Journal of Informatics and Mathematical Sciences
Vol. 9, No. 3, pp. 781–790, 2017
ISSN 0975-5748 (online); 0974-875X (print)
Published by RGN Publications

http://www.rgnpublications.com

Proceedings of the Conference
Current Scenario in Pure and Applied Mathematics
December 22-23, 2016
Kongunadu Arts and Science College (Autonomous)
Coimbatore, Tamil Nadu, India

Research Article

Image Compression of 2-D Continuous Exponential Functions, Continuous Periodic Functions and Product of Sine and Cosine Functions using Discrete Wavelet Transform

G.K. Jagatheswari[1,*], Murugesan R.[2] and Gajanan V. Honnavar[1]

[1] *Department of Science and Humanities, PES Institute of Technology-Bangalore South Campus, Bangalore 560100, India*
[2] *Department of Physical Sciences, REVA University, Bangalore 560064, India*
***Corresponding author:** gkjagadees@gmail.com

Abstract. Wavelet transforms plays an important role in image compression techniques that developed recently. Here three 2-D functions are considered which are approximated and compressed using multilevel discrete 2-D wavelet transforms like Haar, Daubechies, Coiflet and Symlet. The images that are compressed are tested for quality using the error metrics like mean square error (MSE), peak to signal noise ratio (PSNR), maximum error (MAXERR), L2RAT, compression ratio and bit per pixel. The evaluation of the above mentioned wavelets is synthesized in terms of experimental results which demonstrates that Haar wavelets provides high compression ratios for 2-D exponential functions and the product of sine and cosine functions whereas Daubechies wavelet gives good compression ratio for 2-D periodic function.

Keywords. Image compression; DWT; MSE; MAXERR; PSNR; L2RAT; Haar; Daubechies; Coiflet; Symlet; Compression Ratio and Bit Error Rate

MSC. 42C40; 65T60

Received: January 15, 2017 **Accepted:** March 16, 2017

1. Introduction

In storage applications and image transmission the prime constraints are memory and channel bandwidth. Different types of images are compressed using various methods in compression techniques. In image compression application, the role of wavelets has become important as the digital images in the recent years has increased rapidly, which give way to high transmission and memory cost [1]. One of the most significant applications of wavelet transform is in medical images and it furnishes a desirable method for compression of biomedical images like magnetic resonance imaging (MRI), X-ray angiogram (XA) and so on which are extensively used in medical diagnosis [2], [3]. The amount of image data generated each day in health care with the improved scanning resolutions and the importance of volumetric image data sets and handling these images provides the requirement for efficient compression archival and transmission techniques [4]. Computed Tomography imaging compression can be used when a medical test obtained through a series of X-ray exposures resulting in 3-D images that aid medical diagnosis [5]. Since digital images have a high degree of Spatial redundancy, Temporal redundancy, Spectral redundancy, it can be compressed by eliminating redundant information. The lossy compression technique gives higher compression rates and the exact data cannot be reconstructed [6]. Also the human visual system has low sensitivity to some distortion in the image. By considering these facts, the compression potential is much higher which saves the amount of storage space as well as the time required for transmission. In reconstructing image we can achieve higher compression ratio, higher peak signal to noise ratio and low mean square error.

2. Wavelet Analysis

Wavelet analysis or simply 'wavelets' have drawn attention in the recent years and successfully applied in many applications such as transient signal analysis, image analysis and other signal processing applications. Wavelet theory has become an active area of research in many fields like fractals and quantum theory in Physics, harmonic analysis and operator theory in mathematics, signal processing and data compression in electrical engineering and so on [7]. A wavelet is a wavelike oscillation with amplitude starts from zero, increases and then decreases again to zero. Wavelet analysis is similar to Fourier analysis. Fourier transform breaks the signal into a series of sine waves of different frequencies, whereas wavelet analysis breaks the signal into 'wavelets' scaled and shifted versions of the mother wavelet. The wavelet properties of being irregular in shape and compactly supported that make wavelets an ideal instrument for analyzing signals of non-stationary nature when compared to the Fourier transform of sine wave which is smooth and of infinite length. Wavelets are mathematical functions that cut up data into different frequency components and then study each component with a resolution matched to its scale.

