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Research Article

The Non-Split Complement Line Domination in Graphs

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Abstract. Harary and Norman introduced the Line graph $L(G)$. We introduced the split complement line domonation number by posting the disconnected property on the dominating sets of $\overline{L(G)}$. In this paper, we study the connectedness property of dominating sets of $\overline{L(G)}$ by defining non-split domination parameter. Also, we studied its graph theoretical properties in terms of elements of G.

Keywords. Graph; Line graph; Domination number; Line domination number; Split line domination number; Split complement line domination number

MSC. 05C69; 05C76

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1. Introduction

The graph we mean $G = (V, E)$ is a finite, simple, undirected and connected graph with p vertices and q edges. Terms not defined here are used in the sense of Harary [1].

A **line graph** $L(G)$ is the graph whose vertices correspond to the edge of G and two vertices in $L(G)$ are adjacent if and only if the corresponding edges in G are adjacent. This was introduced by Harary and Norman [3].

A set $D \subseteq V(G)$ of a graph is a **dominating set** of G, if every vertex in $V \setminus D$ is adjacent to some vertices in D. The domination number is the minimum cardinality taken over all the dominating sets in G and is denoted by $\gamma(G)$. This concept was introduced by Ore in [8].

A dominating set $D \subseteq V(G)$ is a **non-split dominating set**, if the induced subgraph $\langle V \setminus D \rangle$ is connected. This concept was introduced by Kulli and Janakiram in [5].

In [7], a set $D \subseteq V(L(G))$ is said to be **line dominating set** of G, if every vertex not in D is adjacent to some vertices in D. The domination number in line graph is the minimum cardinality taken over all the dominating sets of $L(G)$, and is denoted by $\gamma_l(G)$.

AA set $D \subseteq V(\overline{L(G)})$ is said to be **complement line dominating set** of G, if every vertex not in D is adjacent to some vertices in D. The domination number in complement line graph is the minimum cardinality taken over all the dominating sets in $\overline{L(G)}$, and is denoted by $\gamma_{\overline{l}}(G)$.

In this paper, we introduced this non-split parameter for complement of line graph. Also we found the exact value of this parameter for some standard graphs and obtained the bounds in terms of elements of G.

2. Main Results

Definition 2.1. A dominating set D of a complement line graph $\overline{L(G)}$ is said to be a non-split complement line dominating set (NSCLD-set), if the induced subgraph $\langle V(\overline{L(G)}) \setminus D \rangle$ is connected. The minimum cardinality of NSCLD-set is said to be **non-split complement line domination number** of G and is denoted by $\gamma_{\overline{nsl}}(G)$.

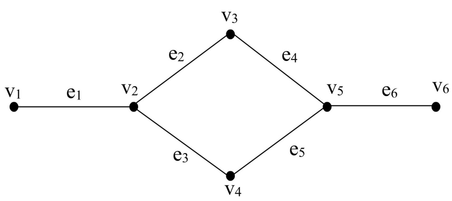

Figure 1. G

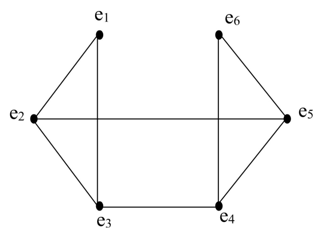

Figure 2. $L(G)$

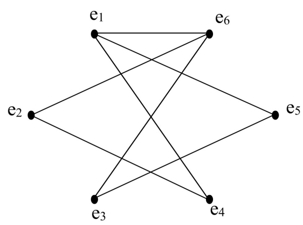

Figure 3. $\overline{L(G)}$

Example 2.1. For the graph $\overline{L(G)}$ in Figure 3, the vertex set $D = \{e_3, e_4\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(G) = 2$.

Remark 2.1. Throughout this paper, we consider the graphs which has atleast one NSCLD-set.

Theorem 2.2. *For the cycle graph C_n,*

$$\gamma_{\overline{nsl}}(C_n) = \begin{cases} 3 & if\ n = 5 \\ 2 & if\ n \geq 6 \end{cases}$$

Proof. Let G be a cycle graph C_n, $n \geq 5$ with $V(G) = \{v_1, v_2, \dots, v_n\}$ and $E(G) = \{e_1, e_2, \dots, e_n\}$.
Then $V(\overline{L(G)}) = \{e_1, e_2, \dots, e_n\}$, $n \geq 5$.

Case i: $n = 5$**.** In this case, the set $D = \{e_1, e_2, e_4\}$ is a NSCLD-set with minimum cardinality. Therefore $\gamma_{\overline{nsl}}(G) = 3$.

Case ii: $n \geq 6$**.** In this case, the set $D = \{e_1, e_2\}$ is a NSCLD-set with minimum cardinality, since $\langle V(\overline{L(G)}) \backslash D \rangle$ is connected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 2, n \geq 6$. □

Example 2.2.

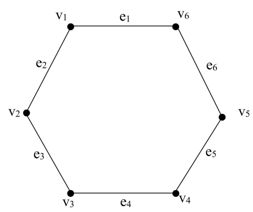

Figure 4. C_6

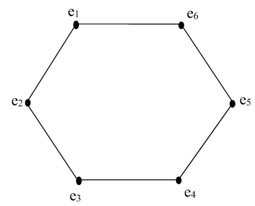

Figure 5. $L(C_6)$

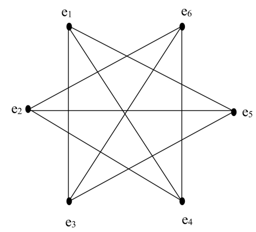

Figure 6. $\overline{L(C_6)}$

For the graph $\overline{L(C_6)}$ in Figure 6, the vertex set $D = \{e_1, e_2\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(C_6) = 2$.

