Journal of Informatics and Mathematical Sciences
Vol. 9, No. 3, pp. 815–825, 2017
ISSN 0975-5748 (online); 0974-875X (print)
Published by RGN Publications

http://www.rgnpublications.com

Proceedings of the Conference
Current Scenario in Pure and Applied Mathematics
December 22-23, 2016
Kongunadu Arts and Science College (Autonomous)
Coimbatore, Tamil Nadu, India

Research Article

Global Domination in Bipolar Fuzzy Graphs

V. Mohanaselvi[1], S. Sivamani[2] and N. Revathi[3,*]

[1] *Department of Mathematics, Nehru Memorial College, Puthanampatti 621007, India*
[2] *Department of Mathematics, Saranathan College of Engineering, Trichy 620012, India*
[3] *Department of Mathematics, Paavendhar College of Arts and Science, Salem 636121, India*
***Corresponding author:** revathinsrm@gmail.com

Abstract. In this paper the concept of global domination in bipolar fuzzy graph is introduced and studied its characterization. The necessary and sufficient condition for the global dominating set is given in terms of domination between the vertices. Also semi complete, purely semi complete, semi complementary and semi global domination concepts are defined and some results are obtained.

Keywords. Bipolar fuzzy graph (BFG); Strong edge; Dominating set; Domination number; Global dominating set; Global domination number; Semi complete; Purely semi complete; Semi complementary; Semi global dominating set and semi global domination number

MSC. 05C69; 05C72; 03E72

Received: January 6, 2017 **Accepted:** March 16, 2017

1. Introduction

In 1962, the study of dominating sets in graphs was begun by mathematicians Ore and Berge. The domination number, independent domination number concepts are introduced by Cockayne and Hedetniemi in 1977. In 1989 the Global domination number of a Graph was discussed by Sampathkumar [8]. Zadeh [9] introduced the notion of fuzzy subset of a set as a method of presenting uncertainty in 1965. The basic idea of fuzzy graph was introduced by mathematician Kauffman in the year 1973. After two years, Rosenfeld [7] introduced the concept of fuzzy graphs. Further, in [1] Akram extended fuzzy graph into bipolar fuzzy graph. In [4], Karunambigai *et al*. defined the domination, the domination number in bipolar fuzzy graphs.

Recently in 2012, Nirmala and Sheela [6] introduced the concept of global domination in fuzzy graphs. In 2012, Nagoor Gani, Yahya Mohamed and Jahir Hussain [5] introduced the concept of semi global domination in fuzzy graphs. In 2014 Hussain and Mohamed [2], [3] introduced the concepts of global domination and semi global domination in intuitionistic fuzzy graphs.

2. Preliminaries

Definition 2.1 (Dominating set)**.** A set D of vertices in a graph $G=(V,E)$ is called a dominating set of G if every vertex in $V-D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of a graph G is the minimum cardinality of a dominating set in G.

Definition 2.2 (Global dominating set)**.** A dominating set D of a graph G is a global dominating set if D is also a dominating set of $\bar{G}$. The global domination number $\gamma_g(G)$ in the minimum cardinality of a global dominating set of G.

Definition 2.3 (Dominating set in bipolar fuzzy graph)**.** A subset D of V is called a *dominating set* in G if for every $v \in V-S$, there exists $u \in S$ such that u dominates v. The minimum cardinality among all minimal dominating set is called domination number, denoted by γ.

3. Global Domination in Bipolar Fuzzy Graphs

Let G be a bipolar fuzzy graph and $u,v \in V$.

Definition 3.1 (Global dominating set)**.** A dominating set D of G is said to beg lobal dominating set if D is a dominating set of $\bar{G}$. The minimum cardinality among all minimal global dominating set is called global domination number denoted by $\gamma_g(G)$.

Example 3.2.

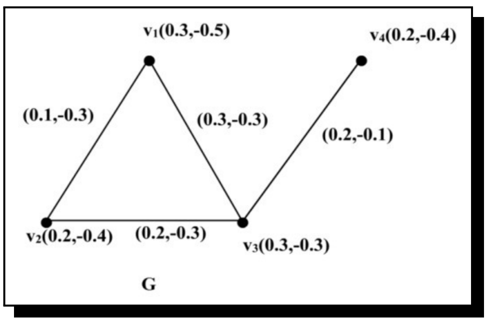


Figure 3.1

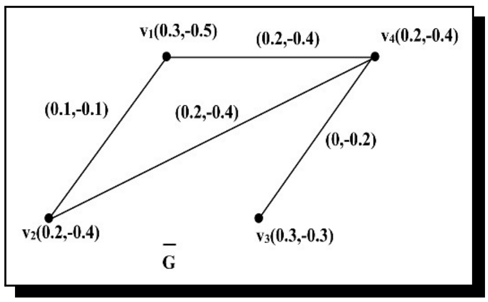


Figure 3.2

For the bipolar fuzzy graph G in Figure 3.2, $D = \{v_3, v_4\}$ is a γ_g-set and hence $\gamma_g(G) = 0.9$.

