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Research Article

# Lower Bounds for Gas Turbines Aircraft Engines

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**Abstract.** Turbine maintenance process is performed periodically at predefined time slots to replace certain turbine parts by new or refurbished parts. The developed heuristics will address the scheduling of turbine maintenance problem to maximize crafts operation time. Scheduling is based on the life span of the replaced parts. Mathematical modeling for the lower bounds of the aircraft turbine maintenance problem will be presented to achieve the desired goal. this study is based on three heuristic categories, the randomized lower bounds, the utilization of the iterative methods solving the subset sum problems and the repeating of the resolution of the knapsack problems.

**Keywords.** Heuristic; Scheduling; Randomization algorithms; Parallel Turbines; Gas turbines aircraft engines

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## 1. Introduction

The aviation sector is one of the most important sectors that directly affect the lives of individuals and the growth and progress of countries. It is also one of the sectors that are most affected by a set of contradictory criteria and one of these are the cost, due to the tight profit margin, cost management is considered a critical factor in the success of aviation sector. The cost in the aviation sector includes but not limited to, Operational cost and maintenance cost. In the maintenance process, turbine maintenance has the highest cost and direct effect on the operation of aircraft and thus on the sustainability of the companies owning these aircraft. profits and sustainability are specified directly by aircraft operation time, which in turn is specified by the assurance of maintenance free-errors process. Scheduling in this research will be deployed to ensure aircraft longest operation period and lowest downtime thus the desired financial gain. The main goal of this research is to maximize aircrafts operation time without affecting turbine maintenance process schedule. This goal will be presented by a set of heuristics that are mathematically modeled and proofed. Cost management in turbine maintenance is used with $F100-PW-100$ engine maintenance in [2].

Many researchers discussed Maximization of the minimum problem such as [1, 3, 5, 6]. The authors in [1] analyze the *LPT* algorithm to characterize the worst-case behavior of the *LPT* algorithm applied to the nonpreemptive tasks scheduling to maximize the minimum processor completion time.

Maximization of the minimum was presented for the first time by [7] as an approximation scheme. In 1997 [7] presented the problem of maximizing the minimum machine completion time, the researchers derived the first polynomial-time approximation scheme, they also, stated that this result can be applied for modular gas turbine aircraft engines, as an algorithm to sequence the maintenance process steps. A two objectives on-line ordinal assignment problem, is another study in the same area presented by [5]. the objectives were, minimizing the $lp$ norm of the makespan and maximizing the minimum machine completion time.

Other researchers presented the optimal solution problem,such as the research introduced by [3], in this work the authors proposed the first optimal solution based on the branch and the bound method while utilizing the tight lower and upper bounds, these objectives were tested by developing several algorithmic features. Another makespan optimal solutions on identical parallel machines are viewed by [6], the given experimental results derived dominance rules that outperform existing approaches (at that time). To enhance the effective dominance criteria for small ratios of $n$ to $m$ that resulted in their research; the authors propose an exact branch-and-bound algorithm based on structural patterns of optimal schedules.

The structure of this paper consist of the following sections. Section 2, describes the proposed problem. While Section 3, details the proposed lower bounds resulted in the new heuristics. Finally, conclusion is elaborated in Section 4.

## 2. Problem Description

Maximization the minimum cumulative lifespan problem is explained as follows. Let $P$ be a set with a deterministic number of spare parts $Sp_n$, that has to be assigned to a fixed number of turbines $Tu_n$. Each turbine is indexed by $i$ and denoted by $Tu_i$. Each part lifespan $j$ is denoted by $lp_j$. The turbine that has some parts to be changed and that for each turbine only one part is permitted at a time. Available parts are with no release date i.e. parts needed for the maintenance process are available for immediate delivery (delivery time equals to zero). This problem aims to maximizing the minimum turbines working time denoted by $Tu_{\min}$. We denote by $Cl_j$ the total parts lifespan $j$.

**Example 1.** Let $SP_n = 6$ and $Tu_n = 2$. Table 1 present the lifespan $lp_j$ for each part $j$.

**Table 1.** Lifespan-parts distribution for Example 1

| $j$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $lp_j$ | 2 | 4 | 3 | 10 | 5 | 1 |

Assume that we have to choose a heuristic that assigns the parts as follows. On turbine 1 we schedule the parts 1, 5 and 6. On turbine 2 we schedule the parts 2, 3 and 4. Based on the latter schedule, The total lifespan of the first turbine equals to 8. while the total lifespan of the second turbine is 17. The minimum working time is $Tu_{\min} = 8$ and the maximum working time is $Tu_{\max} = 17$. The main objective here is to find the schedule that maximizes the minimum working time of $Tu_{\min}$. Thats why another more efficient sequences are needed to assign parts to turbines with a minimum operating time that is more than 8.