Theorem 2.3. *For the path graph* P_n*,* $\gamma_{\overline{nsl}}(P_n) = 2, n \geq 5$.

Proof. Let G be a path graph P_n, $n \geq 5$ with $V(G) = \{v_1, v_2, \dots, v_n\}$ and $E(G) = \{e_1, e_2, \dots, e_{n-1}\}$. Then $V(\overline{L(G)}) = \{e_1, e_2, \dots, e_{n-1}\}$, $n \geq 5$.

Here the set $D = \{e_2, e_3\}$ is a NSCLD-set with minimum cardinality, since $\langle V(\overline{L(G)}) \backslash D \rangle$ is connected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 2, n \geq 5$. □

Example 2.3.

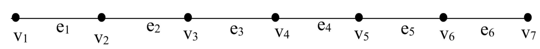

Figure 7. P_7

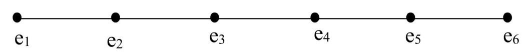

Figure 8. $L(P_7)$

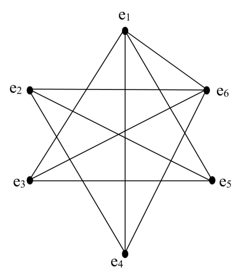

Figure 9. $\overline{L(P_7)}$

For the graph $\overline{L(P_7)}$ in Figure 9, the vertex set $D = \{e_2, e_3\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(P_7) = 2$.

Theorem 2.4. *For the complete bipartite graph $K_{m,n}$,*

$$\gamma_{\overline{nsl}}(K_{m,n}) = \begin{cases} 2 & \text{if } m \text{ (or) } n = 2 \\ 3 & \text{if } m \geq 3,\ n \geq 3 \end{cases}$$

Proof. Let G be a complete bipartite graph $K_{m,n}$, $m,n \geq 2$ with

$$V(G) = \{u_i, v_j / i = 1 \text{ to } m, j = 1 \text{ to } n\} \text{ and } E(G) = \{u_i v_j / i = 1 \text{ to } m, j = 1 \text{ to } n\}.$$

Then $V(\overline{L(G)}) = \{u_i v_j / i = 1 \text{ to } m, j = 1 \text{ to } n\}$, $m,n \geq 2$.

Case i: m (or) $n = 2$. In this case, the set $D = \{u_1v_1, u_2v_1\}$ is the NSCLD-set with minimum cardinality.

Therefore $\gamma_{\overline{nsl}}(G) = 2$.

Case ii: $m,n \geq 3$. In this case, the set $D = \{u_1v_1, u_1v_2, u_2v_1\}$ is the NSCLD-set with minimum cardinality, since $\langle V(\overline{L(G)}) \backslash D \rangle$ is connected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 3$, $m,n \geq 3$. □

Example 2.4.

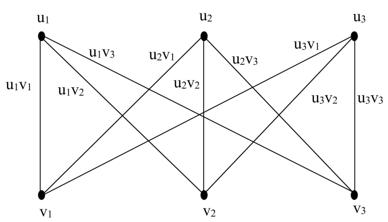

Figure 10. $K_{3,3}$

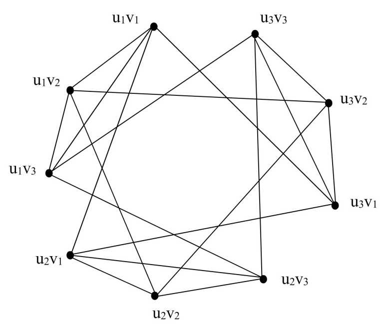

Figure 11. $L(K_{3,3})$

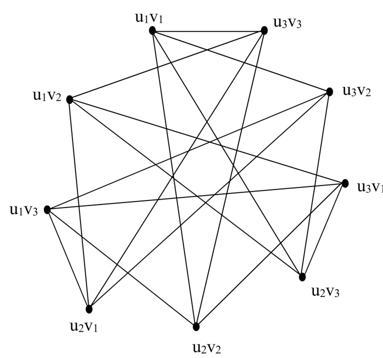

Figure 12. $\overline{L(K_{3,3})}$

For the graph $\overline{L(K_{3,3})}$ in Figure 12, the vertex set $D = \{u_1v_1, u_1v_2, u_2v_1\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(K_{3,3}) = 3$.

Theorem 2.5. *For the wheel graph W_n,*

$$\gamma_{\overline{nsl}}(W_n) = \begin{cases} 4 & \text{if } n = 4 \\ 3 & \text{if } n = 5 \\ 2 & \text{if } n \geq 6 \end{cases}$$

Proof. Let G be a wheel graph W_n, $n \geq 4$ with $V(G) = \{u, v_1, v_2, \ldots, v_n\}$ and $E(G) = \{e_1, e_2, \ldots, e_{2n}\}$. Then $V(\overline{L(G)}) = \{e_1, e_2, \ldots, e_{2n}\}$, $n \geq 4$.

Case i: $n = 4$. In this case, the set $D = \{e_1, e_3, e_5, e_6\}$ is the NSCLD-set with minimum cardinality. Therefore $\gamma_{\overline{nsl}}(G) = 4$.

Case ii: $n = 5$**.** In this case, the set $D = \{e_1, e_2, e_3\}$ is the NSCLD-set with minimum cardinality. Therefore $\gamma_{\overline{nsl}}(G) = 5$.