Theorem 3.3. *The global dominating set D in G is not singleton.*

Proof. The global dominating set D is a dominating set for both G and $\bar{G}$. Both the sets are non-empty gives the result. □

Example 3.4. For the bipolar fuzzy graph G in Figure 3.3, the vertex set $D = \{V_1, V_4\}$ is a global dominating set and hence $\gamma_g(G) = 1.3$.

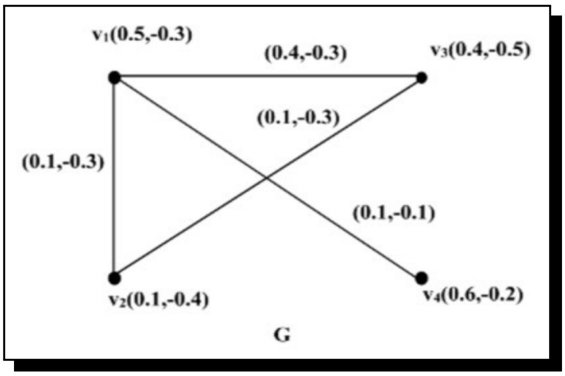


Figure 3.3

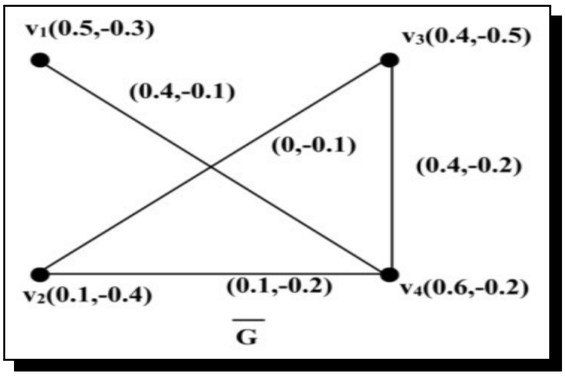


Figure 3.4

Theorem 3.5. *For bipolar fuzzy graph G with effective edges,* $\min\{|v_i| + |v_j|\} \le \gamma_g(G) \le p$*, $i \neq j$.*

Proof. Since the global dominating set is not a singleton set and suppose that it has two vertices say $\{v_i, v_j\}$, then $\min\{|v_i| + |v_j|\} = \gamma_g(G)$.

If the set contains more than $\{v_i, v_j\}$ then $\min\{|v_i| + |v_j|\} < \gamma_g(G)$, $i \neq j$.

If G is complete then the global dominating set contains all the vertices of G.

In this case, $\gamma_g(G) \le p$.

Therefore in general, $\min\{|v_i| + |v_j|\} \le \gamma_g(G) \le p$, $i \neq j$. □

Example 3.6. For the bipolar fuzzy graph G in Figure 3.5, the vertex set $D = \{v_2, v_3, v_4\}$ is a dominating set and hence $\gamma_g(G) = 1.45$ and $p = 1.9$.

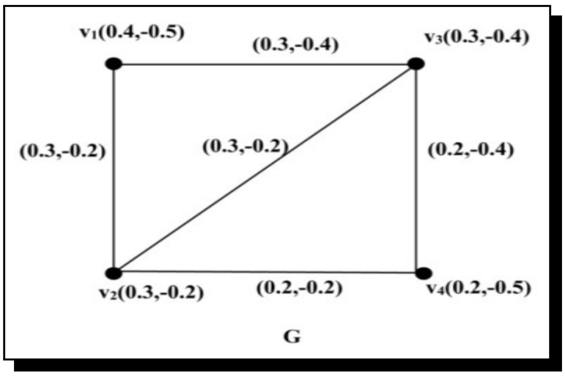


Figure 3.5

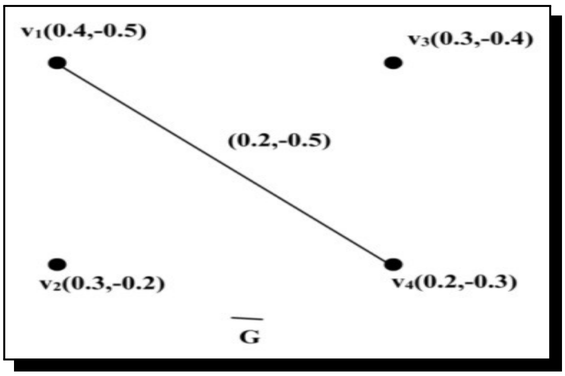


Figure 3.6

Hence $\min\{|v_i|+|v_j|\} = 0.35 < \gamma_g(G) < p$.