Based on the standard three-field notation described in [4], the studied problem can be denoted as $P\|C_{min}$.

## 3. Lower Bounds Heuristics

For this part of the research, the presented heuristics were initiated based on the comparison between the proposed heuristics and the $LPT$ dispatching rule given in [3]. The proposed heuristics consist of 6 lower bounds. The first 4 heuristics are based on choice randomization of the part which will be scheduled for the turbine, another one is based on repeating subset-sum problems resolutions heuristic, while the last one is based on the repeating Knapsack problem resolution heuristic.

### 3.1 Decreasing order of lifespan heuristic ($DOL$)

The parts that will be scheduled for the maintenance problem will be ordered in a decreasing order based on its lifespan. After that, the parts with the highest lifespan are assigned to the most available turbine.

### 3.2 Iteratively Randomized Selection heuristic $IRS_k$

For this type of heuristics, this paper is based on the selection of the part with the highest lifespan to be scheduled for the turbine having the minimum total operating time with some probabilistic method. The proposed heuristics are articulated on probabilistic choice between the $k$ highest part with $k \in \{2,3,4,5\}$ respectively for the heuristics $IRS_1$, $IRS_2$, $IRS_3$ and $IRS_4$. The chosen part is selected among the $k$ first available parts with the highest lifespan that has the probability $\beta$. This probability is fixed as follows:

- We chose randomly a number $r$ in $[1-k]$. The selected part will be the $r^{th}$ largest unscheduled part. We schedule the selected part on the most available turbine.

- Denoted by $U_p$ the number of unassigned parts. If $U_p < k$, then $r$ will be chosen randomly between $[1 - U_p]$.

For a fixed $k$, the iteration number is fixed to $limit$. For this heuristic we can adopt $limit = 1000$. The algorithm of the randomized parts-turbines is given as follows:

**Algorithm 1** Turbine Parts Randomized algorithm: $TPR(k)$

1: Set $it = 1$.
2: Set $P_k = P$.
3: Select randomly $r$ between $[1 - k]$.
4: Assign the $r^{th}$ highest parts, which will be denoted by $L_p$, on the more available turbine.
5: $P_k = P_k \setminus L_p$, if $P_k \neq \{\}$ goto 3.
6: Find $Tu^{it}_{\min}$
7: $it = it + 1$.
8: if $it \leq limit$ goto 2.
9: Stop, return $RPT(k) = \max\limits_{1 \leq it \leq limit} Tu^{it}_{\min}$.

Algorithm 1 has not obtained the results of the proposed lower bounds due to the results obtained for a fixed $k$. Thus, the algorithm described above requires the iteration from 2 to $k$. The value obtained by the proposed lower bound $IRS_k$ for a fixed $k$ will be represented by the coming algorithm.

**Algorithm 2** Iterative Randomized Selection algorithm $IRS_k$

1: **for** $j = 2$ to $k$ **do**.
2: Find $TPR(j)$.
3: **end for**
4: $IRS_k = \max\limits_{2 \leq j \leq k} TPR(j)$.

### 3.3 Repeating subset-sum problems resolutions ($RSS$)

For this heuristic, a repeating of resolution of subset-sum problems will be applied. Firstly, we constitute the set of jobs denoted by $\tilde{J}$ which is resulted by the union of the jobs scheduled on the most charged machine and the jobs scheduled on the least charged machine. Now, we call the following system $S$ to find the solution of the problem of 2 machines $P2\|C_{\min}$:

$$S : \begin{cases} Z = \max \sum\limits_{j \in \tilde{J}} p_j x_j, & (1) \\ \text{s. t.} \sum\limits_{j \in \tilde{J}} p_j x_j \leq \left\lfloor \dfrac{\sum_{j \in \tilde{J}} p_j}{2} \right\rfloor, & (2) \\ x_j \in \{0,1\}, \ \forall\, j \in \tilde{J}. & (3) \end{cases}$$