3. Discrete Wavelet Transform

Wavelet based coding provides substantial improvement in picture quality at high compression ratios mainly due to better energy compaction property wavelet transforms [8]. The discrete wavelet transform (DWT) gives sufficient knowledge both for analysis and synthesis of the original signal with a significant decrease in the computation time. The DWT is an implementation transform that uses a discrete set of the wavelet scales and translations of the wavelet transform following some defined rules. The transform decomposes the signal into a mutually orthogonal set of wavelets, which is the main difference from continuous wavelet transforming or its implementation for the discrete time series known as discrete-time continuous wavelet transform [9]. Wavelet coefficients calculated at each scale generate a lot of awful data. If scales and positions based on powers of 2-called dyadic scales and positions-are chosen the analysis become more efficient and accurate. Such an analysis is obtained from the discrete wavelet transform (DWT) [10]. The analysis start from signal 'f' and results in the coefficients $S(a,b)$ and is given by

$$S(a,b) = S(j,k) = \sum_{n\in z} f(n) g_{j,k}(n).$$

The DWT of an image f(n) is calculated by passing it through a series of filters. First the samples are passed through a low pass filter with impulse response 'g' resulting in a convolution of the two: $(f * g)[n]$.

The low frequency content is the most important part in many signals. It identifies the signal. The high frequency content has less importance. A great deal of study has been done on image compression and still it is going on. Image compression performance and analysis of different wavelets have been done in [11]. Yi Zhang and Xing Yuan have proposed fractal image compression based on wavelet using diamond search and has a drawback in coding of fractal image which needs more time [12]. A Neuro-Wavelet based approach for image compression is proposed in [13]. Combination of vector quantization and wavelet transform using RBF neural network has been proposed in [14]. Bi-orthogonal wavelet based image compression combined with hierarchical back propagation neural network is presented in [15]. The work on Neural based image compression gives large compression ratios on reconstructed images, but the complexity of such techniques is also more. Most of the authors have taken the test images for their research. This proposed paper approaches simpler technique on image compression using discrete wavelet transform presented in [16]. The paper extends the study of compression which is based on different RGB color images which is resized, converted into gray scale image and performed compression using multilevel discrete wavelet transform. The quality of the original image and the compressed image is measured using image quality metrics.

4. Image Quality Metric

Peak to Signal Ratio (PSNR)

PSNR is the peak signal to noise ratio in decibels and is given as the ratio between the maximum possible power of a signal and the power of the distorting noise which affects the quality of its representation defined by

$$\mathrm{PSNR} = 20\log_{10}\left(\frac{\mathrm{MAX}_f}{\sqrt{\mathrm{MSE}}}\right)$$

Mean Square Error (MSE)

The mean square error (MSE) is the squared norm of the cumulative difference between the compressed image and the original image. It is mathematically given by

$$\mathrm{MSE} = \frac{1}{mn}\sum_{0}^{m-1}\sum_{0}^{n-1}\|f(i,j)-g(i,j)\|^2$$

where f is the original image and g is the compressed image. The dimension of the images is $m \times n$. This MSE should be as low as possible for effective compression.

MAXERR

It is the maximum absolute squared deviation of the data from the approximation.

L2RAT

L2RAT is defined as the ratio of the squared norm of the signal or image approximation to the input signal or image.

Compression Ratio (CR)

The compression ratio is defined as

CR = the size of the original image/ the size of the compressed image. The ratio provides a hint of how much compression is achieved for a particular image.

Bit Per Pixel (BPP)

It is defined as the number of bits required to compress each pixel. It should be low to reduce storage requirements.

5. Image Compression

A wide variety of wavelet based image compression scheme have been reported in the literature from simple entropy to more complex techniques such as vector quantization adaptive transform, tree encoding, edge based coding and Huffman coding. Compression is of two types-Lossless compression and Lossy compression. Lossless compression can be used for compressing binary data, such as executable, document, etc., and it involves in compressing the data which, when compressed together will be an accurate reproduction of the original data. In this paper, we use Lossy compression since image need not be reproduced exactly. As long as the error between the original image and the compressed image is adequate, an estimate of the original image serves for most uses. The proposed work is based on the compression performed on three functions using Haar, Daubechies, Symlet and Coiflet wavelet techniques.

2-D Decaying Continuous Exponential Function

Consider the two dimensional continuous decaying exponential function $f(x,y) = \frac{-\exp(x+y)}{2}$. The image is compressed using different wavelet transforms and the results are simulated using Octave which is an alternate open source to Matlab. The multilevel decomposition is used for different wavelets and comparative analysis of Coiflet, Symlet, Daubechies and Haarwavelets are displayed.