Theorem 3.7. *A dominating set D is a global dominating set if and only if for each $v \in V-D$ there exist $u \in D$ such that u and v are not dominating each other.*

Proof. Let G be a bipolar fuzzy graph with a global dominating set D. Suppose u in D is dominating to v in $V-D$ then D is not a dominating set, which contradicts D is a dominating set of G.

Conversely, for each $v \in V-D$ and u is not dominating to v, then the set D is dominating both G and $\bar{G}$ which gives D is a global dominating set of G and hence the result. □

Example 3.8. In the bipolar fuzzy graph G in Figure 3.1, $D = \{v_3, v_4\}$ and $V-D = \{v_1, v_2\}$. Here for each $v \in V-D$ there exist $u \in D$ such that u and v are not dominating each other.

Theorem 3.9. *For the connected G, $\gamma_g(G) = \gamma_g(\bar{G})$.*

Proof. Since from the construction of $\overline{G}$ in bipolar fuzzy graph, we obtain both the global dominating set of G and $\bar{G}$ are same and hence the result. □

Example 3.10.

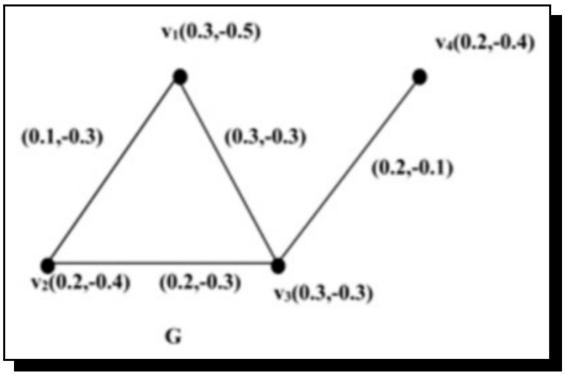


Figure 3.7

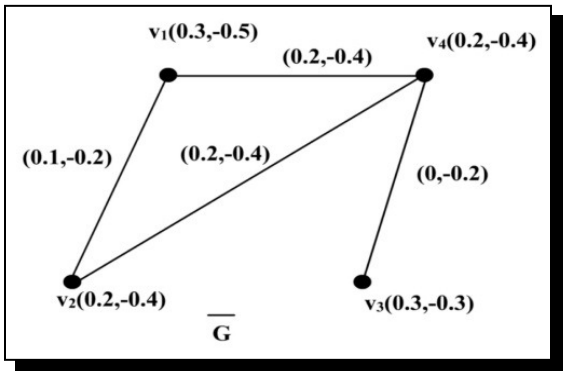


Figure 3.8

For the bipolar fuzzy graph G in Figure 3.7, $\gamma_g(G) = 0.9$ and the bipolar fuzzy graph in Figure 3.8, $\gamma_g(\bar{G}) = 0.9$. Hence $\gamma_g(G) = \gamma_g(\bar{G})$.

Theorem 3.11. *For any connected bipolar fuzzy graph G, (i) $\gamma(G) \le \gamma_g(G)$ (ii) $\gamma(G) \le \gamma_g(\bar{G})$.*

Proof. Since every global dominating set of G is a dominating set of G and hence $\gamma(G) \le \gamma_g(G)$. Similarly, every global dominating set of G is a dominating set of $\bar{G}$ gives $\gamma(\bar{G}) \le \gamma_g(\bar{G})$. □

Example 3.12. For the bipolar fuzzy graph G in Figure 3.9, $\gamma(G) = 0.5$ and $\gamma_g(G) = 0.9$. Hence $\gamma(G) < \gamma_g(G)$. Also $\gamma(\bar{G}) = 0.4$ and $\gamma_g(\bar{G}) = 0.9$ gives $\gamma(\bar{G}) < \gamma_g(\bar{G})$.

