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# Topological Relations between Fuzzy Regions with Holes in a General Fuzzy Topological Space

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**Abstract.** In this paper, we have proposed the definition of a fuzzy region with holes in a general fuzzy topological space in such a way that the holes and the generalized region are disjoint with each other. In particular, the topological relations between fuzzy regions each with a single hole have been obtained.

## 1. **Introduction**

Topological relations between spatial objects is one of the main concerns in the study of spatial analysis. Crisp spatial objects and their relations have been extensively studied in [3, 4, 5, 6, 8]. Study of topological relations between fuzzy spatial objects has emerged as one of the major challenges in GIS. Out of the three basic types of fuzzy spatial objects, fuzzy points and fuzzy lines are well studied, whereas fuzzy regions remain to be an issue of discussion and various mathematical frameworks [1, 13, 15] have been proposed for the same. A fuzzy region in a topological space was defined by Schneider [14] in terms of open sets. In 2004, Tang [18] proposed two definitions of fuzzy regions, one in a crisp fuzzy topological space and the other in a general fuzzy topological space both in terms of closed sets. The later appeared to be more consistent due to the fact that a region in a crisp topological space has been usually considered to be a closed set.

The main purpose behind the development of fuzzy regions is the derivation of topological relations. Zhan [20] formulated fuzzy analogues of the 4-intersection model in terms of $\alpha$-cut operation. Du et al. [2] proposed the fuzzy extension of the 9-intersection matrix by defining the membership grades for interior, boundary and exterior and by partitioning the space accordingly. Tang [18, 19, 17] derived the topological relations between such regions for various application purpose. Sobrevilla et al. [16] shows the application of fuzzy region in the detection of white blood cell in the bone marrow.

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A large number of real life phenomena exhibit discontinuity at boundary and exterior in the form of cavities giving rise to regions with holes [7, 9, 10]. In order to capture these real phenomena, it becomes essential to incorporate them in the formulation of fuzzy regions. In our earlier work [11], we provided a formal definition of a fuzzy region with hole in the setting of a crisp fuzzy topological space. Since a general fuzzy topological space allows flexibility and relaxation of membership grades of points in space, a fuzzy region with a hole in such a space cannot be defined analogous to its definition in a crisp topological space as it is required to ensure that the hole/s and the generalized region should be disjoint. As shown by Tang [18], additional topological invariants are required to deal with the situation. In this paper, we have proposed a definition of fuzzy regions with holes in the setting of general fuzzy topological space utilizing the same framework. Topological relations between them have been obtained in general and under restricted conditions.

## 2. **Fuzzy sets and Fuzzy topology**

The following definitions and results can be found in [12].

**Definition 2.1.** Let $X$ be a set. A fuzzy set in $X$ is a function from $X$ into the closed unit interval $[0,1]$.

**Definition 2.2.** A fuzzy topology is a family $T$ of fuzzy sets in $X$ which satisfies the following:

(i) $0_X, 1_X \in T$

(ii) If $A, B \in T$ then $A \cap B \in T$

(iii) If $\{A_i : i \in J\} \subset T$, where J is an index set then $\cup_{i \in J} A_i \in T$.

Here $0_X$ denotes the empty set and $1_X$ is the whole set $X$. $T$ is called a fuzzy topology for $X$ and the pair $(X, T)$ is a fuzzy topological space (fts). The elements in $T$ are the open sets and their complements are the closed sets.

**Definition 2.3.** Let $A$ be a fuzzy set in $(X, T)$ then

(i) The interior of $A$ is the union of all the open sets contained in $A$, denoted by $A^\circ$.

(ii) The closure of $A$ is the intersection of all closed sets containing $A$, denoted by $\overline{A}$.

**Definition 2.4.** The exterior of $A$ is the complement of the closure of $A$ which is obviously an open set.

**Definition 2.5.** A mapping from an fts to another fts is said to be a fuzzy homeomorphism if it is bijective, continuous and open. Fuzzy homeomorphism is union preserving and crisp subset preserving. Those properties of fuzzy set that are invariant under fuzzy homeomorphism are said to be (fuzzy) topological invariants.

**Definition 2.6.** Let $X, Y$ be universal sets, then

$$R = \{((x, y), \mu_R(x, y)) | (x, y) \in X \times Y\}$$

where $\mu_R : X \times Y \to [0, 1]$, is called a binary fuzzy relation on $X \times Y$.