Case iii: $n \geq 6$**.** In this case, the set $D = \{e_1, e_7\}$ is the NSCLD-set with minimum cardinality, since $\langle V(\overline{L(G)}) \backslash D \rangle$ is connected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 2$, $n \geq 6$. □

Example 2.5.

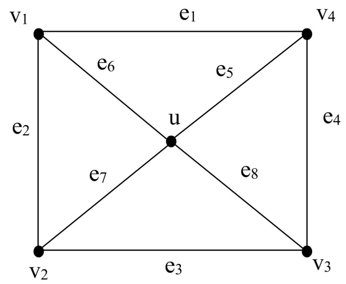

Figure 13. W_4

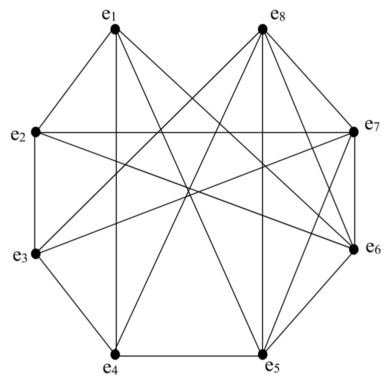

Figure 14. $L(W_4)$

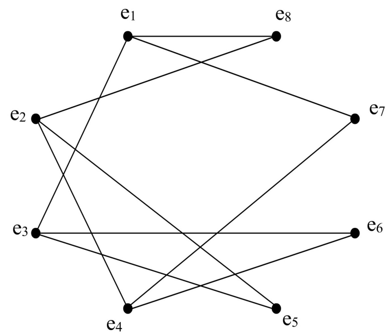

Figure 15. $\overline{L(W_4)}$

For the graph $\overline{L(W_4)}$ in Figure 15, the vertex set $D = \{e_1, e_3, e_5, e_6\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(W_4) = 4$.

Theorem 2.6. *For the bistar tree* $B_{n,n}$, $\gamma_{\overline{nsl}}(B_{n,n}) = 3$, $n \geq 2$.

Proof. Let G be a bistar tree $B_{n,n}$, $n \geq 2$ with $V(G) = \{v_1, v_2, \dots, v_{2n+2}\}$ and $E(G) = \{e_1, e_2, \dots, e_{2n+1}\}$. Then $V(\overline{L(G)}) = \{e_1, e_2, \dots, e_{2n+1}\}$, $n \geq 2$. Here the set $D = \{e_1, e_{n+1}, e_{n+2}\}$ is a NSCLD-set with minimum cardinality, since $\langle V(\overline{L(G)}) \backslash D \rangle$ is connected. Which gives, $\gamma_{\overline{nsl}}(G) = |D| = 3$, $n \geq 2$. □

Example 2.6.

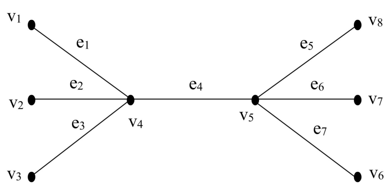

Figure 16. $B_{3,3}$

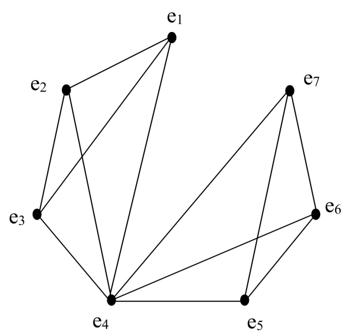

Figure 17. $L(B_{3,3})$

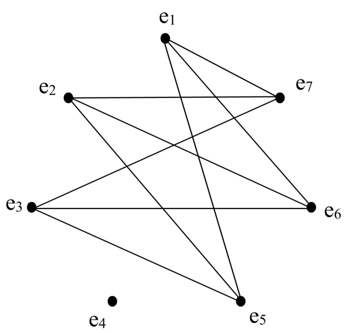

Figure 18. $\overline{L(B_{3,3})}$

For the graph $\overline{L(B_{3,3})}$ in Figure 18, the vertex set $D = \{e_1, e_4, e_5\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(B_{3,3}) = 3$.

Theorem 2.7. *For the crown graph C_n^+,*

$$\gamma_{\overline{nsl}}(C_n^+) = \begin{cases} 3 & if\ n = 3 \\ 2 & if\ n \geq 4 \end{cases}$$

Proof. Let G be a crown graph C_n^+, $n \geq 3$ with $V(G) = \{v_1, v_2, \dots, v_{2n}\}$ and $E(G) = \{e_1, e_2, \dots, e_{2n}\}$. Then $V(\overline{L(G)}) = \{e_1, e_2, \dots, e_{2n}\}$, $n \geq 3$.

Case i: $n = 3$**.** In this case, the set $D = \{e_2, e_4, e_6\}$ is a NSCLD-set with minimum cardinality. Therefore $\gamma_{\overline{nsl}}(G) = 3$.

Case ii: $n \geq 4$**.** In this case, the set $D = \{e_1, e_{n+1}\}$ is a NSCLD-set with minimum cardinality, since $\langle V(\overline{L(G)}) \setminus D \rangle$ is connected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 2$, $n \geq 4$. □

Example 2.7.

