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Research Article

Stochastic Integrals and Power Contractions in Bernoulli Selections

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Abstract. Random contractions and Bernoulli selections are recognized as strong analytical tools of probability distributions theory. The paper investigates the distribution of a Bernoulli selection incorporating a stochastic integral and a random contraction. Moreover, the paper establishes a practical interpretation of the formulated Bernoulli selection.

Keywords. Stochastic integral; Random contraction; Bernoulli selection

MSC. 97K60; 97K50; 60E05

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1. Introduction

We consider the positive random variable H and the random variable V with values in the interval $(0,1)$. We also consider the random variable

$$J = HV.$$

If the random variables H, V are independent then the random variable J is said a random contraction of the random variable H via the random variable V [7]. If the random variable V follows the power distribution then the random variable J is said a power contraction. Random contractions are generally recognized as strong analytical tools of probability theory for investigating unimodality [11], infinite divisibility [13], stability [5], selfedecomposability

[8] and other important properties of probability distributions. Moreover, random contractions have practical applications in income distributions analysis [10], cindynics [9], continuous discounting [4], reliability theory [3], inventory control [15], operations research [14], proactive risk management [1], engineering [12], systemics [16], and informatics [2].

The present paper is mainly devoted to the characterization of the distribution of a Bernoulli selection incorporating a random contraction and a stochastic integral.

2. Formulation of a Stochastic Model

The present section of the paper makes use of two positive random variables, a Bernoulli random variable and a stochastic integral in order to formulate a stochastic model.

Let $\{X(t), t \geq 0\}$ be a stochastic process with stationary, independent, and positive increments. We assume that $E(X(t)) = \mu t$ and $V(X(t)) = \sigma^2 t$. We also assume $\{X(t), t \geq 0\}$ is continuous in probability and that its sample paths are right continuous and have left limits. Moreover, we assume that the increment $L = X(t+1) - X(t)$ has characteristic function $\varphi_L(u)$. The stochastic integral

$$C = \int_0^\infty e^{-rt} dX(t), \quad r > 0$$

exists in the sense of convergence in probability and is finite almost surely. In addition, the distribution function of C is continuous and

$$\varphi_C(u) = \exp\left\{\int_0^\infty \log \varphi_L\left(ue^{-rt}\right) dt\right\}$$

is its characteristic function [6]. The characteristic function $\varphi_C(u)$ is easily shown to be given by

$$\varphi_C(u) = \exp\left\{\frac{1}{r}\int_0^u \frac{\log \varphi_L(w)}{w} dw\right\}.$$

Let N be a Bernoulli random variable with probability generating function

$$P_N(z) = q + pz, \quad 0 < p < 1, \ q = 1 - p,$$

S is a positive random variable with characteristic function $\varphi_S(u)$ and T is a positive random variable with distribution function $F_T(t)$. We consider the random variable $\Pi = Se^{-rT}$ and the stochastic model

$$Y = \begin{cases} \Pi, & N = 0, \\ C, & N = 1. \end{cases}$$

The following sections of the paper are devoted to the theoretical investigation and the practical interpretation of the formulated stochastic model.

3. Characteristic Function of a Stochastic Model

In general, an explicit evaluation of the characteristic function $\varphi_Y(u)$ of the stochastic model Y is very difficult. The present section is devoted to the establishment of conditions for an explicit evaluation of a particular case of $\varphi_Y(u)$.

Theorem. *We assume that the random variables N, C, S, T are independent and that $F_T(t) = 1 - e^{-\lambda t}$, $\lambda > 0$. The characteristic function of the stochastic model Y has the form*

$$\varphi_Y(u) = p\exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_L(w)}{w}dw\right) + q\left[\frac{ap}{u^{ap}}\int_0^u \exp\left(\frac{1}{r}\int_0^w \frac{\log\varphi_L(\theta)}{\theta}d\theta\right)w^{ap-1}dw\right]$$

with $a = \lambda/r$ if and only if,

$$Y \stackrel{d}{=} S, \tag{1}$$

where $\stackrel{d}{=}$ denotes equality in distribution.