**Definition 2.7.** Let $R$ be a binary fuzzy relation from a fuzzy set $A \subset X$ to a fuzzy set $B \subset X$ on an fts $X$, then $R$ is called a fuzzy topological relation from $A$ to $B$ on $X$ if $R$ is a fuzzy topological invariant (or simply topological invariant) under a fuzzy homeomorphism.

**Definition 2.8.** Two fuzzy sets $A$ and $B$ in $(X, T)$ are said to be separated if there exist $U, V \in T$ such that $U \supset A$, $V \supset B$ and $U \cap B = V \cap A = \phi$ and Q-separated if there exist closed sets $H$, $K$ such that $H \supset A$, $K \supset B$ and $H \cap B = K \cap A = \phi$.

**Definition 2.9.** A fuzzy topological space $X$ is called connected if there are no separated sets $C$ and $D$ such that $X = C \cup D$. Further, a fuzzy set $A$ is said to be open-connected if there are no separated sets $C$ and $D$ such that $A = C \cap D$ and is said to be closed connected if there are no $Q$-separated sets $C$ and $D$ such that $A = C \cup D$. A fuzzy set is double connected if it is both open-connected and closed-connected.

**Definition 2.10.** A connected component in an fts is a maximal connected subset.

**Definition 2.11** (Warren)**.** The fuzzy boundary of $A$ (denoted by $\partial A$) in an fts $(X, T)$ is the infimum of all the closed fuzzy sets $D$ in $X$ with the property $D(x) \geqslant \overline{A(x)}$ for all $x \in X$ for which $(\overline{A} \wedge \overline{A^c})(x) > 0$ or $A^\circ(x) \neq 1$.

**Definition 2.12.** A fuzzy set is said to be regular closed if $\overline{A} = \overline{(\overline{A})^\circ}$. The complement of the fuzzy regular closed set is fuzzy regular open.

**Definition 2.13** ([18])**.** Let $(X, T)$ be an fts and $A$ be a fuzzy set in $X$. Then

(i) The core (denoted by $A^\oplus$) is the subset of $\overline{A}$ where $(\overline{A} \cap \overline{A^c})(x) = 0$ for all $x \in X$ or $A^\oplus(x) = \overline{A(x)}$ iff $(\overline{A} \cap \overline{A^c})(x) = 0$.

(ii) The fringe (denoted by $lA$) is the subset of $\overline{A}$ where $(\overline{A} \cap \overline{A^c})(x) > 0$ for all $x \in X$ or $lA(x) = \overline{A(x)}$ iff $(\overline{A} \cap \overline{A^c})(x) > 0$.

**Remark 2.14.** (i) $A^\oplus$ is the only crisp subset of $A^\circ$.

(ii) $lA$ is a subset of the $\partial A$, if $A$ is closed then $lA = \partial A$.

**Definition 2.15** ([18])**.** Let $(X, T)$ be an fts. The subset of the closure of fuzzy set $A$, where $\overline{A(x)} > A^\circ(x)$ for all $x \in X$ is called the frontier of $A$ in $X$ and denoted by $l^c A$. The subset of the closure of fuzzy set $A$ where $\overline{A(x)} = A^\circ(x)$ for all $x \in X$ is called the internal of $A$ and denoted by $A^i$.

In other words, $A^i(x) = \overline{A(x)}$ iff $\overline{A(x)} = A^\circ(x)$.

$$l^c A(x) = \overline{A(x)} \text{ iff } \overline{A(x)} > A^\circ(x) \text{ for all } x \in X.$$

**Remark 2.16.** (i) $A^{\oplus} \subset A^{i} \subset A^{\circ}$ and
(ii) $\partial A \supset lA \supset l^{c}A$.

**Definition 2.17** ([18])**.** Internal fringe is the subset of $A^{i} \cap A^{\oplus c}$ denoted by $l^{i}A$.

**Definition 2.18** ([18])**.** Outer of the fuzzy set $A$ is the complement of support of the closure of $A$ and is denoted by $A^{=}$ which is obviously a crisp set.

**Theorem 2.19** ([18])**.** *Let $A$ be a fuzzy set in an fts $(X, T)$. $A^{=}$, $l^{c}A$, $l^{i}A, A^{\oplus}$ are mutually disjoint and they are topological invariant.*

**Definition 2.20** ([18])**.** A fuzzy set is called a simple fuzzy region in a connected fts if it satisfies the following conditions:

(i) It is a non-empty proper double-connected closed set.
(ii) The interior, the core and the outer are double-connected regular open.
(iii) The support is equal to the support of the closure of the interior.
(iv) The fringe is double connected and the internal fringe is a double-connected open set.
(v) The frontier is a non-empty closed set.