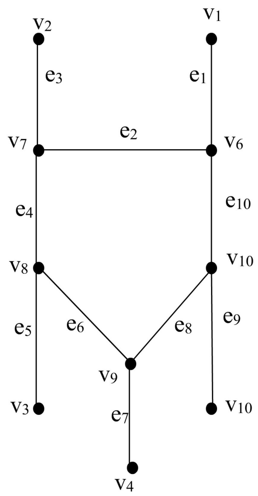

Figure 19. C_5^+

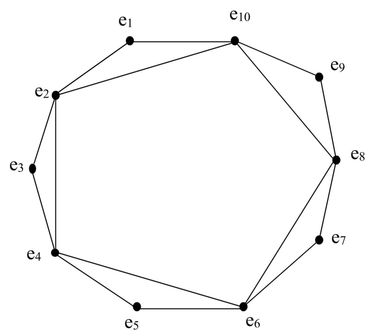

Figure 20. $L(C_5^+)$

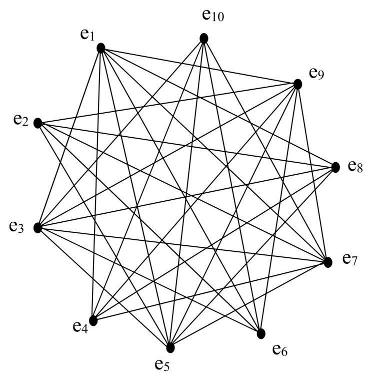

Figure 21. $\overline{L(C_5^+)}$

For the graph $\overline{L(C_5^+)}$ in Figure 21, the vertex set $D = \{e_1, e_6\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(C_5^+) = 2$.

Theorem 2.8. *For the comb tree P_n^+,*

$$\gamma_{\overline{nsl}}(P_n^+) = \begin{cases} 3 & if\ n = 3 \\ 2 & if\ n \geq 4 \end{cases}$$

Proof. Let G be a comb tree P_n^+, $n \geq 3$ with $V(G) = \{v_1, v_2, \dots, v_{2n}\}$ and $E(G) = \{e_1, e_2, \dots, e_{2n-1}\}$. Then $V(\overline{L(G)}) = \{e_1, e_2, \dots, e_{2n-1}\}$, $n \geq 3$.

Case i: $n = 3$**.** In this case, the set $D = \{e_2, e_3, e_4\}$ is a NSCLD-set with minimum cardinality. Therefore $\gamma_{\overline{nsl}}(G) = 3$.

Case ii: $n \geq 4$**.** In this case, the set $D = \{e_1, e_2\}$ is a NSCLD-set with minimum cardinality, since $\langle V(\overline{L(G)}) \setminus D \rangle$ is connected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 2$, $n \geq 4$. □

Example 2.8.

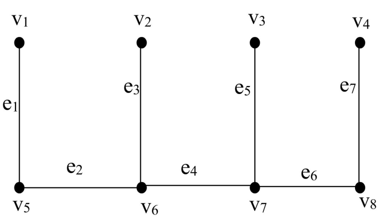

Figure 22. P_4^+

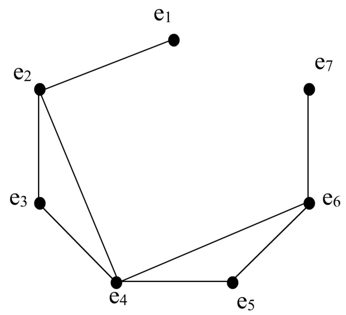

Figure 23. $L(P_4^+)$

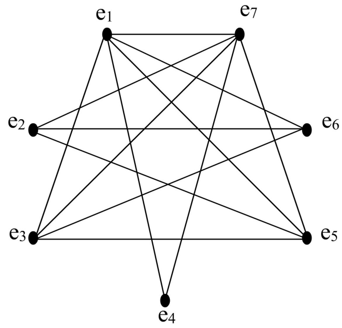

Figure 24. $\overline{L(P_4^+)}$

For the graph $\overline{L(P_4^+)}$ in Figure 24, the vertex set $D = \{e_1, e_2\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(P_4^+) = 2$.

Theorem 2.9. *For the helm graph* W_n^+,

$$\gamma_{\overline{nsl}}(W_n^+) = \begin{cases} 3 & if\ n = 2,3 \\ 2 & if\ n \geq 4 \end{cases}$$

Proof. G be a helm graph W_n^+, $n \geq 2$ with $V(G) = \{u, u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$ and $E(G) = \{e_1, e_2, \dots, e_{3n+1}\}$. Then $V(\overline{L(G)}) = \{e_1, e_2, \dots, e_{3n+1}\}$, $n \geq 2$.

Case i: $n = 2,3$**.** In this case, the set $D = \{e_4, e_5, e_6\}$ is a NSCLD-set with minimum cardinality. Therefore $\gamma_{\overline{nsl}}(G) = 3$.

Case ii: $n \geq 4$**.** In this case, the set $D = \{e_1, e_{n-1}\}$ is a NSCLD-set with minimum cardinality, since $\langle V(\overline{L(G)}) \backslash D \rangle$ is connected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 2$, $n \geq 4$. □

Example 2.9.