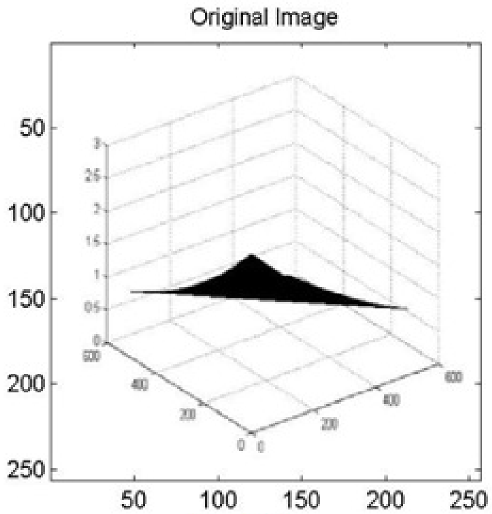

Fig. 1(a)

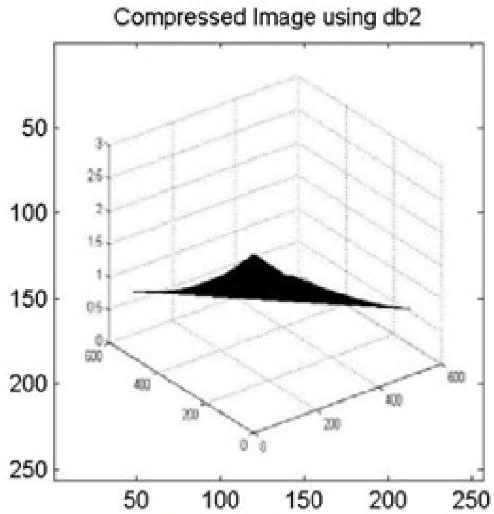

Fig. 1(b)

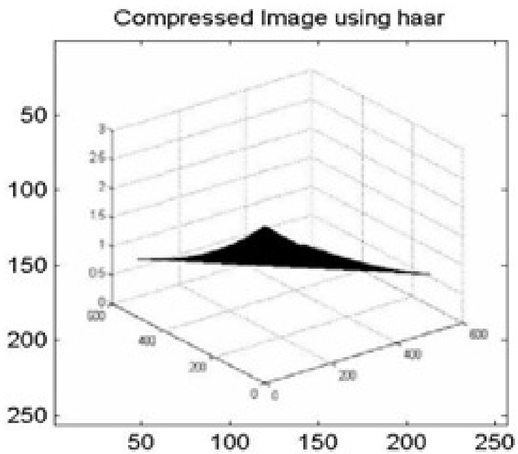

Fig. 1(c)

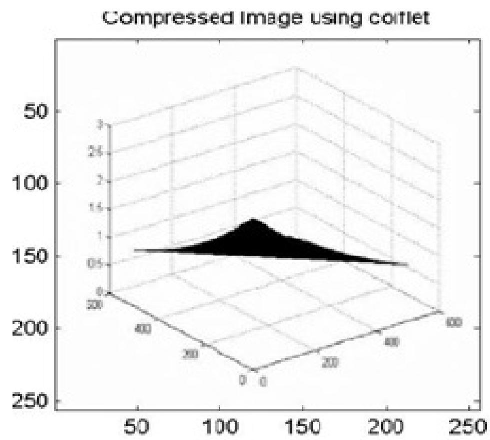

Fig. 1(d)

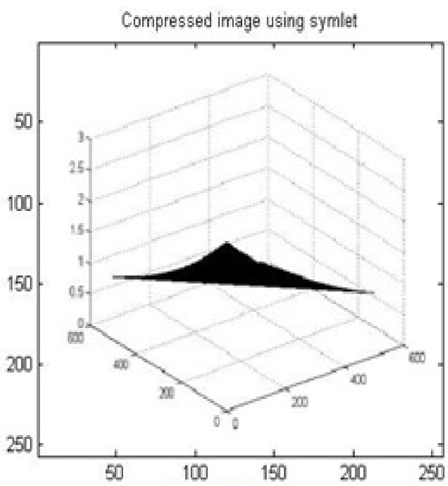

Fig. 1(e)