Proof. Only the sufficiency condition will be proved since the necessity condition can be proved by reversing the argument. It is readily shown that the characteristic function of the stochastic model Y has the form

$$\varphi_Y(u) = p\exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_L(w)}{w}dw\right) + q\frac{a}{u^a}\int_0^u \varphi_S(w)w^{a-1}dw. \tag{2}$$

If we use (1) in (2) we get the integral equation

$$\varphi_Y(u) = p\exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_L(w)}{w}dw\right) + q\frac{a}{u^a}\int_0^u \varphi_Y(w)w^{a-1}dw. \tag{3}$$

If we multiply both sides of (3) by u^a, $u \neq 0$, and then differentiating we get the differential equation

$$\varphi_Y(u) + \frac{u}{ap}\frac{d\varphi_Y(u)}{du} = \exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_L(w)}{w}dw\right) + \frac{u}{ar}\frac{\log\varphi_L(u)}{u}\exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_Y(w)}{w}dw\right). \tag{4}$$

From (4) we get that

$$\begin{aligned}\frac{ap}{u^{ap}}&\int_0^u \varphi_Y(w)w^{ap-1}dw + \frac{ap}{u^{ap}}\int_0^u \frac{w}{ap}\frac{d\varphi_Y(w)}{dw}w^{ap-1}dw \\ &= \frac{ap}{u^{ap}}\int_0^u \exp\left(\frac{1}{r}\int_0^w \frac{\log\varphi_L(\theta)}{\theta}d\theta\right)w^{ap-1}dw \\ &\quad + \frac{ap}{u^{ap}}\int_0^u \frac{w\log\varphi_L(w)}{arw}\exp\left(\frac{1}{r}\int_0^w \frac{\log\varphi_L(\theta)}{\theta}d\theta\right)w^{ap-1}dw.\end{aligned} \tag{5}$$

Moreover, it is readily shown that

$$\frac{ap}{u^{ap}}\int_0^u \frac{w}{ap}\frac{d\varphi_Y(w)}{dw}w^{ap-1}dw = \varphi_Y(u) - \frac{ap}{u^{ap}}\int_0^u \varphi_Y(w)w^{ap-1}dw \tag{6}$$

and that

$$\begin{aligned}\frac{ap}{u^{ap}}&\int_0^u \frac{w\log\varphi_L(w)}{arw}\exp\left(\frac{1}{r}\int_0^w \frac{\log\varphi_L(\theta)}{\theta}d\theta\right)w^{ap-1}dw \\ &= p\exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_L(w)}{w}dw\right) - \frac{ap^2}{u^{ap}}\int_0^u \exp\left(\frac{\log\varphi_L(\theta)}{\theta}d\theta\right)w^{ap-1}dw.\end{aligned} \tag{7}$$

If we use (6) and (7) in (5) we get the characteristic function

$$\varphi_Y(u) = p\exp\left(\frac{1}{r}\int_0^u \frac{\log\varphi_L(w)}{w}dw\right) + q\left[\frac{ap}{u^{ap}}\int_0^u \exp\left(\frac{1}{r}\int_0^w \frac{\log\varphi_L(\theta)}{\theta}d\theta\right)w^{ap-1}dw\right]. \qquad \square$$

It is obvious that the above characteristic function belongs to a Bernoulli selection incorporating the stochastic integral C and a power contraction of C.

4. Application

The theory of finance is concerned with the determination of the value of the firm as a going concern, the identification and analysis of factors with direct and indirect influence on this value, and with the valuation of investment opportunities. The economic value of the firm as a going concern is the present value of income that the firm will generate in the future. Assuming that the income is given by the stochastic process $\{X(t), t \geq 0\}$ and since the corporate firm has an indefinite life, its economic value can be approximated by the stochastic integral C where r is the force of interest. Moreover, we assume that the random variable S denotes a cash flow arising at the random time T, then the random variable Π denotes the present value of S as viewed at time 0. Hence, the stochastic model formulated by the second section is suitable for making selection between two present values.

Competing Interests

The author declares that he has no competing interests.

Authors' Contributions

The author wrote, read and approved the final manuscript.

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