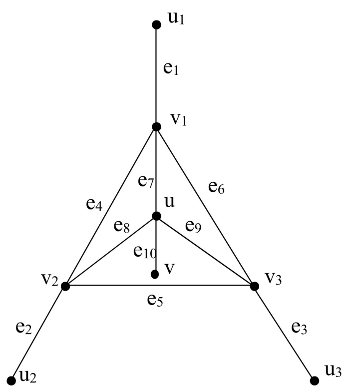

Figure 25. W_3^+

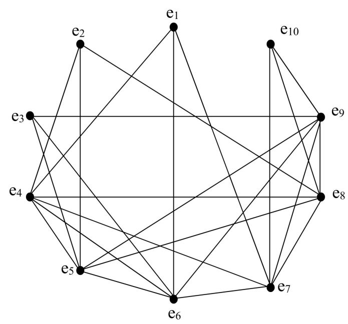

Figure 26. $L(W_3^+)$

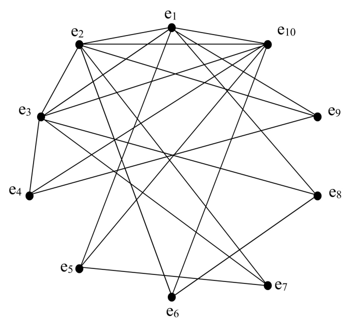

Figure 27. $\overline{L(W_3^+)}$

For the graph $\overline{L(W_3^+)}$ in Figure 27, the vertex set $D = \{e_4, e_5, e_6\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(W_3^+) = 3$.

Theorem 2.10. *For the graph K_n^+, $\gamma_{\overline{nsl}}(K_n^+) = 3$, $n \geq 3$.*

Proof. Let G be a K_n^+ graph, $n \geq 3$ with $V(G) = \{v_1, v_2, \ldots, v_{2n}\}$ and $E(G) = \{e_1, e_2, \ldots, e_{\frac{n(n+1)}{2}}\}$. Then $V(\overline{L(G)}) = \{e_1, e_2, \ldots, e_{\frac{n(n+1)}{2}}\}$, $n \geq 3$.

Case i: $n = 3$**.** In this case, the set $D = \{e_2, e_4, e_5\}$ is a NSCLD-set with minimum cardinality. Therefore $\gamma_{\overline{nsl}}(G) = 3$.

Case ii: $n = 4$**.** In this case, the set $D = \{e_1, e_2, e_9\}$ is a NSCLD-set with minimum cardinality, since $\langle V(\overline{L(G)}) \backslash D \rangle$ is connected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 3$.

Case iii: $n \geq 5$**.** In this case, the set $D = \{e_1, e_2, e_3\}$ is a NSCLD-set with minimum cardinality, since $\langle V(\overline{L(G)}) \backslash D \mathring{a}ngle$ is connected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 3$, $n \geq 5$. □

Example 2.10.

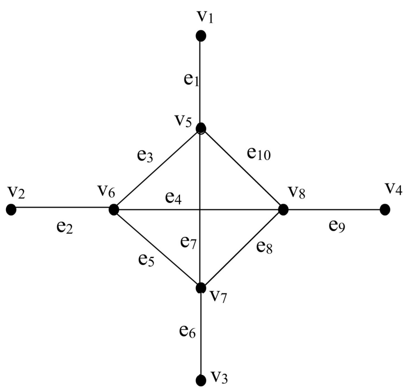

Figure 28. K_4^+

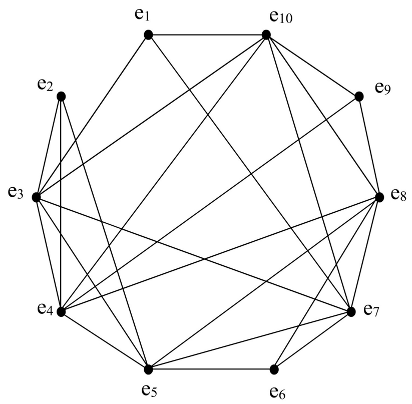

Figure 29. $L(K_4^+)$

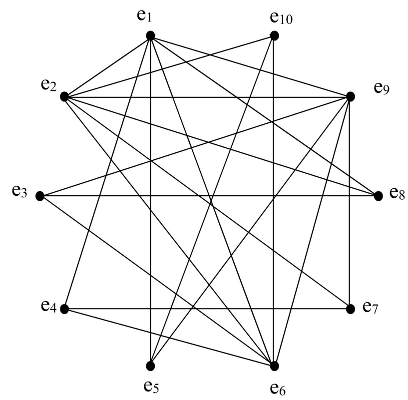

Figure 30. $\overline{L(K_4^+)}$

For the graph $\overline{L(K_4^+)}$ in Figure 30, the vertex set $D = \{e_1, e_2, e_9\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(K_4^+) = 3$.

Theorem 2.11. *For the book graph B_n, $\gamma_{\overline{nsl}}(B_n) = 2$, $n \geq 2$.*

Proof. Let G be a book graph B_n, $n \geq 2$ with $V(G) = \{u, v, v_1, v_2, \ldots, v_{2n}\}$ and $E(G) = \{e, e_1, e_2, \ldots, e_{3n}\}$. Then $V(\overline{L(G)}) = \{e, e_1, e_2, \ldots, e_{3n}\}$, $n \geq 2$. Here the set $D = \{e_1, e_{2n}\}$ is the NSCLD-set with minimum cardinality, since $\langle V(\overline{L(G)}) \backslash D \rangle$ is connected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 2$, $n \geq 2$. □

Example 2.11.