## 3. Fuzzy region with holes

A fuzzy region with holes in a general fts should be consistent with the definition of a crisp region with holes in a crisp topological space as every notion in fuzzy topology may be considered as an extension of the notion in the general topology. So for defining a fuzzy region with holes in a general fts, it should be kept in view that

(i) A crisp subset of a simple fuzzy region should exhibit the same behavior as a simple crisp region in a general topological space.
(ii) A crisp region with holes in a fts should exhibit behavior similar to a region with hole in a general topological space.
(iii) When holes are eliminated, it should be a fuzzy region without holes in a general fts.

We shall first need to define the structure of crisp region with holes in a fuzzy topological space.

**Definition 3.1.** A subset is called a simple crisp region with hole in the fts if it meet the following conditions

(i) Its interior is a non-empty proper double connected open crisp set.
(ii) It boundary and exterior are not closed as a whole but are the union of disjoint connected component of the fringe and inner outer which are itself double connected regular crisp set.

We further require the following definitions:

**Definition 3.2.** Main outer is the outer of the fuzzy region.

**Definition 3.3.** Inner outer is the outer inside the interior of the fuzzy region.

**Definition 3.4.** Hole of the fuzzy region is the closure of the inner outer.

**Definition 3.5.** Connected component of fringe (frontier) is a fringe (frontier) which separates interior (internal) of the fuzzy region and the inner outer.

**Definition 3.6.** Generalized region is the union of the double connected interior, internal and connected component of the fringe.

### 3.1. *Formal definition of a fuzzy region with holes*

For convenience, we consider that holes are disjoint from each other which are not along the boundary of the region with holes and the region should be composed of finite number of holes.

**Definition 3.7.** A fuzzy set in a connected fts is called a simple fuzzy region with holes if it meets the following requirements:

(i) The interior and the core are double-connected regular open sets.
(ii) The outer as a whole is not double connected but is the union of main outer and disjoint inner outer which are double connected.
(iii) Inner outers are double connected regular open sets.
(iv) The support is equal to the support of the closure of the interior.
(v) The fringe and the frontier as a whole are not double connected but the union of disjoint connected components such that each component are itself closed.
(vi) The internal fringe is a double connected open set.

Condition (i) is a direct extension of crisp region in fuzzy setting as connectedness is extended into double connectedness (i.e. both open connectedness and closed connectedness) in fuzzy topology. Conditions (ii) and (v) signify the existence of hole as outer is equal to an exterior of the fuzzy region and fringe as well as frontier is equal to the boundary of the region. Double connectedness of inner outer and fringe signifies the non existence of cuts and punctures. Condition (iii) removes the irregular points or spikes etc. Condition (iv) signifies that shadow of the region will be a crisp region with hole in the crisp topological space.

Figure 1 shows the example of a fuzzy region with hole in a general fuzzy topological space which allow the membership of a point in the space in relation to the space.

## 4. **Topological relations between fuzzy regions with holes**

The definition cannot be formulated purely in terms of interior, boundary and exterior for then, the hole and the generalized region ceases to be disjoint. our definition ensure that these are disjoint topological parts so that we can use this definition to derive the topological relational matrix. We shall consider the generalized region and holes as mutually disjoint topological

**Figure 1.** Fuzzy region with hole in general fuzzy topological space

invariant to determine topological relations between two fuzzy region with holes. From regional connection calculus [3, 20], we know that there are only 8 recognizable relations between two spatial objects with connected boundary. As each generalized regions and holes can be considered as a spatial scene to determine topological relations. Let $A^*$ and $B^*$ be two generalized regions with $n$ and $m$ holes $H_{A1}, H_{A2}, \ldots, H_{An}$ and $H_{B1}, H_{B2}, \ldots, H_{Bm}$ respectively. Then the topological relations between these two regions are given by the following relational matrix:

| $t(R,R)$ | $A^*$ | $H_{A1}$ | . | . | $H_{An}$ | $B^*$ | $H_{B1}$ | . | . | $H_{Bm}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $A^*$ | $t(A^*,A^*)$ | $t(A^*,H_{A1})$ | . | . | $t(A^*,H_{An})$ | $t(A^*,B^*)$ | $t(A^*,H_{B1})$ | . | . | $t(A^*,H_{Bm})$ |
| $H_{A1}$ | $t(H_{A1},A^*)$ | $t(H_{A1},H_{A1})$ | . | . | $t(H_{A1},H_{An})$ | $t(H_{A1},B^*)$ | $t(H_{A1},H_{B1})$ | . | . | $t(H_{A1},H_{Bm})$ |
| . | . | . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . |
| $H_{An}$ | $t(H_{An},A^*)$ | $t(H_{An},H_{A1})$ | . | . | $t(H_{An},H_{An})$ | $t(H_{An},B^*)$ | $t(H_{An},H_{B1})$ | . | . | $t(H_{An},H_{Bm})$ |
| $B^*$ | $t(B^*,A^*)$ | $t(B^*,H_{A1})$ | . | . | $t(B^*,H_{An})$ | $t(B^*,B^*)$ | $t(B^*,H_{B1})$ | . | . | $t(B^*,H_{Bm})$ |
| $H_{B1}$ | $t(H_{B1},A^*)$ | $t(H_{B1},H_{A1})$ | . | . | $t(H_{B1},H_{An})$ | $t(H_{B1},B^*)$ | $t(H_{B1},H_{B1})$ | . | . | $t(H_{B1},H_{Bm})$ |
| . | . | . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . |
| $H_{Bm}$ | $t(H_{Bm},A^*)$ | $t(H_{Bm},H_{A1})$ | . | . | $t(H_{Bm},H_{An})$ | $t(H_{Bm},B^*)$ | $t(H_{Bm},H_{B1})$ | . | . | $t(H_{Bm},H_{Bm})$ |

where $t(A^*,H_{A1})$ denotes the topological relations between the object parts. Further, $t(A^*,H_{A1})$ is implied by $t(H_{A1},A^*)$ as a kind of symmetric relation. Therefore the above intersection matrix can be reduced to an equivalent upper triangular or lower triangular matrix.

| $t(R,R)$ | $A^*$ | $H_{A1}$ | . | . | $H_{An}$ | $B^*$ | $H_{B1}$ | . | . | $H_{Bm}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $A^*$ | $t(A^*,A^*)$ | . | . | . | . | . | . | . | . | . |
| $H_{A1}$ | $t(H_{A1},A^*)$ | $t(H_{A1},H_{A1})$ | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . |
| $H_{An}$ | $t(H_{An},A^*)$ | $t(H_{An},H_{A1})$ | . | . | $t(H_{An},H_{An})$ | . | . | . | . | . |
| $B^*$ | $t(B^*,A^*)$ | $t(B^*,H_{A1})$ | . | . | $t(B^*,H_{An})$ | $t(B^*,B^*)$ | . | . | . | . |
| $H_{B1}$ | $t(H_{B1},A^*)$ | $t(H_{B1},H_{A1})$ | . | . | $t(H_{B1},H_{An})$ | $t(H_{B1},B^*)$ | $t(H_{B1},H_{B1})$ | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . |
| $H_{Bm}$ | $t(H_{Bm},A^*)$ | $t(H_{Bm},H_{A1})$ | . | . | $t(H_{Bm},H_{An})$ | $t(H_{Bm},B^*)$ | $t(H_{Bm},H_{B1})$ | . | . | $t(H_{Bm},H_{Bm})$ |

Since the relation between each pair of the holes is disjoint, relation to itself is equal and each hole is inside the generalized region. Therefore, we can further reduce the number of redundant relations in the relational matrix. Under this assumption the number of distinct relations in $1^{\text{st}}$ row, $2^{\text{nd}}$ row,..., $n^{\text{th}}$ row are zero but the number of distinct elements in $(n+1)$th, $(n+2)$th, ..., $(m+n+1)$th row are given by

Number of distinct elements in the $1^{\text{st}}$ row $= 0$
Number of distinct elements in the $2^{\text{nd}}$ row $= 0$
..............................................
..............................................
Number of distinct elements in the $n^{\text{th}}$ row $= 0$
Number of distinct elements in the $(n+1)^{\text{th}}$ row $= 8^{n+1}$
Number of distinct elements in the $(n+2)^{\text{th}}$ row $= 8^{n+1}$
......................................................
......................................................
Number of distinct elements in the $(m+n+2)^{\text{th}}$ row $= 8^{n+1}$

Now, the number of necessary relations will be equal to the sum of the number distinct relations in each row of the relation matrix.