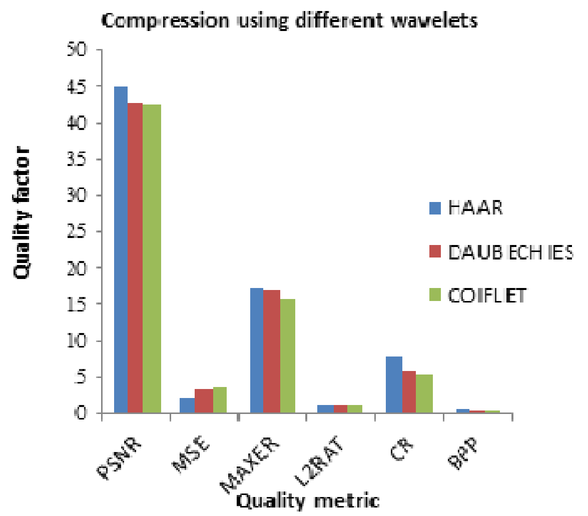

Fig. 1(F)

Figure 1

The Octave Pseudo code is given below.

- Load the image.
- Perform single level wavelet decomposition on the image.
- Construct approximation and coefficients.
- Perform multilevel wavelet decomposition and extract the coefficients. Reconstruct the decomposed values.
- Reconstruct the image from multilevel decomposition.
- Compress the image and calculate the PSNR, MSE, MAXERR and L2RAT.
- Calculate CR and BPP between the original image and compressed image using different wavelets.

The original image of 2-D decaying exponential function and the compressed images using Haar, Daubechies, Symlet and Coiflet wavelets are plotted in Figure 1. The image of this function is generated using octave by considering 201 points which uniformly partitioning the domain $[-1,1]$ based on a grid measurement of 0.01. By implementing the above code to the image, compression is done using different wavelets which are shown by Figures 1(b) to 1(e).

Figures 1(a)-(e) represents image compression of the 2-D exponential function using Haar, db2, Coiflet and Symlet respectively. Figures 1(f) shows comparison of the quality metrics of different wavelets.

Table 1

Wavelet families	PSNR	MSE	MAXERR	L2RAT	CR	BPP
HAAR	44.8641	2.1216	17.2500	1	7.8278	0.6262
DAUBECHIES	42.7892	3.4210	16.8984	0.9999	5.9143	0.4731
COIFLET	42.4586	3.6916	15.6135	0.9999	5.3223	0.4258
SYMLET	42.7892	3.4210	16.8984	0.9999	5.9143	0.4731

Table 1 clearly indicates that the compression results are good for the continuous exponential function by Haar wavelet transform as a lesser value for MSE means lower error and a high value of PSNR. Also the compression ratio is more and Bit per pixel is less for Haar wavelet transform than other wavelet families. This shows that Haar wavelet is best suited for compressing the decaying exponential function.

Image Compression of 2-D Continuous Periodic Functions

Consider the 2-D continuous periodic function $f(x,y) = \exp(x+y)\sin(x+y)$. The original image of 2-D continuous periodic function is plotted in Figure 2 using octave by considering 201 points on a grid measurement of 0.01. Quantitative analysis has been demonstrated at first decomposition levels of wavelet families by measuring the values of PSNR, MSE, MAXERR and L2RAT. Qualitative analysis has been done by performing the compressed version of the input image by DWT technique and comparing it to the original image. The image is compressed using different wavelets and the results are tabulated below.

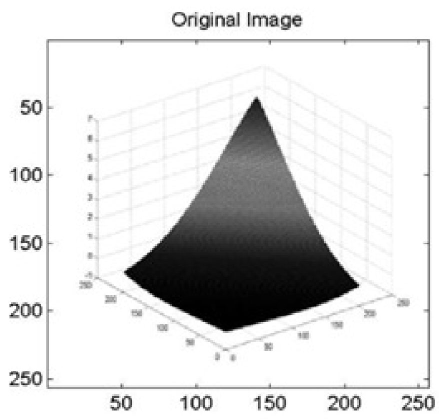

Fig. 2(a)

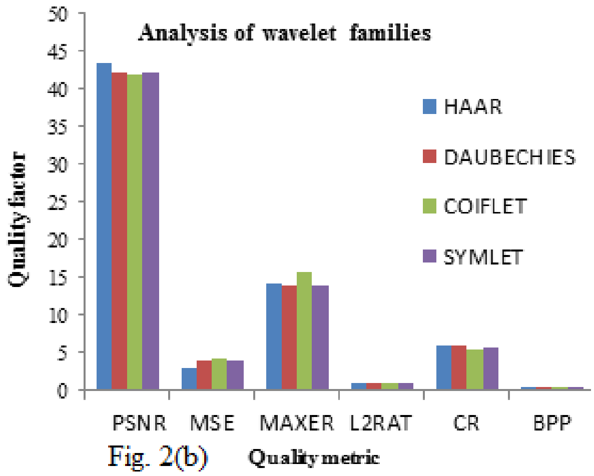

Fig. 2(b)