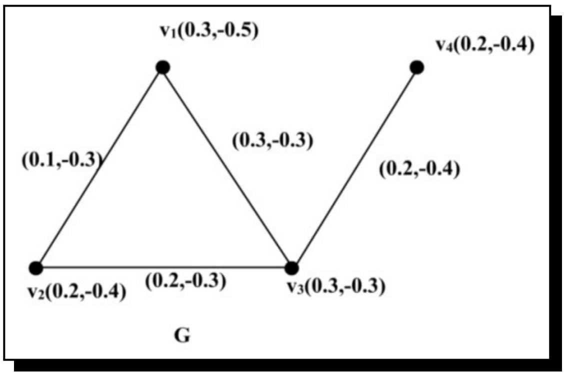


Figure 3.9

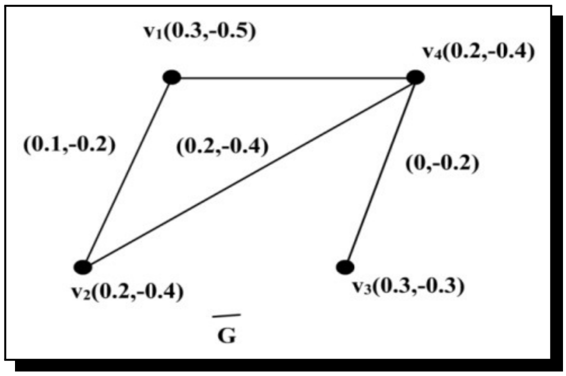


Figure 3.10

Theorem 3.13. *In a bipolar fuzzy graph G, a dominating set D contains k vertices and there exists a $v \in V-D$ dominates only one vertex in D then the global dominating set of G contains at most $k+1$ vertices.*

Proof. Since D is the dominating set and $v \in V-D$ dominates only one vertex in D. Then we get, $D \cup \{v\}$ is a global dominating set of G and hence the result. □

Example 3.14. For the graph G in Figure 3.5, the dominating set $D = \{v_2, v_3, v_4\}$ and $v \in V-D$ dominates $\{v_2, v_3\}$ vertex in D and hence $D_1 = \{v_2, v_3, v_4, v_1\}$ is a global dominating set of G and $\gamma_g(G) = 1.45$.

Theorem 3.15. *For a bipolar fuzzy graph with strong edge $\gamma_g(G) = \min\{|v_i| + |v_j|\}$, $i \neq j$ iff there is a strong edge between u and v such that each vertex in $V-(u,v)$ is adjacent to u or v but not both.*

Proof. Let $\gamma_g(G) = \min\{|v_i| + |v_j|\}$, $i \neq j$.
We assume $D = \{u, v\}$ be the global dominating set in G.
Since $\langle D \rangle$ is connected and uv is a strong edge in G, then there exist a vertex w in $V - \{u, v\}$ is dominating to both u and v, then it contradict to $\gamma_g(G)$. Hence w cannot be adjacent to both u and v and hence the necessary part.
Conversely, each vertex in $V - \{u, v\}$ is dominating to u or v but not both then we get, $\gamma_g(G) = \min\{|v_i| + |v_j|\}$. □

Example 3.16. For the bipolar fuzzy graph G in Figure 3.5, the dominating set $D = \{v_2, v_3, v_4\}$ and $V - D = \{v_1\}$.
Here v_2 and v_1 are dominating each other and each vertex in $V - \{v_2, v_1\}$ is adjacent to v_2.

Definition 3.17 (Semi complete bipolar fuzzy graph)**.** A connected bipolar fuzzy graph with effective edges is said to be *semi complete*, if every pair of vertices have a common neighbour in G.

Example 3.18. The following bipolar fuzzy graph is semi compete, since $\{v_i, v_j\}$ have a common neighbor.

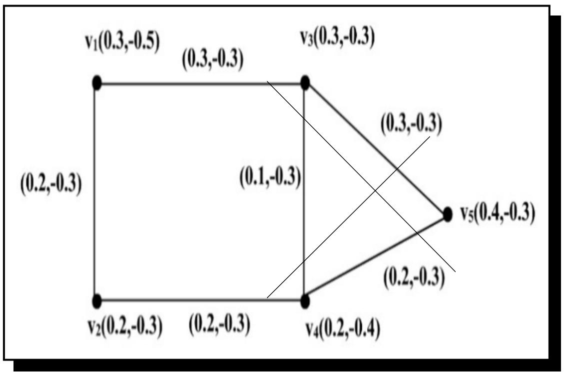


Figure 3.11

Definition 3.19 (Purely semi complete bipolar fuzzy graph)**.** A semi complete bipolar fuzzy graph G is said to be *purely semi complete* if G is not a complete graph.

Example 3.20. The following bipolar fuzzy graph is purely semi complete, since $\{v_i, v_j\}$ have a common neighbour but not complete.