Therefore, the total number of necessary relations will be $8^{(n+1)(m+1)}$.

**For example,** if $n = 1$ and $m = 1$ i.e. each region with a single hole there will be $8^4 = 4096$ relations but under certain conditions only a few subsets of them are realized between two region with a hole.

4.1. *Conditions for reducing redundant relations*

Topological relations between two simple regions each with a hole can be identified by using intersection matrix considering the generalized region and hole as disjoint topological invariants. We list some of the geometric conditions to reduce the number of redundant relations:

(i) If the regions with holes do not intersect each other then the relation between holes of the region is 'disjoint'.
(ii) If both the regions meet each other then too the relation between holes is 'disjoint'.
(iii) If a fuzzy region with hole is completely inside the hole of the other then the relation between generalized region of the second and generalized region of the first as well as relation between hole of the first and generalized region of the second is 'disjoint'.
(iv) If a fuzzy region with hole meets the hole of the other, then the relation between generalized region of the second and generalized region of the first as well as relation between hole of the first and generalized region of the second are non empty.

(v) If one region with hole is inside the generalized region of the other region with hole and does not intersect the hole of the second, then the intersection between generalized regions of the both the regions as well as hole of the first with the generalized region of the second is non-empty. But the intersection between the holes of both the regions is empty.

(vi) If one region with hole is inside the generalized region of the other region with hole and intersects the hole of the second then the intersection between generalized region of both the regions as well as hole of the first with the generalized region of the second is non empty. The intersection between the holes of both the regions is also non-empty.

(vii) If one region with hole is inside the generalized region of the other region with hole and meet the hole of the second then the intersection between generalized regions of the both the regions as well as hole of the first with the generalized region of the second is non empty. The intersection between the holes of both the regions is also non-empty.

(viii) If one region with hole is inside the generalized region of the other region with hole and covers the hole of the second then the intersection between generalized region of the both the regions as well as hole of the first (second) with the generalized region of the second (first) is non empty. The intersection between the holes of both the regions is also non-empty.

(ix) If one region with hole is inside the generalized region of the other region with hole such that the hole of both the region also intersect each other then the intersection between generalized region of the both the regions as well as hole of the first (second) with the generalized region of the second (first) is non empty also the intersection between the hole of both the regions is non-empty.

(x) Conditions (v), (vi), (vii), (viii), (ix) are also applicable if one region with hole intersects the other region with a hole.

(xi) If both the region with hole coincides (or one remains inside) but their holes do not, then the intersection between the generalized region is non empty but the intersection between holes shall depend on whether they are disjoint or meet or intersect each other.

(xii) If both the generalized regions (one inside the other) and holes coincide then all the intersections are non-empty.

Under this set of conditions only 117 relations are recognizable between two fuzzy regions each with a single hole.

## 5. Conclusion

In this paper we have proposed a formal definition of a fuzzy region with holes in a general fuzzy topological space and derived the topological relations between two fuzzy regions with holes. This definition provides a more general framework to deal with complex spatial objects as a better representative of real life situations. We have seen that in case of general fuzzy topological space there are 117 distinct topological relations recognizable between two fuzzy regions each with a single hole. Egenhofer, considered his framework in a crisp topological space and derived topological relations between regions (by considering the generalized region and the hole as topological invariant) each with single hole [10] as well as regions each with 2 and 3 holes [7] respectively. In this paper, we have considered our space to be a general fuzzy topological space and defined a fuzzy region with holes in such a way that the generalized region and the holes should be disjoint topological invariant. We have discussed topological relations between two fuzzy regions each with '$m$' and '$n$' holes respectively and found that there are total of $8^{(n+1)(m+1)}$ relations. But if we consider $m = 1 = n$ i.e. each region with single hole then the number of relations between two regions each with a hole is 4096. After applying the conditions we have seen that the number of relations reduces to only 117 relations which is less than the number of relations determined by Egenhofer et al. in the crisp case.

Fuzzy regions with holes are likely to have wide applications in various real life situations such as detection of occurrence of cancer, crack detection via X-ray etc.

A model of fuzzy region with holes have the potential to be utilized in the detection of white blood cells in bone marrow in the diagnosis and treatment of various forms of cancer.

For instance, in leukemia the contour of bone marrow may be considered as the boundary of the region, white blood cells as holes and red blood cells with platelets as the interior of the region. Therefore, it can be modeled as a fuzzy region with holes and topological relations can be used to analyze various stages of propagation of the disease.

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