The variable $x_j$ is related for every job $j \in \tilde{J}$. $x_j = 1$ if the job $j$ is scheduled on the machine which having the minimum load, otherwise $x_j = 0$. We denote by $X$, the vector contains all $x_j$

values with $j \in \tilde{J}$. Let $V = \{j \in \tilde{J}, x_j = 1\}$. The solution derived by the system $S$ gives two groups of jobs. The first group is the group denoted by $V$ with $V = \{j \in \tilde{J}, x_j = 1\}$. The second one is $\tilde{J} \setminus V$. We schedule now the constructed sets ($V$ and $\tilde{J} \setminus V$) respectively to the least charged machine and to the most charged machine. We calculate the new value of $C_{min}$. We repeat several times the iteration with consideration of the jobs scheduled on the two most-least charged machines and solve the system $S$ and so on until any enhancement occurs.

### 3.4 Repeating Knapsack problem resolution ($RKR$)

This heuristic utilizes the same idea described for $RMSS$. Besides that, we give some performance. The enhancement is focalized on the resolution of the $P2\|C_{\min}$ by a knapsack problem instead of a subset-sum problem. The formulation of the initial problem must have some modification as follows: To apply the knapsack problem correctly we must introduce a variable changing as follows $\hat{p}_j = |\tilde{J}|p_j - 1$ for $j \in \tilde{J}$. We denoted by $\tilde{J}_w$ the set constructed by the new processing time and by $\hat{P}$ the vector given by all elements of $\hat{p}_j$. Now, we can give the formulated following knapsack problem:

$$K: \begin{cases} Q = \max \sum_{j \in \tilde{J}} \hat{p}_j y_j, & (4) \\ \text{s. t.} \sum_{j \in \tilde{J}} p_j y_j \leq \left\lfloor \dfrac{\sum_{j \in \tilde{J}} p_j}{2} \right\rfloor, & (5) \\ y_j \in \{0,1\}, \quad \forall\, j \in \bar{J}. & (6) \end{cases}$$

**Proposition 1.** *The system* ($K$) *is more performed than* ($S$) *to searching result for the studied problem.*

## 4. Conclusion

This research proposed 6 heuristics to present the problem of maximization the minimum cumulative lifespan. The proposed heuristics are classified into three categorize, the first category consists of 4 heuristics based on randomization of the chosen turbine part that will be scheduled (Lower bounds heuristics). The second category, Repeating subset-sum problems resolutions ($RSS$), that will iteratively solve the mathematically formulated set of subproblems resulted from the original problem. The third category, Repeating Knapsack problem resolution ($RKR$), which will handle the mathematical formulation of the original problem. The presented heuristics can be used as a basis to present an approximate good solution of the maximization the minimum problem.

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## Competing Interests

The authors declare that they have no competing interests.

## Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

## References

[1] B. L. Deuermeyer, D. K. Friesen and M. A. Langston, Scheduling to maximize the minimum processor finish time in a multiprocessor system, *SIAM Journal on Algebraic Discrete Methods* **3**(2) (1982), 190 – 196, DOI: 10.1137/0603019.

[2] D.B. Edmunds, Modular engine maintenance concept considerations for aircraft turbine engines, *Aircraft Engineering and Aerospace Technology* **50**(1) (1978), 14 – 17, DOI: 10.1108/eb035417.

[3] M. Haouari and M. Jemmali, Maximizing the minimum completion time on parallel machines, *4OR* **6**(4) (2008), 375 – 392, DOI: 10.1007/s10288-007-0053-5.

[4] E. L. Lawler, J. K. Lenstra, A. H. G. R. Kan and D. B.Shmoys, Sequencing and scheduling: algorithms and complexity, Chapter 9, *Handbooks in Operations Research and Management Science* **4** (1993), 445 – 522, DOI: 10.1016/S0927-0507(05)80189-6.

[5] Z. Tan, Y. He and L. Epstein, Optimal on-line algorithms for the uniform machine scheduling problem with ordinal data, *Information and Computation* **196**(1) (2005), 57 – 70, DOI: 10.1016/j.ic.2004.10.002.

[6] R. Walter, M. Wirth and A. Lawrinenko, Improved approaches to the exact solution of the machine covering problem, *Journal of Scheduling* **20**(2) (2017), 147 – 164, DOI: 10.1007/s10951-016-0477-x.

[7] G. J. Woeginger, A polynomial-time approximation scheme for maximizing the minimum machine completion time, *Operations Research Letters* **20**(4) (1997), 149 – 154, DOI: 10.1016/S0167-6377(96)00055-7.