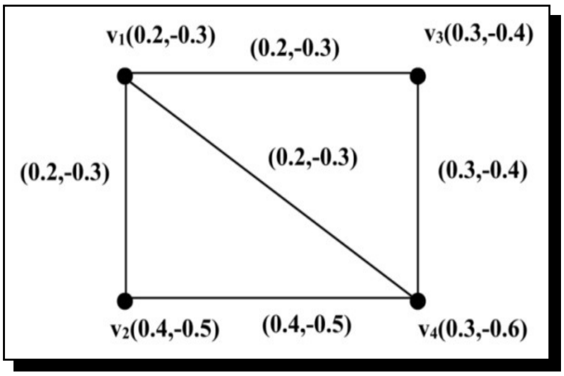


Figure 3.12

Theorem 3.21. *A connected bipolar fuzzy graph G with effective edges is semicomplete then any pair of vertices is lie on the same triangle or lie on two different triangles with atleast one common vertex.*

Proof. If G is semi complete, then every pair of vertices in G have a common neighbour, which make they lie on a triangle. If not, they lie on two different triangles, but the neighbourhood property gives the two triangles have atleast one common vertex which proves the result. □

Example 3.22. For the semi complete bipolar fuzzy graph G in Figure 3.12, $\{v_1, v_3\}$, $\{v_1, v_4\}$, $\{v_2, v_4\}$ are lie on the one triangle. $\{v_1, v_2\}$, $\{v_1, v_4\}$, $\{v_3, v_4\}$ are lie on another triangle. But $\{v_3, v_2\}$ are lie on the two different triangles.

Theorem 3.23. *For the purely semi complete bipolar fuzzy graph, then the global dominating set contains atleast 3 vertices.*

Proof. Since G is purely semi complete bipolar fuzzy graph then it contains a triangle or more than one triangle with atleast one common vertex say v.

Then, v is isolated in $\bar{G}$. Assume that global dominating set has less than 3 vertices.

Since by the Theorem 3.23, we have global dominating set has two vertices say $\{v_1, v_2\} = D$.

Case 1. $\langle D \rangle$ is connected in G.

Then v_1, v_2 is an effective edge in G.

By the definition of semi complete bipolar fuzzy graph, there is a vertex v_3 in G such that $D \cup \{v_3\}$ is a triangle in and $D \cup \{v_3\}$ is the γ_g-set of G, which is a contradiction. Hence D is a global dominating set in G.

Case 2. $\langle D \rangle$ is disconnected in G.

(i.e.) There is no effective edge between v_1 and v_2.

Since G is semi complete bipolar fuzzy graph, there is v_3 in G such that v_1, v_3 and v_3, v_2 are the effective edges in G.

In G^c, v_3 is not dominates by a vertex in D, which gives D is not a global dominating set in G, which is a contradiction. Hence from both the case, we attain the global dominating set contains atleast 3 vertices. □

Example 3.24. For the graph in Figure 3.12, the global dominating sets are $D = \{v_1, v_2, v_3\}$, $\{v_1, v_3, v_4\}$, $\{v_2, v_3, v_4\}$, $\{v_1, v_2, v_4\}$.

Therefore, for purely semi complete bipolar fuzzy graph then the global dominating set contains atleast 3 vertices.

Definition 3.25 (Semi complementary of a bipolar fuzzy graph)**.** A bipolar fuzzy graph $G = (V, E)$ is a *semi complementary bipolar fuzzy graph* $G^{sc} = (V^{sc}, E^{sc})$ if

(i) $\mu_1^{sc}(v) = \mu_1(v)$ and $\gamma_1^{sc}(v) = \gamma_1(v)$, where $\gamma_1^{sc}(v)$, $\mu_1^{sc}(v) \in V^{sc}$, and

(ii) $E^{sc} = \{uv \notin E$ and there exist w such that uw and wv in E then $\mu_2^{sc}(u,v) = \mu_1(u) \wedge \mu_1(v)$, $\gamma_2^{sc}(u,v) = \gamma_1(u) \vee \gamma_1(v)\}$.

Example 3.26.

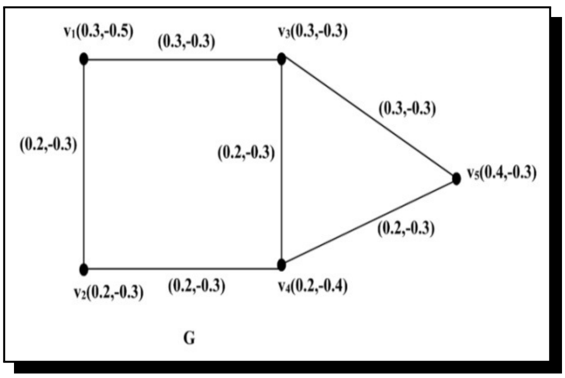


Figure 3.13

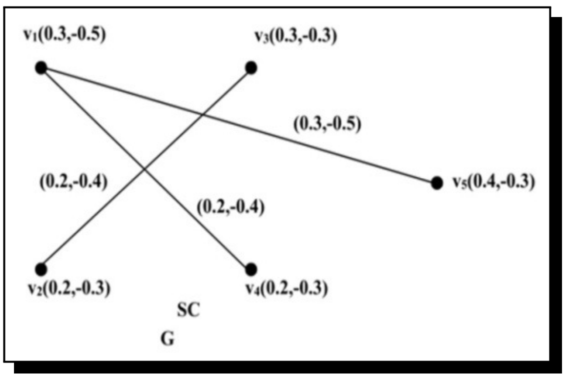


Figure 3.14

The above bipolar fuzzy graph is semi complementary, since $\{v_i, v_j\}$ in G^{sc} is not neighbor in G.

