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Reliability Analysis of A Two Non-Identical Unit System with Repair and Replacements having Correlated Failures and Repairs

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Abstract. A two non-identical unit parallel system model is investigated and analysed. The system consists of two non-identical units, priority and non-priority arranged in parallel configuration. The priority unit is given preference in repair over non-priority unit. The priority unit is repairable but non-priority unit is not repairable and to be replaced after a random period of operation. The non-priority unit gives a signal for its replacement before going into failure mode and may be replaced by a new unit just after giving signal or upon failure. The failure and repair times of the priority unit are taken to be correlated random variables having bivariate exponential distribution. The failure time and the time after which non-priority unit gives signal are exponential random variables with different parameters and the time for replacement of non-priority unit is taken as general. The reliability analysis of this model has been carried out by using regenerative point technique.

1. Introduction

Two unit parallel system models have been studied widely by several authors including Osaki, Nakagawa, Yamashiro, Goel, Gupta, Murari etc. in the field of reliability theory, under different set of assumptions. In most of these papers it is assumed that the failure and repair times are uncorrelated random variables but in real life this assumption does not seem to be realistic and in many practical situations failure and repair times are found to be correlated. The concept of correlation in failure and repair times of a unit was first introduced by Goel and Shrivastva in 1991 in a two unit redundant system with provision of rest. Later on Gupta, Chaudhary, Kishan and Kumar extended the concept of correlated failure and repair as well as correlated life times in complex models.

In the present study we investigate and analyze a two non-identical unit system having priority unit and non-priority unit arranged in parallel configuration. Initially both the units are working. The system failure occurs only when both the units stop functioning. The priority unit is repairable but non-priority unit is not repairable and to be replaced after a random period of operation. If during the repair of the priority unit, the non-priority unit fails or gives signal for its replacement then it has to wait for its replacement till the repair of the priority unit is completed. The failure time and random period of time after which non-priority unit gives signal are exponential random variables with different parameters and the time of replacement of non-priority unit is taken as general. The failure and repair times of the priority unit are taken to be correlated random variables having bivariate exponential distribution.

$$f(x,y)=\alpha\beta(1-r)e^{-(\alpha x+\beta y)}I_0(2\sqrt{\alpha\beta r x y});\quad \alpha,\beta,x,y>0,\ |r|<1$$

where

$x=$ random variable denotes the time to failure of priority unit,

$y=$ random variable denotes the time to repair of priority unit,

$r=$ correlation coefficient (x,y)

and $I_0(z)=\sum\limits_{K=0}^{\infty}\frac{(z/2)^K}{(K!)^2}$ is modified Bessel's function of type one and order zero.

Using regenerative point technique the following important reliability characteristics of interest are obtained:

(i) Reliability and mean time to system failure (MTSF).
(ii) Pointwise and steady-state availabilities of the system.
(iii) Expected up-time of the system and expected busy period of the repairman during $(0,t)$.
(iv) Expected number of repairs during $(0,t)$.
(v) Net expected profit incurred by the system during $(0,t)$ and in steady state.

2. **System Description and Assumptions**

A two non-identical unit system is analysed under following practical assumptions:

- The system is having two non-identical units namely priority unit and non-priority unit arranged in parallel configuration.
- Initially both the units are working.
- The priority unit is repairable but non-priority unit is not repairable and to be replaced after a random period of operation.
- The priority unit is given preference in repair over the replacement of the non-priority unit.
- The non-priority unit gives a signal for its replacement before going into failure mode and may be replaced by a new unit just after giving signal or upon failure.
- The failure and repair times of the priority unit are taken to be correlated random variables having bivariate exponential distribution.
- The failure time and random period of operation after which non-priority unit gives signal are exponential random variables with different parameters and the time of replacement of non-priority unit is taken as general

- Repairs are perfect that is repair facility never does any damage to the system.

