \to 0$ transition.

Table 2. Longitudinal gluon matrix elements vs. transverse gluon matrix elements.

Meson	q_{eff}	$\langle\lvert V_{21}(X)\rvert^2\rangle$	$\langle\lvert V_{20}(X)\rvert^2\rangle$	Q_{eff}	$\langle\lvert V_{21}(X)\rvert^2\rangle/\langle\lvert V_{20}(X)\rvert^2\rangle$
$\Psi(2S)$	0.2924	0.007309	0.034337	0.4305	0.2129
$\Upsilon(2S)$	0.2499	0.003901	0.023834	0.3929	0.1637

From the above it is seen that the average interaction potential responsible for the "soft" longitudinal gluon emission is roughly four ninths that of the "hard" transverse gluon emission (from $\left[\frac{1}{2}(\sqrt{0.2129} + \sqrt{0.1637})\right] = 0.4330 \approx (4/9)$). Hence, it appears that the mitigating factor in the longitudinal gluon emission as compared to transverse gluon emission is electromagnetic in origin. It is easily shown, for example, that if we simply multiply those parts of Figure 4 that represent the $2 \to 0$ transition of the $\Psi(2S)$ resulting in hadronic decay products (after replacing one factor of "$(m_\rho/m_{\Psi(2S)})$" by "$[m_\rho/(m_{\Psi(2S)} - m_{\Psi(1S)})]$" in the GEM formula for $\Gamma_{20}(\Psi : \text{GEM})$) by $q_c^4 = (2/3)^4 = 16/81$, we obtain 160 Kev for the hadronic partial width of the $2 \to 1$ transition of the $\Psi(2S)$, a figure reasonably close to the PDG (2008) report of 178 Kev. Hence, once again, the electromagnetic interaction shows itself to be an important constituent in the formation and decay of vector mesons.

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