Figure 2

Figure 2(a) represents the original function $f(x,y) = \exp(x+y)\sin(x+y)$. Figure 2(b) represents the comparison of the evaluation parameters of the wavelets used.

Table 2

Wavelet families	PSNR	MSE	MAXER	L2RAT	CR	BPP
HAAR	43.3128	3.0325	14.2500	0.9999	5.8563	0.4685
DAUBECHIES	42.3015	3.8276	13.8339	0.9999	5.8838	0.4707
COIFLET	41.9447	4.1554	15.5751	0.9999	5.3528	0.4282
SYMLET	42.3015	3.8276	13.8339	0.9999	5.8838	0.4707

From the experimental results of Table 2, it is evident that Daubechies wavelet provides a higher compression ratio than other wavelets and avoids blocking artifacts. Haar wavelets enable the Haar decomposition to have good time localization, which means that Haar coefficients are effective for locating jump discontinuities and the evaluation of Haar wavelet of the continuous periodic function shows that a higher value of PSNR is good because it signifies that the ratio of Signal to Noise is higher. A compression scheme having a lower MSE and a high PSNR is obtained and in turn indicates that the compression made by Haar and Daubechies is a good one. Here signal is the original image and the noise is the error in reconstruction.

Image Compression of 2-D Product of Sine and Cosine Function

Wavelet decomposition and reconstruction have been carried out for the product of sine and cosine function given by $f(x,y) = \sin(x+y)\cos(x+y)$ which is generated using octave. The data set consisted of 201 points chosen uniformly by partitioning the domain $[-1,1]$. The image of this function is simulated using matlab. The original function is plotted in Figure 3(a).

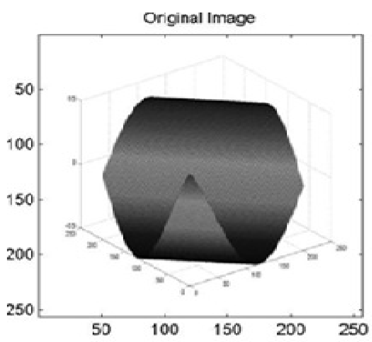

Fig. 3(a)

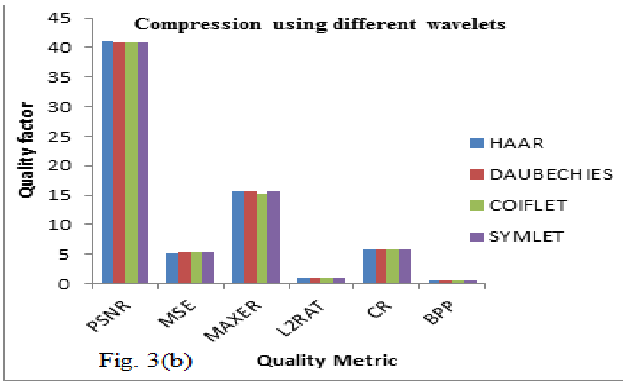

Fig. 3(b)

Figure 3

Figure 3(a) represents the original function $f(x,y) = \sin(x+y) * \cos(x+y)$. Figure 3(b) represents the comparison of the evaluation parameters of the wavelets used.

Table 3

Wavelet families	PSNR	MSE	MAXER	L2RAT	CR	BPP
HAAR	41.0792	5.0718	15.7500	0.9999	5.8487	0.4679
DAUBECHIES	40.7634	5.4543	15.6961	0.9999	5.7877	0.4630
COIFLET	40.8501	5.3465	15.2476	0.9999	5.8075	0.4646
SYMLET	40.7634	5.4543	15.6961	0.9999	5.7877	0.4630

By evaluating the image quality metric, it is found that Haar wavelet gives high PSNR and high compression ratio when compared with other wavelets like Coiflet, Symlet and Daubechies as given in Table 3. So Haar wavelet is found suitable for compressing the sine and cosine function. Also Daubechies and Symlet wavelets give exact identical values for the calculated quality metrics and this shows that Symlets are another family of Daubechies wavelets thus are constructed in the same way as Daubechies wavelts. These wavelets can be used interchangeably while synthesizing the continuous periodic function.