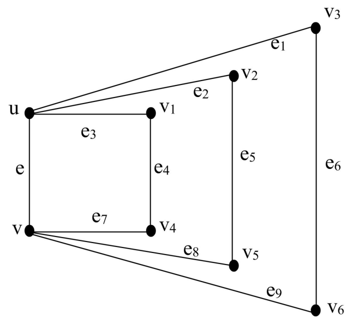

Figure 31. B_3

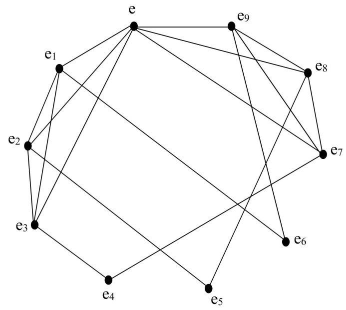

Figure 32. $L(B_3)$

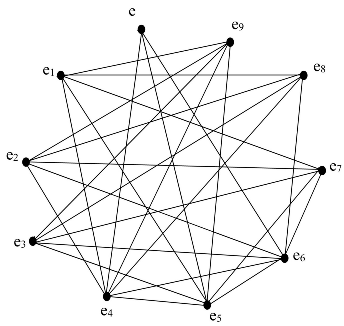

Figure 33. $\overline{L(B_3)}$

For the graph $\overline{L(B_3)}$ in Figure 33, the vertex set $D = \{e_1, e_6\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(B_3) = 2$.

Theorem 2.12. *For the friendship graph $C_3^{(m)}$,*

$$\gamma_{\overline{nsl}}(C_3^{(m)}) = \begin{cases} 3 & \text{if } m = 2,3 \\ 2 & \text{if } m \geq 4 \end{cases}$$

Proof. Let G be a friendship graph $C_3^{(m)}$, $m \geq 2$ with $V(G) = \{u, v_1, v_2, \ldots, v_{3m}\}$ and $E(G) = \{e_1, e_2, \ldots, e_{3m}\}$. Then $V(\overline{L(G)}) = \{e_1, e_2, \ldots, e_{3m}\}$, $m \geq 2$.

Case i: $m = 2,3$**.** In this case, the set $D = \{e_1, e_2, e_{3m}\}$ is a NSCLD-set with minimum cardinality. Therefore $\gamma_{\overline{nsl}}(G) = 3$.

Case ii: $n \geq 4$**.** In this case, the set $D = \{e_1, e_4\}$ is a NSCLD-set with minimum cardinality, since $\langle V(\overline{L(G)}) \backslash D \rangle$ is connected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 2$, $m \geq 4$. □

Example 2.12.

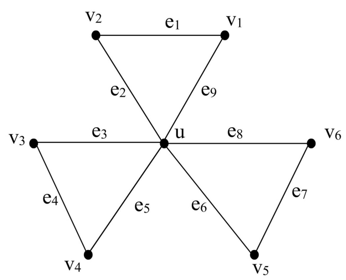

Figure 34. C_3^3

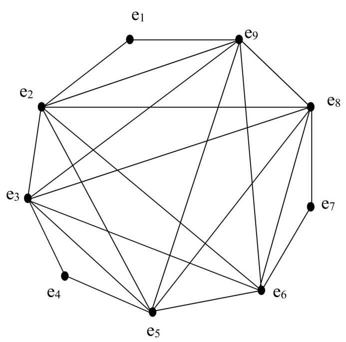

Figure 35. $L(C_3^3)$

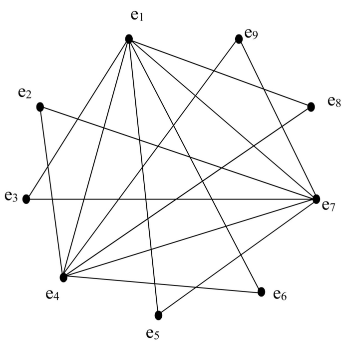

Figure 36. $\overline{L(C_3^3)}$

For the graph $\overline{L(C_3^3)}$ in Figure 36, the vertex set $D = \{e_1, e_2, e_9\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(C_3^3) = 3$.

Theorem 2.13. *For the triangular snake graph mC_3,*

$$\gamma_{\overline{nsl}}(mC_3) = \begin{cases} 4 & if\ m = 2 \\ 2 & if\ m \geq 3 \end{cases}$$

Proof. Let G be a triangular snake graph mC_3, $m \geq 2$ with $V(G) = \{v_1, v_2, \dots, v_{2m+1}\}$ and $E(G) = \{e_1, e_2, \dots, e_{3m}\}$. Then $V(\overline{L(G)}) = \{e_1, e_2, \dots, e_{3m}\}$, $m \geq 2$.

Case i: $m = 2$. In this case, the set $D = \{e_2, e_3, e_5, e_6\}$ is a NSCLD-set with minimum cardinality. Therefore $\gamma_{\overline{nsl}}(G) = 4$.

Case ii: $m \geq 3$. In this case, the set $D = \{e_1, e_{3m}\}$ is a NSCLD-set with minimum cardinality, since $\langle V(\overline{L(G)}) \setminus D \rangle$ is connected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 2$, $m \geq 3$. □

Example 2.13.

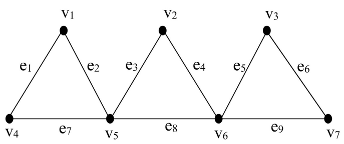

Figure 37. $3C_3$

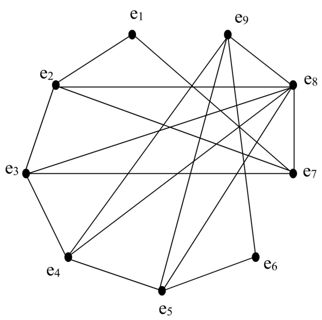

Figure 38. $L(3C_3)$

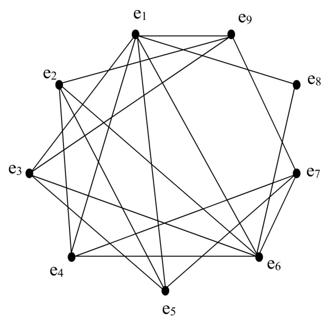

Figure 39. $\overline{L(3C_3)}$

For the graph $\overline{L(3C_3)}$ in Figure 39, the vertex set $D = \{e_1, e_9\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(3C_3) = 2$.