3. **Notations and States of the System**

X	:	random variable representing failure time of the priority unit
Y	:	random variable representing repair time of the priority unit
$F(\cdot)$	:	d.f. of repair time of non-priority unit
$f(x,y)$	:	the joint p.d.f. of X and Y i.e. $f(x,y) = \alpha\beta(1-r)e^{-(\alpha x+\beta y)}I_0(2\sqrt{\alpha\beta rxy})$; $\alpha,\beta,x,y>0$, $\|r\|<1$
$g(x)$	:	marginal p.d.f. of X i.e. $g(x) = \alpha(1-r)e^{-\alpha(1-r)x}$
$k(y/x)$	:	conditional p.d.f. of $(y/X=x)$ i.e. $k(y/x) = \beta e^{-\beta y-\alpha rx}I_0(2\sqrt{\alpha\beta rxy})$
θ_1	:	failure rate of non-priority unit.
θ_2	:	parameter of the signal time distribution.
θ	:	parameter of repair time distribution of non-priority unit.
$q_{ij}(t)$	:	p.d.f. of transition time from state S_i to S_j in time $(0,t)$
$Q_{ij}(t)$	:	c.d.f. of transition time from state S_i to S_j in time $(0,t)$
p_{ij}	:	unconditional steady state probability of direct transition from the regenerative state S_i to S_j.
$p_{ij}^{(m)}$	:	unconditional steady state probability of transition from the regenerative state S_i to S_j via non-regenerative state S_m.
$p_{ij\|x}$	:	conditional steady state probability of transition from the regenerative state S_i to S_j given that the unit under repair in state S_i entered failure mode after an operation of time x.
μ_i	:	unconditional mean sojourn time in state S_i.
$\mu_{i\|x}$	:	conditional mean sojourn time in state S_i given that the unit under repair in this state entered failure mode after an operation of time x.*
$Z_i(t)$	:	probability that the system sojourns in state S_i upto time t.
$*$	:	Symbol for Laplace transform, i.e. $f^*(s) = \int e^{-st}f(t)dt$
$\sim$	:	Symbol for Laplace-Stieltjes transform i.e $\tilde{F}(s) = \int e^{-st}dF(t)$

Symbols for the state of the system

N_{10}	:	priority unit is in normal mode and operative.
N_{20}	:	non-priority unit is in normal mode and operative.
N_{1r}	:	priority unit is under repair.
N_{2sR}	:	non-priority unit is operative and gives signal for its replacement.
N_{2ure}	:	non-priority unit is under its replacement.
N_{2wfrep}	:	non-priority unit is waiting for its replacement.

*Limits of integration are not mentioned whenever they are zero and infinity

With the help of the above symbols the possible states of the system are:

$$S_0 = [N_{10}, N_{20}], \quad S_1 = [N_{1r}, N_{20}], \quad S_2 = [N_{10}, N_{2ure}],$$
$$S_3 = [N_{10}, N_{2SR}], \quad S_4 = [N_{1r}, N_{2SR}], \quad S_5 = [N_{1r}, N_{2wfrep}].$$

The transition diagram along with all transitions is shown in Figure 1.

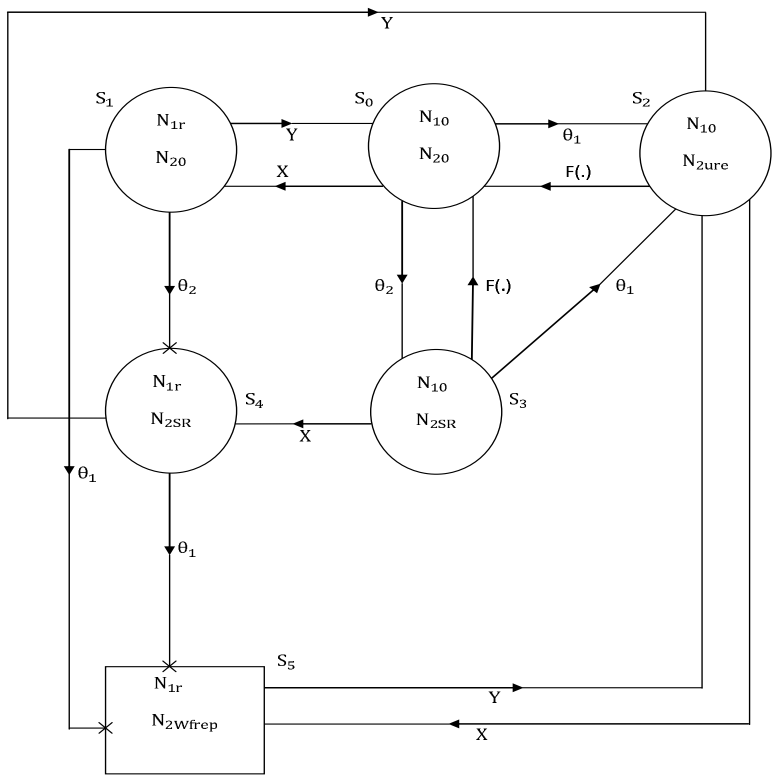

Figure 1. Transition diagram

4. **Transition Probabilities and Sojourn Times**

First we find the following conditional direct and indirect steady-state probabilities of transition:

$$p_{10|x} = \int dK(u|x)e^{-(\theta_1+\theta_2)u} = k^*[(\theta_1+\theta_2)|x] = \frac{\beta}{\theta_1+\theta_2+\beta}e^{-\frac{\alpha r(\theta_1+\theta_2)x}{\theta_1+\theta_2+\beta}},$$

$$p_{42|x} = \int e^{-\theta_1 u} dK(u|x) = k^*(\theta_1|x),$$

$$p_{12|x}^{(4)} = \theta_2 \int e^{-(\theta_1+\theta_2)u} \bar{K}(u|x)du \int_u^\infty \frac{dK(v|x)e^{-\theta_1 v}}{\bar{K}(u|x)e^{-\theta_1 u}}$$
$$= \int dK(v|x)e^{-\theta_1 v}(1-e^{-\theta_2 v}) = k^*(\theta_1|x) - k^*[(\theta_1+\theta_2)|x].$$

Similarly

$$p_{12|x}^{(5)} = \frac{\theta_1}{\theta_1+\theta_2}[1-k^*\{(\theta_1+\theta_2)|x\}],$$
$$p_{12|x}^{(4,5)} = 1-k^*(\theta_1|x) - \frac{\theta_1}{\theta_1+\theta_2}[1-k^*\{(\theta_1+\theta_2)|x\}],$$
$$p_{42|x}^{(5)} = \theta_1 \int e^{-\theta_1 u}\bar{K}(u|x)du \int_u^\infty \frac{dK(v|x)}{\bar{K}(u|x)}$$
$$= \int dK(v|x)(1-e^{-\theta_1 v}) = 1-k^*(\theta_1|x).$$

Unconditional steady state probabilities of transition are

$$p_{01} = \frac{\alpha(1-r)}{\alpha(1-r)+\theta_1+\theta_2}, \qquad p_{02} = \frac{\theta_1}{\alpha(1-r)+\theta_1+\theta_2},$$
$$p_{03} = \frac{\theta_2}{\alpha(1-r)+\theta_1+\theta_2}, \qquad p_{20} = \tilde{F}[\alpha(1-r)],$$
$$p_{25} = 1-\tilde{F}[\alpha(1-r)], \qquad p_{30} = \tilde{F}[\alpha(1-r)+\theta_1],$$
$$p_{32} = \frac{\theta_1}{[\alpha(1-r)+\theta_1]}[1-\tilde{F}\{\alpha(1-r)+\theta_1\}],$$
$$p_{34} = \frac{\alpha(1-r)}{[\alpha(1-r)+\theta_1]}[1-\tilde{F}\{\alpha(1-r)+\theta_1\}],$$

$$p_{10} = \int p_{10|x}g(x)d(x) = \frac{\beta\alpha(1-r)}{\theta_1+\theta_2+\beta}\int e^{-\frac{\alpha r(\theta_1+\theta_2)x}{\theta_1+\theta_2+\beta}} e^{-\alpha(1-r)x}dx$$
$$= \frac{\beta(1-r)}{[\theta_1+\theta_2+(1-r)\beta]}.$$

Similarly

$$p_{12}^{(5)} = \frac{\theta_1}{[\theta_1+\theta_2+(1-r)\beta]},$$
$$p_{12}^{(4)} = \left\{\frac{\beta(1-r)}{[\theta_1+(1-r)\beta]} - \frac{\beta(1-r)}{[\theta_1+\theta_2+(1-r)\beta]}\right\},$$
$$p_{12}^{(4,5)} = \left\{\frac{\theta_1}{[\theta_1+(1-r)\beta]} - \frac{\theta_1}{[\theta_1+\theta_2+(1-r)\beta]}\right\},$$
$$p_{42} = \frac{\beta(1-r)}{[\theta_1+(1-r)\beta]}, \qquad p_{42}^{(5)} = \frac{\theta_1}{[\theta_1+(1-r)\beta]}.$$