6. Conclusion

In this paper an efficient compression technique based on discrete wavelet transform (DWT) is proposed and developed. Wavelets are better suited to time-limited data and wavelet based compression technique maintains better image quality by reducing errors and allows good localization in both spatial and frequency domain. From the experimental results it is found that Haar wavelet is best suited for compressing the continuous exponential functions and the product of sine and cosine functions, whereas Daubechies wavelet families are best suited for continuous periodic functions. However the Coiflet and Symlet functions do perform better in statistical terms.It can be concluded that since each wavelet filter gives a different performance for different fidelity metrics and different images, the compression performance depends on the size and content of the image and therefore it is appropriate to choose the choice of wavelet based on image size and content for the desired quality of reconstructed image. In all the wavelet that are used for analyzing various 2-D functions, it is observed thatDaubechies and Symlet wavelet gives exactly identical values for the calculated quality metrics. This is interesting because the generating functions of the two wavelets are different. So we conclude that the wavelets of Dabuechies and Symlet can be used interchangeably while analyzing the above discussed functions.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

The authors wrote, read and approved the final manuscript.

References

[1] R.N. Khatke, Image compression using wavelet transform, *Imperial Journal of Interdisciplinary Research* **2** (9) (2016), 82 – 84.

[2] R.J. Sapkal and S.M. Kulkarni, Image fusion based on wavelet transform for medical application, *International Journal of Engineering Research and Applications* **2** (5) (September-October 2012), 624 – 627.

[3] Ruchika, M. Singh and A.R. Singh, Compression of medical images using wavelet transforms, *International Journal of Soft Computing and Engineering* **2** (2) (May 2012), 2231 – 2307.

[4] T. Bruylants, A. Munteanu and P. Schelkens, Wavelet based volumetric medical image compression, *Signal Processing: Image Communication* **31** (February 2015), 112 – 133.

[5] J. Munoz-Gomez, J. Bartrina-Rapesta, M.W. Marcellin and J. Serra-Sagrista, Correlation modeling for compression of computed tomography, *IEEE Journal of Biomedical and Health Informatics* **17** (5) (2013), 928 – 935.

[6] K.S. Thyagarajan, *Still Image and Video Compression with Matlab*, John Wiley and Sons, Inc., New Jersey (2011).

[7] I. Daubechies, *Ten Lectures on Wavelets*, SIAM (in Wavelet Transforms-Image Compression) **5** (1992).

[8] R.C. Gonzalec, R.E. Woods and S.L. Eddins, *Digital Image Processing using Matlab*, Prentice Hall (2004).

[9] S.G. Chang, B. Yu and M. Vetterli, Adaptive wavelets thresholding for image denoising and compression, *IEEE Trans. Image Processing* **9**(9) (2000), 1532 – 1546.

[10] S. Mallat, A theory of mulitiresolution signal decomposition; the wavelet representation, *IEEE Pattern Anal. And Machine Intel.* **11** (7) (1989), 674 – 693.

[11] P. Telagarapu et al., Image compression using DCT and wavelet transformations, *International Journal of Signal Processing, Image Processing and Pattern recognition* **4** (3) (September, 2011), 61 – 74.

[12] Y. Zhang and X.Y. Wang, Fractal compression coding based on wavelet transform with diamond search, *Nonlinear Analysis: Real World Applications* **13** (1) (February 2012), 106 – 112.

[13] V. Singh, N. Rajpal and K.S. Murthy, Neuro Wavelet based Approach for Image Compression, *Computer Graphics, Imaging and Visualization*, CGIV APOS'2007, 280 – 286.

[14] G. Boopathi and S. Arockiasamy, Image compression: wavelet transform using Radial Basis Function (RBF) neural network, in *2012 Annual IEEE India Conference* (INDICON) (2012), 340 – 344.

[15] A. Rani and M. Mehta, Image compression of radiograph using neural network and wavelet, *International Journal of Engineering Science and Technology* **5** (4) (April 2013), 803 – 809.

[16] R.C. Gonzalez and R. Eugene, *Digital Image Processing*, 3rd edition, Pearson Prentice Hall, Pearson Education, Inc. Upper Saddle River, New Jersey (2008).