Theorem 2.14. *For the dragon graph $C_m@P_n$, $m \geq 3$, $n \geq 1$,*

$$\gamma_{\overline{nsl}}(C_m@P_n) = \begin{cases} 3 & if\ m+n = 5 \\ 2 & otherwise \end{cases}$$

Proof. Let G be a dragon graph $C_m@P_n$, $m \geq 3$, $n \geq 1$ with $V(G) = \{v_1, v_2, \dots, v_{m+n}\}$ and $E(G) = \{e_1, e_2, \dots, e_{m+n}\}$. Then $V(\overline{L(G)}) = \{e_1, e_2, \dots, e_{m+n}\}$.

Case i: $m + n = 5$. In this case, the set $D = \{e_{m-2}, e_m, e_{m+1}\}$ is a NSCLD-set with minimum cardinality. Therefore $\gamma_{\overline{nsl}}(G) = 3$.

Case ii: $m = 3$, $n \geq 3$. In this case, the set $D = \{e_{m+n-1}, e_{m+n}\}$ is a NSCLD-set with minimum cardinality. Therefore $\gamma_{\overline{nsl}}(G) = 2$.

Case iii: $m > 3$, $n \geq 3$. In this case, the set $D = \{e_{m-2}, e_{m-1}\}$ is a NSCLD-set with minimum cardinality, since $\langle V(\overline{L(G)}) \setminus D \rangle$ is connected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 2$. □

Example 2.14.

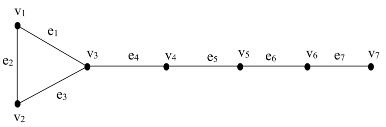

Figure 40. $C_3@P_4$

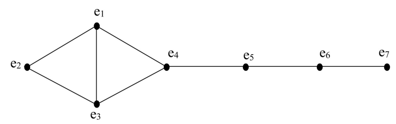

Figure 41. $L(C_3@P_4)$

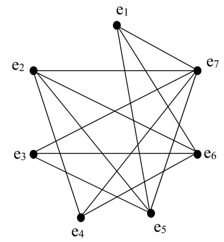

Figure 42. $\overline{L(C_3@P_4)}$

For the graph $\overline{L(C_3@P_4)}$ in Figure 42, the vertex set $D = \{e_6, e_7\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(C_3@P_4) = 2$.

Theorem 2.15. *For the quadrilateral snake graph mC_4, $\gamma_{\overline{nsl}}(mC_4) = 2$, $m \geq 2$.*

Proof. Let G be a quadrilateral snake graph mC_4, $m \geq 2$ with $V(G) = \{v_1, v_2, \ldots, v_{3m+1}\}$ and $E(G) = \{e_1, e_2, \ldots, e_{4m}\}$. Then $V(\overline{L(G)}) = \{e_1, e_2, \ldots, e_{4m}\}$, $m \geq 2$. Here the set $D = \{e_1, e_2\}$ is a NSCLD-set with minimum cardinality, since $\langle V(\overline{L(G)}) \backslash D \rangle$ is connected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 2$, $m \geq 2$. □

Example 2.15.

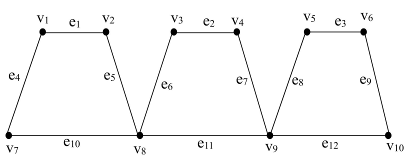

Figure 43. $3C_4$

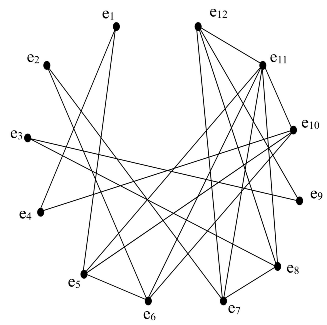

Figure 44. $L(3C_4)$

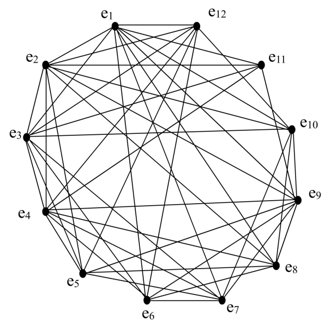

Figure 45. $\overline{L(3C_4)}$

For the graph $\overline{L(3C_4)}$ in Figure 45, the vertex set $D = \{e_1, e_2\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(3C_4) = 2$.

Theorem 2.16. *For the graph $K^+_{m,n}$,*

$$\gamma_{\overline{nsl}}(K^+_{m,n}) = \begin{cases} 0 & \textit{if } m = n = 1 \\ 3 & \textit{if } m + n = 3 \\ 2 & \textit{otherwise} \end{cases}$$

Proof. Let G be a $K^+_{m,n}$ graph, $m, n \geq 1$ with $V(G) = \{u_1, u_2, \ldots, u_{2m}, v_1, v_2, \ldots, v_{2n}\}$ and $E(G) = \{e_1, e_2, \ldots, e_{m+n}, u_i v_j / i = 1 \text{ to } m, j = 1 \text{ to } n\}$. Then $V(\overline{L(G)}) = \{e_1, e_2, \ldots, e_{m+n}, u_i v_j / i = 1 \text{ to } m, j = 1 \text{ to } n\}$.