It can be easily seen that the following results hold good:

$$p_{01}+p_{02}+p_{03}=1, \tag{1}$$

$$p_{01}+p_{12}^{(4)}+p_{12}^{(5)}+p_{12}^{(4,5)}=1, \tag{2}$$

$$p_{20}+p_{25}=1, \tag{3}$$

$$p_{30}+p_{32}+p_{34}=1, \tag{4}$$

$$p_{42}+p_{42}^{(5)}=1, \tag{5}$$

$$p_{52}=1\,. \tag{6}$$

To obtain mean sojourn time in a state, let T_i be the sojourn time in state S_i then mean sojourn time in that state is given by

$$\mu_i=\int P(T_i>t)dt\,.$$

First we obtain the following conditional mean sojourn times

$$\mu_{1|x}=\int e^{-(\theta_1+\theta_2)u}\bar{K}(u|x)du=\frac{1}{\theta_1+\theta_2}[1-\tilde{K}(\theta_1+\theta_2)|x]\,,$$

$$\mu_{5|x}=\int \bar{K}(u|x)du=\frac{1+\alpha r x}{\beta}\,.$$

Unconditional mean sojourn times are given by

$$\mu_0=\int e^{-[\theta_1+\theta_2+\alpha(1-r)]x}dx=\frac{1}{\theta_1+\theta_2+\alpha(1-r)},$$

$$\mu_1=\frac{1}{\theta_1+\theta_2+\beta(1-r)}, \qquad \mu_2=\frac{1}{\alpha(1-r)}[1-\tilde{F}\{\alpha(1-r)\}],$$

$$\mu_3=\frac{1}{\theta_1+\alpha(1-r)}[1-\tilde{F}\{\theta_1+\alpha(1-r)\}], \qquad \mu_4=\frac{1}{\theta_1},$$

$$\mu_5=\int \psi_{5|x}g(x)dx=\frac{1}{\beta(1-r)}\,.$$

5. **Analysis of Reliability and MTSF**

Let T_i be the time to system failure when system starts functioning from regenerative state S_i at time $t=0$. Then the reliability of the system is given by

$$R_i(t)=P[T_i>t]\,.$$

To obtain $R_i(t)$, we regard failed state S_5 as absorbing state. Using basic probabilistic arguments, the recursive relations among $R_i(t)$; $(i=0,1,2,3,4)$ can be easily developed and taking L.T. of the relations and solving for $R_0^*(s)$, we get

$$R_0^*(s)=\frac{N_1(s)}{D_1(s)} \tag{7}$$

where

$$N_1(s) = Z_0^* + q_{01}^* Z_1^* + (q_{01}^* q_{12}^{(4)*} + q_{02}^* + q_{03}^* q_{34}^* q_{42}^*) Z_2^* + q_{03}^* Z_3^* + q_{03}^* q_{34}^* Z_4^*$$

and

$$D_1(s) = 1 - q_{01}^* q_{10}^* - q_{01}^* q_{12}^{(4)*} q_{20}^* - q_{02}^* q_{20}^* - q_{03}^* q_{30}^* - q_{03}^* q_{34}^* q_{42}^* q_{20}^* .$$

Here the argument 's' has been omitted for brevity.
Where Z_0^*, Z_1^*, Z_2^*, Z_3^* and Z_4^* are the L.T.'s of

$$Z_0(t) = e^{-[\theta_1+\theta_2+\alpha(1-r)]t}, \quad Z_1(t) = e^{-(\theta_1+\theta_2)t}\bar{K}(t|x),$$
$$Z_2(t) = e^{-\alpha(1-r)t}\bar{F}(t), \qquad Z_3(t) = e^{-[\theta_1+\alpha(1-r)]t}\bar{F}(t),$$
$$Z_4(t) = e^{-\theta_1 t}\bar{K}(t|x).$$