Theorem 3.27. *For the connected bipolar fuzzy graph G with effective edges if* $G^c = G^{sc}$ *then every pair of non-dominating vertices there must be atleast two effective edges.*

Proof. Since G is connected bipolar fuzzy graph, we have $G^c = G^{sc}$.

Implies $V(G^c) = V(G^{sc})$. If $(uv) \in G^c$ then $uv \in G^{sc}$.

Since G is connected bipolar fuzzy graph, we have $G^c = G^{sc}$.

$\Rightarrow E(G^c) = E(G^{sc})$.

If $(uv) \in G^c \Rightarrow (uv) \in G^{sc}$.

Hence between u and v there must be atleast two effective edges. □

Example 3.28. For the bipolar fuzzy graph in Figure 3.11, $G^c = G^{sc}$ and the not dominating edges in graph G, $D = \{v_1, v_3, v_5\}$, $V - D = \{v_2, v_4\}$ are effective.

Theorem 3.29. *If G is a semi complete bipolar fuzzy graph with effective edges then the complement and semi complementary graph of G are same.*

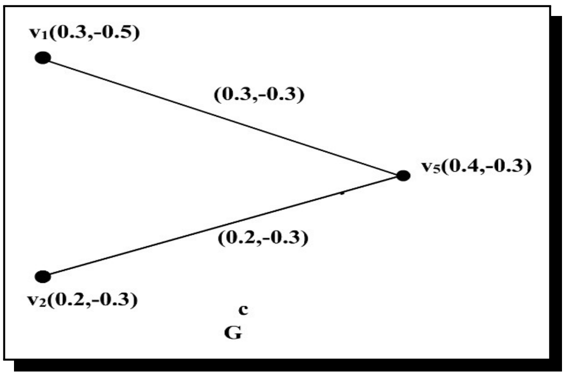
Proof. Since G is a semi complete bipolar fuzzy graph, we have for every pair of non-dominating vertices there must be atleast two effective edges.

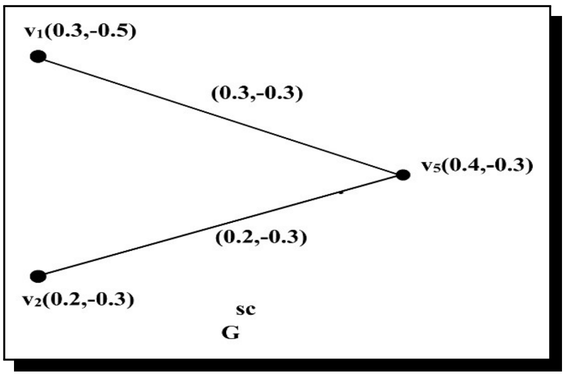
Hence G^{sc} is a subgraph of G^c.

Similarly, for every pair of dominating vertices in G^c there must be a path between them, which gives G^c is a subgraph of G^{sc}.

Hence the result. □

Example 3.30. For the bipolar fuzzy graph G in Figure 3.11, the dominating set $D = \{v_1, v_3, v_5\}$.





Remark 3.31. The converse of the above theorem is true.

Example 3.32.

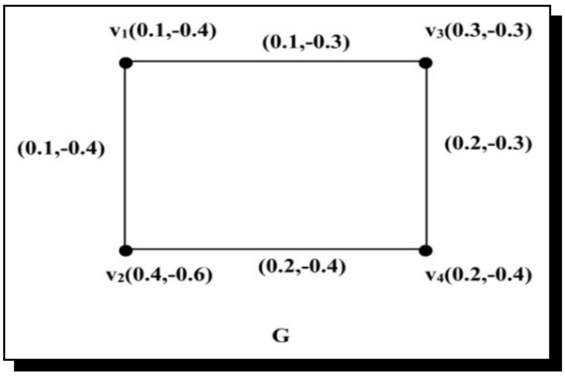


Figure 3.15

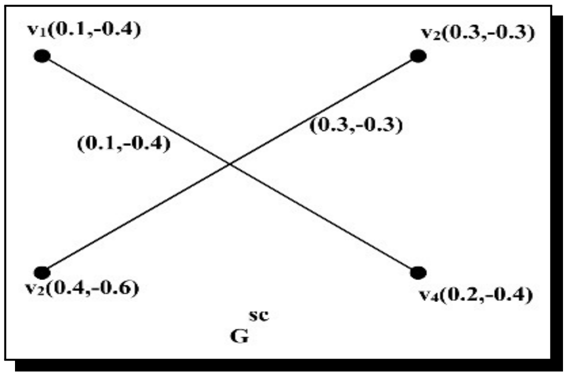


Figure 3.16

In the above bipolar fuzzy graph G in Figure 3.15, $G^{sc} = G^c$.