Case i: $m = n = 1$**.** In this case, the NSCLD does not exist.

Case ii: $m + n = 3$**.** In this case, the set $D = \{e_2, u_i v_j / i = 1,2; j = 1,2\}$ is a NSCLD-set of G with minimum cardinality. Therefore $\gamma_{\overline{nsl}} = 3$.

Case iii: $m = 1, n = 3$**.** In this case, the set $D = \{e_2, u_1 v_1\}$ is a NSCLD-set of G with minimum cardinality. Therefore $\gamma_{\overline{nsl}}(G) = 2$.

Case iv: $m = 2$ **and** $n = 2$**.** In this case, the set $D = \{e_{m+n-1}, e_{m+n}\}$ is a NSCLD-set of G with minimum cardinality, since $\langle V(\overline{L(G)}) \backslash D \rangle$ is disconnected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 2$.

Case v: $m = 3, n = 1$**.** In this case, the set $D = \{e_1, u_1 v_1\}$ is a NSCLD-set of G with minimum cardinality, since $\langle V(\overline{L(G)}) \backslash D \rangle$ is disconnected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 2$.

Case vi: $m > 3$ **and** $n > 3$**.** In this case, the set $D = \{e_{m+n-1}, e_{m+n}\}$ is a NSCLD-set of G with minimum cardinality, since $\langle V(\overline{L(G)}) \backslash D \rangle$ is disconnected. Hence, $\gamma_{\overline{nsl}}(G) = |D| = 2$. □

Example 2.16.

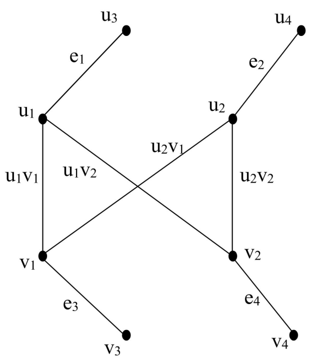

Figure 46. $K^+_{2,2}$

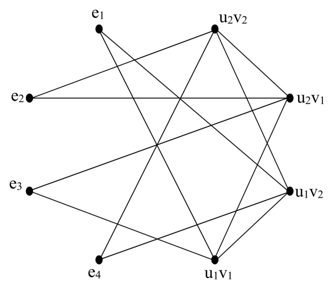

Figure 47. $L(K^+_{2,2})$

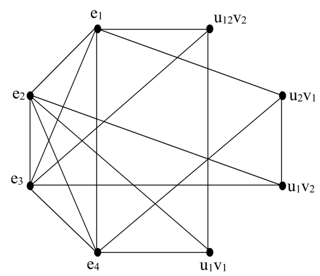

Figure 48. $\overline{L(K^+_{2,2})}$

For the graph $\overline{L(K^+_{2,2})}$ in Figure 48, the vertex set $D = \{e_3, e_4\}$ is a $\gamma_{\overline{nsl}}$-set and hence $\gamma_{\overline{nsl}}(K^+_{2,2}) = 2$.

3. Bounds

Theorem 3.1. *For any graph G, $\gamma_{\overline{l}}(G) \le \gamma_{\overline{nsl}}(G)$.*

Proof. Since every non-split complement line dominating set is necessarily a complement line dominating set, and hence we have $\gamma_{\overline{l}}(G) \le \gamma_{\overline{nsl}}(G)$.

The following result is obvious from the bounds of standard simple graphs.

Theorem 3.2. *For any graph G, $2 \le \gamma_{\overline{nsl}}(G) \le 4$.*

Theorem 3.3. *For any graph G, $\gamma_{\overline{l}}(G) = \gamma_{\overline{nsl}}(G)$, if $\delta(\overline{L(G)}) \ge 4$.*

Proof. For the graph $\overline{L(G)}$ with minimum degree≥ 4, every non-split complement line dominating set is a line dominating set, hence

$$\gamma_{\bar{l}}(G) \leq \gamma_{\overline{nsl}}. \tag{3.1}$$

Also, every complement line dominating set is a non-split complement line dominating set, and hence

$$\gamma_{\bar{l}}(G) \geq \gamma_{\overline{nsl}}. \tag{3.2}$$

The result is followed from (3.1) and (3.2).

Theorem 3.4. *For any graph G, $\gamma_{\overline{nsl}}(G) \leq q - \Delta(\overline{L(G)}) + 1$.*

Proof. Let V be a vertex set of $\overline{L(G)}$ with maximum degree ≥ 2 implies there exist two vertices v_1 and v_2 adjacent to v.

Consider the vertex set $D = \{V \setminus N(v)\} \cup \{v_1, v_2\}$, clearly v and the vertices $N(v)$ are dominated by v_1 and v_2. So, D is a vertex set of $\overline{L(G)}$. Also, $V \setminus D = N(v) \setminus \{v_1, v_2\}$ which is connected.

Therefore,

$$\begin{aligned} \gamma_{\overline{nsl}}(G) &\leq |D| = q - (\Delta(\overline{L(G)}) + 1) + 2 \\ &= q - \Delta(\overline{L(G)}) + 1. \end{aligned}$$

□

4. Conclusion

In this paper, we found the non-split complement line domination number for the standard graphs Cycle, Path, Complete bipartite graph, Wheel graph, Banana graph, Crown graph, Comb tree, Helm graph, K_n^+ graph, $K_{m,n}^+$ graph, Book graph, Friendship graph, Triangular snake graph, Dragon graph and Quadrilateral snake graph. Also we studied the relationship with other domination parameters.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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