Taking inverse L.T. of (7), we get the reliability of the system. To get MTSF, we use the well known formula

$$E(T_0) = \int R_0(t)dt = \lim_{s\to 0} R_0^*(s) = \frac{N_1(0)}{D_1(0)} \tag{8}$$

where

$$N_1(0) = \mu_0 + p_{01}\mu_1 + (p_{01}p_{12}^{(4)} + p_{02} + p_{03}p_{34}p_{42})\mu_2 + p_{03}\mu_3 + p_{03}p_{34}\mu_4$$

and

$$D_1(0) = 1 - p_{01}p_{10} - p_{01}p_{12}^{(4)}p_{20} - p_{02}p_{20} - p_{03}p_{30} - p_{03}p_{34}p_{42}p_{20} .$$

Here we have used the relations

$$q_{ij}^*(0) = p_{ij} \quad \text{and} \quad Z_i^*(0) = \mu_i .$$

6. **Availability Analysis**

Let $A_i(t)$ be the probability that the system initially up in regererative state S_i remains up continuously till time t without passing through any other regenerative state or returning to itself. Using the definition of $A_i(t)$, the recursive relations among $A_i(t)$, $(i = 0, 1, 2, 3, 4, 5)$ can be easily developed and taking their L.T. and solving for $A_0^*(s)$, we get

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)} \tag{9}$$

where

$$\begin{aligned} N_2(s) = {} & (1 - q_{25}^* q_{52}^*) Z_0^* + q_{01}^*(1 - q_{25}^* q_{52}^*) Z_1^* + q_{03}^* q_{34}^* (1 - q_{25}^* q_{52}^*) Z_4^* \\ & + \{q_{01}^*(q_{12}^{(4)*} + q_{12}^{(4,5)*} + q_{52}^* q_{12}^{(5)*}) + q_{02}^* + q_{03}^*[q_{32}^* + q_{34}^*(q_{42}^{(5)*} + q_{42}^*)]\} Z_2^* \\ & + q_{03}^*(1 - q_{25}^* q_{52}^*) Z_3^* \end{aligned}$$

and

$$D_2(s) = (1 - q_{25}^* q_{52}^*)(1 - q_{01}^* q_{10}^* - q_{03}^* q_{30}^*) - q_{01}^* q_{20}^* (q_{12}^{(4)*} + q_{12}^{(4,5)*} + q_{12}^{(5)*} q_{52}^*)$$
$$- q_{02}^* q_{20}^* - q_{03}^* q_{20}^* [q_{32}^* + q_{34}^* (q_{42}^* + q_{42}^{(5)*})] .$$

The steady state availability of the system is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) = \lim_{s\to 0} \frac{sN_2(s)}{D_2(s)} = \lim_{s\to 0} N_2(s) \lim_{s\to 0} \frac{s}{D_2(s)} .$$

Since, $D_2(0) = 0$, by using L. Hospital rule, we have

$$A_0 = \frac{N_2(0)}{D_2'(0)} \tag{10}$$

where

$$N_2(0) = \mu_0 p_{20} + \mu_1 p_{01} p_{20} + \mu_2 (1 - p_{01} p_{10} - p_{03} p_{30})$$
$$+ \mu_3 p_{03} p_{20} + \mu_4 p_{03} p_{20} p_{34}$$

and

$$D_2'(0) = \mu_0 p_{20} + \mu_1 p_{01} p_{20} + \mu_2 (1 - p_{01} p_{10} - p_{03} p_{30})$$
$$+ \mu_3 p_{03} p_{20} + \mu_4 p_{03} p_{20} p_{34}$$
$$+ \mu_5 [p_{25}(1 - p_{01} p_{10} - p_{03} p_{30}) + p_{01} p_{12}^{(5)} p_{20}] .$$

The expected up time of the system during $(0, t)$ is given by

$$\mu_{up}(t) = \int_0^t A_0(u) du \tag{11}$$

so that

$$\mu_{up}^*(s) = A_0^*(s)/s . \tag{12}$$

7. Busy Period Analysis

$B_i(t)$ is the probability that the system having started initially from regenerative state $S_i \in E$ is under repair at time t due to failure of the unit. Using basic probabilistic arguments the recursive relations among $B_i(t)$, $(t = 0, 1, 2, 3, 4, 5)$ can be easily developed and taking L.T. of the relations and solving for $B_0^*(s)$, we have

$$B_0^*(s) = \frac{N_3(s)}{D_2(s)} \tag{13}$$

where

$$N_3(s) = (1 - q_{25}^* q_{52}^*)(q_{01}^* Z_1^* + q_{03}^* q_{34}^* Z_4^*) + \{q_{01}^* (q_{12}^{(4,5)*} + q_{12}^{(4)*} + q_{12}^{(5)*})$$
$$+ q_{02}^* + q_{03}^* [q_{32}^* + q_{34}^* (q_{42}^* + q_{42}^{(5)*})]\}(Z_2^* + q_{25}^* Z_5^*) .$$