Theorem 3.33. *If G and G^{sc} are connected bipolar fuzzy graph with effective edges then G has a cycle.*

Proof. Let $e = uv \in E(G)$, which implies u, v are the vertices of G and G^{sc}.
Since G^{sc} is connected there is shortest uv path in G^{sc}.
This induces a path p in G. Now $p \cup \{e\}$ is a cycle in G.
Thus G has a cyclic. □

Remark 3.34. If G is cyclic bipolar fuzzy graph, then G^{sc} need not be connected.

Example 3.35.

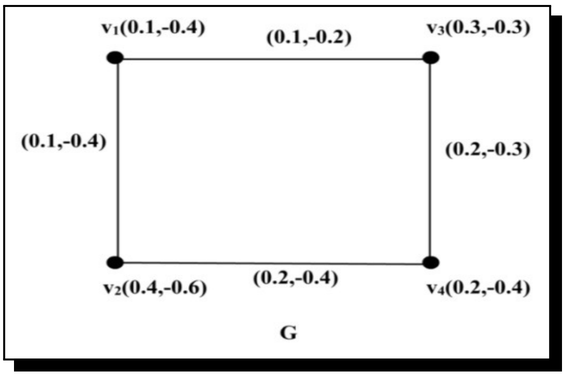


Figure 3.17

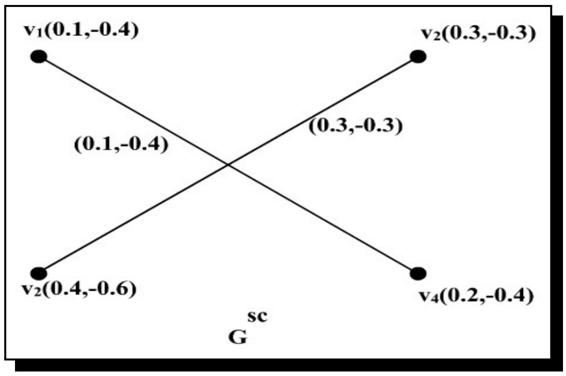


Figure 3.18

For the above cyclic bipolar fuzzy graph G in Figure 3.17, the G^{sc} is not connected.

Definition 3.36. Let $G = (V, E)$ be connected bipolar fuzzy graph. The set $D \subseteq V$ is said to be *semi global dominating set* of G if D is a dominating set for both G and G^{sc}.
The minimum cardinality among all semi global bipolar fuzzy dominating sets of G is called *semi global domination number* and is denoted by $\gamma_{sg}(G)$.

Example 3.37. For the bipolar fuzzy graph G in Figure 3.19, $D = \{v_3, v_2, v_5\}$ is a γ_{sg}-set and hence $\gamma_{sg}(G) = 1.65$.

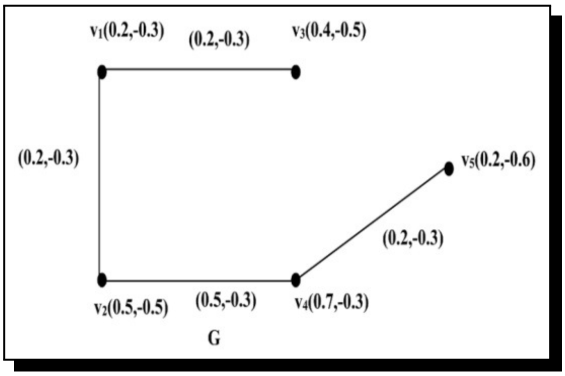


Figure 3.19

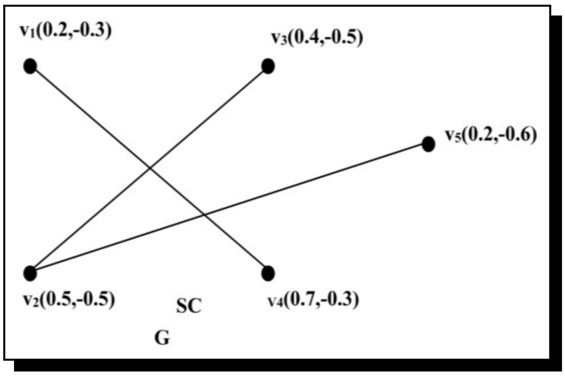


Figure 3.20

Theorem 3.38. *The semi global bipolar fuzzy dominating set is not singleton.*

Proof. Since semi global bipolar fuzzy dominating set contain dominating set for both G and G^{sc} then atleast two vertices are in the set.
(i.e.) The semi global bipolar fuzzy dominating set containing more than two vertices. □

Example 3.39. For the graph G in Figure 3.19, the vertex set $D = \{v_3, v_2, v_5\}$ is not singleton.

Theorem 3.40. *If* $G = (V, E)$ *is a connected bipolar fuzzy graph with effective edges then* $\min\{|v_i| + |v_j|\} \le \gamma_{sg}(G) \le p$, $i \ne j$ *and for every* v_i, $v_j \in V$.

Proof. Suppose the semi global dominating set D has atleast two vertices say $\{v_1, v_2\}$ then $\min\{|v_1| + |v_2|\} = \gamma_{sg}(G)$ and for the remaining vertices in D, $\min\{|v_i| + |v_j|\} < \gamma_{sg}(G)$.
Since G is complete bipolar fuzzy graph then semi global bipolar fuzzy dominating set contains all the vertices of G.
Hence $\gamma_{sg}(G) \le p$ which gives, $\min\{|v_i| + |v_j|\} \le \gamma_{sg}(G) \le p$. □

Example 3.41.

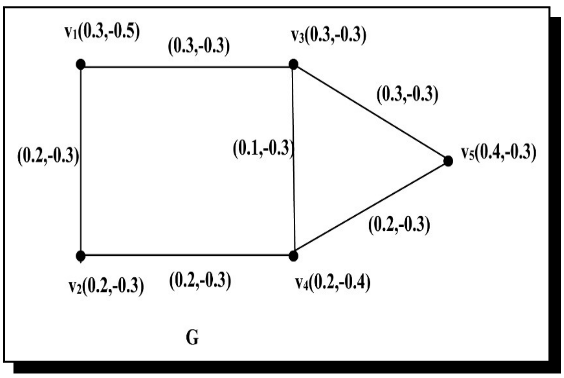


Figure 3.21

For the graph G in Figure 3.21, $\min\{|v_i| + |v_j|\} = 0.8$, $\gamma_{sg}(G) = 1.45$ and $p = 2.3$.
Hence $\min\{|v_i| + |v_j|\} \le \gamma_{sg}(G) \le p$.

4. Conclusion

In this paper for bipolar fuzzy graph the concept of global domination is introduced and studied its characterization. The necessary and sufficient condition for the global dominating set is given in terms of domination between the vertices. Also semi complete, purely semi complete, semi complementary and semi global domination concepts are defined and some results are obtained. Further, we are going to investigate global domination number with some other domination parameters in bipolar fuzzy graph.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] M. Akram, Bipolar fuzzy graphs, *Information Sciences* **181** (2011), 5548 – 5564.

[2] R.J. Hussain and S.Y. Mohamed, Global domination set in intuitionistic fuzzy graph, *International Journal of Computational Engineering Research* **4**(9) (2014), 55 – 58

[3] R.J. Hussain and S.Y. Mohamed, Semi global domination set in intuitionistic fuzzy graph, *IOSR Journal of Mathematics* **10**(4) (Version II) (2014), 23 – 27

[4] M.G. Karunambigai, M. Akram, K. Palanivel and S. Sivasankar, Domination in bipolar fuzzy graph, in *Proceedings of the International Conference on Fuzzy Systems* FUZZ-IEEE-2013, Hyderabad, India (2013), 1 – 6.

[5] G.A. Nagoor, S.Y. Mohamed and J. Hussain, Semi global dominating set in fuzzy graphs, *International Journal of Fuzzy Mathematical Archive* **4**(2) (2014), 115 – 122

[6] G. Nirmal and M. Sheela, Global and factor domination in fuzzy graph, *International Journal of Scientific and Research Publications* **2**(6) (2012), 1 – 4.

[7] A. Rosenfeld, Fuzzy graphs, *Fuzzy Sets and their Applications to Cognitive and Decision Processes* (Proc. U.S.-Japan Sem., Univ. Calif., Berkeley, Calif. (1974)), Academic Press, New York (1975), 77 – 95,

[8] E. Sampathkumar, The global domination number of a graph, *Jour. Math. Phy. Sc.* **23** (5), 377 – 385.

[9] L.A. Zadeh, Fuzzy sets, *Information and Control* **8** (1965), 338 – 353.