Thus in the long run, the fraction of time for which system is under repair is given by

$$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} sB_0^*(s) = \frac{N_3(0)}{D_2'(0)} \tag{14}$$

where

$$N_3(0) = p_{20}(p_{01}\mu_1 + p_{03}p_{34}\mu_4) + (1 - p_{03}p_{30} - p_{01}p_{10})(\mu_2 + p_{25}\mu_5)\,.$$

The expected busy period of the repairman during $(0,t)$ is given by

$$\mu_b(t) = \int_0^t B_0(u)du \tag{15}$$

so that

$$\mu_b^*(s) = B_0^*(s)/s\,. \tag{16}$$

8. Expected Number of Repairs

Let us define $V_i(t)$ as the expected number of repairs of the priority unit during the time interval $(0,t)$ when the system initially starts from regenerative state S_i. Using the definition of $V_i(t)$, the recursive relations among $V_i(t)$ can be easily developed and taking their L.S.T and solving for $\tilde{V}_0(s)$, we get

$$\tilde{V}_0(s) = \frac{N_4(s)}{D_2(s)} \tag{17}$$

where

$$\begin{aligned} N_4(s) = {} & (1-\tilde{Q}_{25}\tilde{Q}_{52})[(\tilde{Q}_{10} + \tilde{Q}_{12}^{(4)} + \tilde{Q}_{12}^{(5)} + \tilde{Q}_{12}^{(4,5)})\tilde{Q}_{01} + (\tilde{Q}_{42} + \tilde{Q}_{42}^{(5)})\tilde{Q}_{03}\tilde{Q}_{34}] \\ & + \tilde{Q}_{25}\tilde{Q}_{52}\tilde{Q}_{01}(\tilde{Q}_{12}^{(4)} + \tilde{Q}_{12}^{(5)} + \tilde{Q}_{12}^{(4,5)}) + \tilde{Q}_{25}\tilde{Q}_{52}\tilde{Q}_{01} \\ & + \tilde{Q}_{25}\tilde{Q}_{52}\tilde{Q}_{03}[\tilde{Q}_{32} + \tilde{Q}_{34}(\tilde{Q}_{42} + \tilde{Q}_{42}^{(5)})]\,. \end{aligned}$$

In steady state the expected number of repairs of the priority unit per unit of time is given by

$$V_0 = \lim_{t\to\infty} V_0(t) = \lim_{s\to 0} sV_0^*(s) = \frac{N_4(0)}{D_2'(0)} \tag{18}$$

where

$$N_4(0) = p_{01} + p_{25}(1 - p_{02} - p_{01}p_{10} - p_{03}p_{30}) + p_{20}p_{03}p_{34}\,.$$

9. Profit Function Analysis

Two profit functions $P_1(t)$ and $P_2(t)$ can easily be obtained for the system model under study with the help of characteristics obtained earlier. The expected total profits incurred during $(0,t)$ are

$$\begin{aligned} P_1(t) &= \text{expected total revenue in } (0,t)\text{-expected total expenditure in } (0,t) \\ &= K_0\mu_{up}(t) - K_1\mu_b(t)\,. \end{aligned}$$

Similarly

$$P_2(t) = K_0\mu_{up}(t) - K_2V_0(t).$$

where K_0 is the revenue per unit up time, K_1 is the cost per unit time for which repair man is busy in repair of the failed unit and K_2 is per unit repair cost.

The expected total profits per unit time, in steady state, is

$$P_1 = \lim_{t\to\infty}[P_1(t)] = \lim_{s\to 0} s^2 P_1^*(s),$$

$$P_2 = \lim_{t\to\infty}[P_2(t)] = \lim_{s\to 0} s^2 P_2^*(s)$$

so that

$$P_1 = K_0A_0 - K_1V_0, \quad (19)$$

$$P_2 = K_0A_0 - K_2V_0. \quad (20)$$

10. **Graphical Study of System Model**

For a more concrete study of the system, we plot the graphs for MTSF and profit functions with respect to α (failure rate) for different values of r as shown in the Figures 2 and 3 respectively assuming that the joint distribution of failure and repair times follow bivariate exponential distribution.

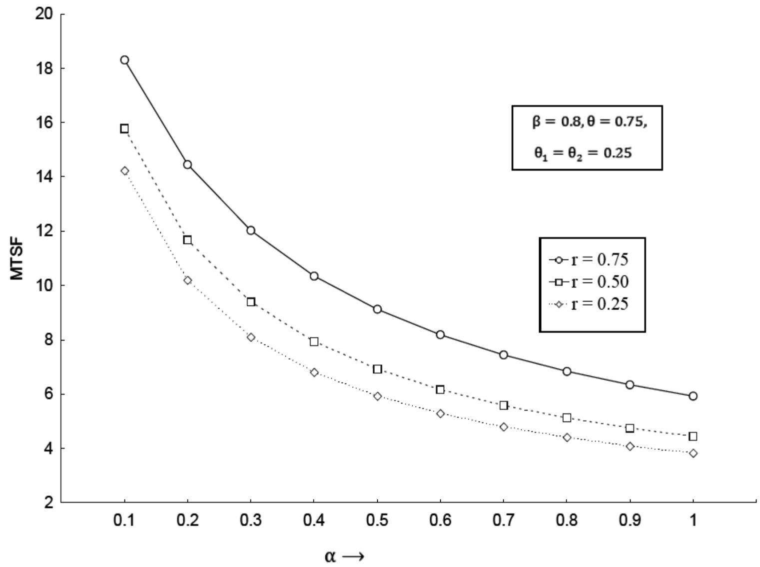

Figure 2. Behaviour of MTSF with respect to α for different values of r

Figure 2 depicts the behaviour of the MTSF with respect to α for $r = 0.25$, 0.50 and 0.75, when the values of other parameters are kept fixed $\beta = 0.8$, $\theta_1 = \theta_2 = 0.25$, $\theta = 0.75$. From the figure it is observed that the MTSF decreases almost exponentially with increase in the value of failure rate α. It is observed that, for higher value of r, the MTSF is higher, irrespective of other parameters meaning that for higher value of r, the repair facility has a better understanding of failure phenomenon resulting in longer lifetime of the system.

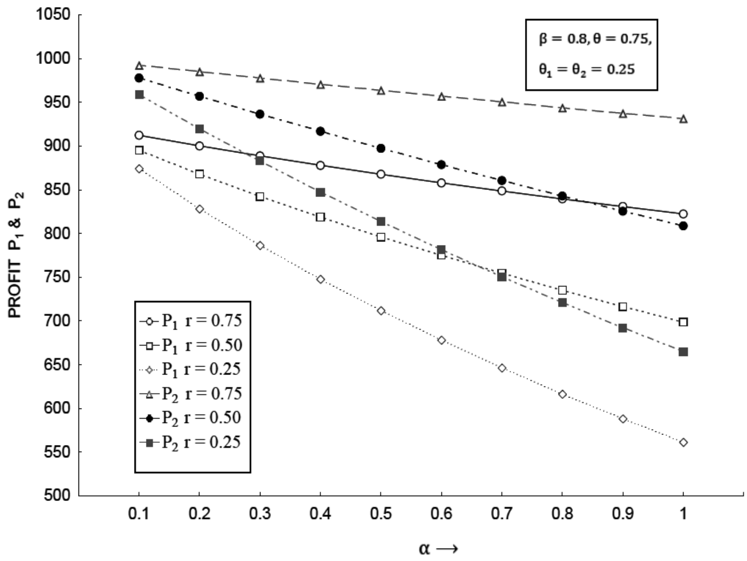

Figure 3. Behaviour of Profit functions P_1 and P_2 with respect to α for different values of r

In Figure 3 graphs are plotted for profit functions P_1 and P_2 with respect to α and varying values of $r = 0.25$, 0.50 and 0.75. It is observed from the graph that both the profit functions decrease with increase in failure parameter and increase with the increase in correlation coefficient r. Also profit function P_2 is higher as compared to P_1. Thus the better understanding of failure phenomenon by the repairman results in better system performance.

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